sectionproperties Documentation

Release 2.1.5

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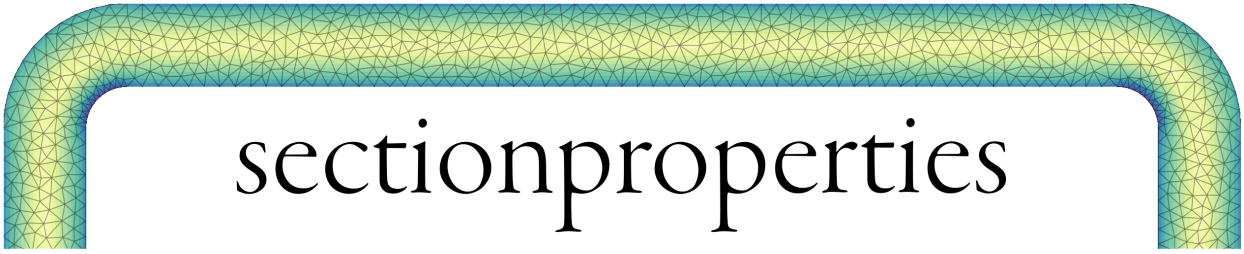

sectionproperties is a python package for the analysis of arbitrary cross-sections using the finite element method written by Robbie van Leeuwen. *sectionproperties* can be used to determine section properties to be used in structural design and visualise cross-sectional stresses resulting from combinations of applied forces and bending moments.

A list of the current features of the package and implementation goals for future releases can be found in the README file on github.

CHAPTER

ONE

INSTALLATION

These instructions will get you a copy of *sectionproperties* up and running on your local machine. You will need a working copy of python 3.8, 3.9 or 3.10 on your machine.

1.1 Installing *sectionproperties*

sectionproperties uses shapely to prepare the cross-section geometry and triangle to efficiently generate a conforming triangular mesh in order to perform a finite element analysis of the structural cross-section.

sectionproperties and all of its dependencies can be installed through the python package index:

```
pip install sectionproperties
```

Note that dependencies required for importing from rhino files are not included by default. To obtain these dependencies, install using the *rhino* option:

```
pip install sectionproperties[rhino]
```

1.2 Testing the Installation

Python *pytest* modules are located in the *sectionproperties.tests* package. To see if your installation is working correctly, install *pytest* and run the following test:

```
pytest --pyargs sectionproperties
```

CHAPTER

TWO

STRUCTURE OF AN ANALYSIS

The process of performing a cross-section analysis with `sectionproperties` can be broken down into three stages:

1. Pre-Processor: The input geometry and finite element mesh is created.
2. Solver: The cross-section properties are determined.
3. Post-Processor: The results are presented in a number of different formats.

2.1 Creating a Geometry and Mesh

The dimensions and shape of the cross-section to be analysed define the *geometry* of the cross-section. The *Section Library* provides a number of functions to easily generate either commonly used structural sections. Alternatively, arbitrary cross-sections can be built from a list of user-defined points, see *Geometry from points, facets, holes, and control points*.

The final stage in the pre-processor involves generating a finite element mesh of the *geometry* that the solver can use to calculate the cross-section properties. This can easily be performed using the *`create_mesh()`* method that all *`Geometry`* objects have access to.

The following example creates a geometry object with a circular cross-section. The diameter of the circle is 50 and 64 points are used to discretise the circumference of the circle. A finite element mesh is generated with a maximum triangular area of 2.5:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections

geometry = primitive_sections.circular_section(d=50, n=64)
geometry.create_mesh(mesh_sizes=[2.5])
```

If you are analysing a composite section, or would like to include material properties in your model, material properties can be created using the *`Material`* class. The following example creates a steel material object:

```
from sectionproperties.pre.pre import Material

steel = Material(name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, density=7.85e-
↪6,
                 yield_strength=500, color='grey')
```

Refer to *Creating Geometries, Meshes, and Material Properties* for a more detailed explanation of the pre-processing stage.

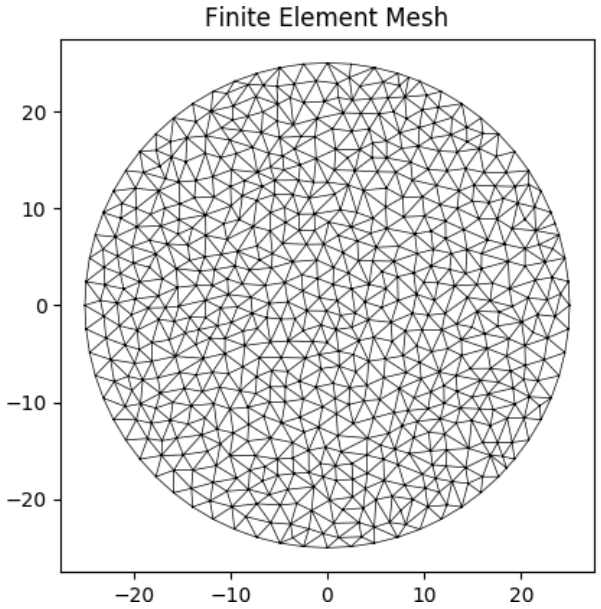

Fig. 1: Finite element mesh generated by the above example.

2.2 Running an Analysis

The solver operates on a *Section* object and can perform four different analysis types:

- Geometric Analysis: calculates area properties.
- Plastic Analysis: calculates plastic properties.
- Warping Analysis: calculates torsion and shear properties.
- Stress Analysis: calculates cross-section stresses.

The geometric analysis can be performed individually. However in order to perform a warping or plastic analysis, a geometric analysis must first be performed. Further, in order to carry out a stress analysis, both a geometric and warping analysis must have already been executed. The program will display a helpful error if you try to run any of these analyses without first performing the prerequisite analyses.

The following example performs a geometric and warping analysis on the circular cross-section defined in the previous section with steel used as the material property:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.analysis.section import Section
from sectionproperties.pre.pre import Material


steel = Material(name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, density=7.85e-
↪6,
                 yield_strength=500, color='grey')
geometry = primitive_sections.circular_section(d=50, n=64, material=steel)
geometry.create_mesh(mesh_sizes=[2.5])  # Adds the mesh to the geometry

section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
```

Refer to *Running an Analysis* for a more detailed explanation of the solver stage.

2.3 Viewing the Results

Once an analysis has been performed, a number of methods belonging to the *Section* object can be called to present the cross-section results in a number of different formats. For example the cross-section properties can be printed to the terminal, a plot of the centroids displayed and the cross-section stresses visualised in a contour plot.

The following example analyses a 200 PFC section. The cross-section properties are printed to the terminal and a plot of the centroids is displayed:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12, n_r=8)
geometry.create_mesh(mesh_sizes=[2.5])  # Adds the mesh to the geometry

section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
section.calculate_warping_properties()

section.plot_centroids()
section.display_results()
```

Output generated by the *display_results()* method:

```
Section Properties:
A       = 2.919699e+03
Perim.  = 6.776201e+02
Qx      = 2.919699e+05
Qy      = 7.122414e+04
cx      = 2.439434e+01
cy      = 1.000000e+02
Ixx_g   = 4.831277e+07
Iyy_g   = 3.392871e+06
Ixy_g   = 7.122414e+06
Ixx_c   = 1.911578e+07
Iyy_c   = 1.655405e+06
Ixy_c   = -6.519258e-09
Zxx+    = 1.911578e+05
Zxx-    = 1.911578e+05
Zyy+    = 3.271186e+04
Zyy-    = 6.786020e+04
rx      = 8.091461e+01
ry      = 2.381130e+01
phi     = 0.000000e+00
I11_c   = 1.911578e+07
I22_c   = 1.655405e+06
Z11+    = 1.911578e+05
Z11-    = 1.911578e+05
Z22+    = 3.271186e+04
```

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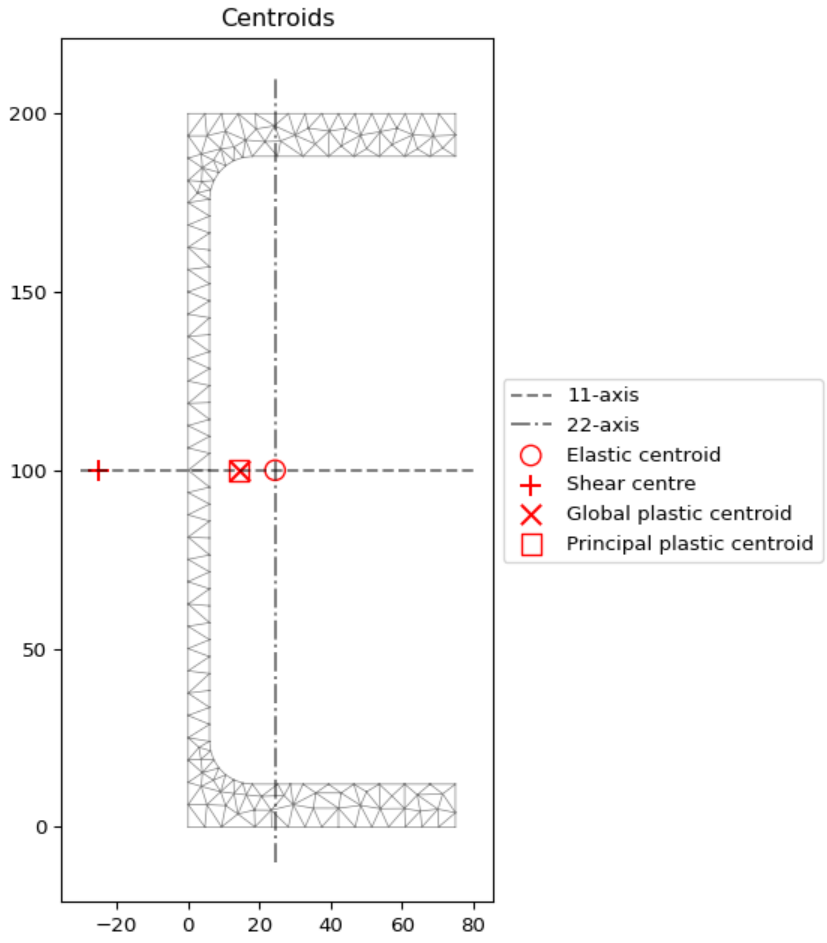

Fig. 2: Plot of the elastic centroid and shear centre for the above example generated by *plot_centroids()*

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```
Z22-    = 6.786020e+04
r11     = 8.091461e+01
r22     = 2.381130e+01
J       = 1.011522e+05
Iw      = 1.039437e+10
x_se    = -2.505109e+01
y_se    = 1.000000e+02
x_st    = -2.505109e+01
y_st    = 1.000000e+02
x1_se   = -4.944543e+01
y2_se   = 4.905074e-06
A_sx    = 9.468851e+02
A_sy    = 1.106943e+03
A_s11   = 9.468854e+02
A_s22   = 1.106943e+03
betax+  = 1.671593e-05
betax-  = -1.671593e-05
betay+  = -2.013448e+02
betay-  = 2.013448e+02
beta11+ = 1.671593e-05
beta11- = -1.671593e-05
beta22+ = -2.013448e+02
beta22- = 2.013448e+02
x_pc    = 1.425046e+01
y_pc    = 1.000000e+02
Sxx     = 2.210956e+05
Syy     = 5.895923e+04
SF_xx+  = 1.156613e+00
SF_xx-  = 1.156613e+00
SF_yy+  = 1.802381e+00
SF_yy-  = 8.688337e-01
x11_pc  = 1.425046e+01
y22_pc  = 1.000000e+02
S11     = 2.210956e+05
S22     = 5.895923e+04
SF_11+  = 1.156613e+00
SF_11-  = 1.156613e+00
SF_22+  = 1.802381e+00
SF_22-  = 8.688337e-01
```

Refer to *Viewing the Results* for a more detailed explanation of the post-processing stage.

CHAPTER

THREE

CREATING GEOMETRIES, MESHES, AND MATERIAL PROPERTIES

Before performing a cross-section analysis, the geometry of the cross-section and a finite element mesh must be created. Optionally, material properties can be applied to different regions of the cross-section. If materials are not applied, then a default material is used to report the geometric properties.

3.1 Section Geometry

New in `v2.0.0` - There are two types of geometry objects in sectionproperties:

- The *Geometry* class, for section geometries with a single, contiguous region
- The *CompoundGeometry* class, comprised of two or more *Geometry* objects

3.1.1 Geometry Class

class sectionproperties.pre.geometry.**Geometry**(*geom: Polygon*, *material:* pre.Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *control_points: Optional[Union[Point, List[float, float]]] = None*, *tol=12*)

Class for defining the geometry of a contiguous section of a single material.

Provides an interface for the user to specify the geometry defining a section. A method is provided for generating a triangular mesh, transforming the section (e.g. translation, rotation, perimeter offset, mirroring), aligning the geometry to another geometry, and designating stress recovery points.

Variables

- **geom** (`shapely.geometry.Polygon`) – a Polygon object that defines the geometry
- **material** (Optional[`Material`]) – Optional, a Material to associate with this geometry
- **control_point** – Optional, an *(x, y)* coordinate within the geometry that represents a pre-assigned control point (aka, a region identification point) to be used instead of the automatically assigned control point generated with `shapely.geometry.Polygon.representative_point()`.
- **tol** – Optional, default is 12. Number of decimal places to round the geometry vertices to. A lower value may reduce accuracy of geometry but increases precision when aligning geometries to each other.

A *Geometry* is instantiated with two arguments:

1. A `shapely.geometry.Polygon` object

2. An optional *Material* object

Note: If a *Material* is not given, then the default material is assigned to the `Geometry.material` attribute. The default material has an elastic modulus of 1, a Poisson's ratio of 0 and a yield strength of 1.

3.1.2 CompoundGeometry Class

class sectionproperties.pre.geometry.**CompoundGeometry**(*geoms: Union[MultiPolygon, List[*Geometry*]]*)

Class for defining a geometry of multiple distinct regions, each potentially having different material properties.

CompoundGeometry instances are composed of multiple Geometry objects. As with Geometry objects, CompoundGeometry objects have methods for generating a triangular mesh over all geometries, transforming the collection of geometries as though they were one (e.g. translation, rotation, and mirroring), and aligning the CompoundGeometry to another Geometry (or to another CompoundGeometry).

CompoundGeometry objects can be created directly between two or more Geometry objects by using the + operator.

Variables

geoms (Union[`shapely.geometry.MultiPolygon`, List[*Geometry*]]) – either a list of Geometry objects or a `shapely.geometry.MultiPolygon` instance.

A *CompoundGeometry* is instantiated with **one argument** which can be one of **two types**:

1. A list of *Geometry* objects; or
2. A `shapely.geometry.MultiPolygon`

Note: A *CompoundGeometry* does not have a `.material` attribute and therefore, a *Material* cannot be assigned to a *CompoundGeometry* directly. Since a *CompoundGeometry* is simply a combination of *Geometry* objects, the Material(s) should be assigned to the individual *Geometry* objects that comprise the *CompoundGeometry*. *CompoundGeometry* objects created from a `shapely.geometry.MultiPolygon` will have its constituent *Geometry* objects assigned the default material.

3.1.3 Defining Material Properties

Materials are defined in *sectionproperties* by creating a *Material* object:

class sectionproperties.pre.pre.**Material**(*name: str*, *elastic_modulus: float*, *poissons_ratio: float*, *yield_strength: float*, *density: float*, *color: str*)

Class for structural materials.

Provides a way of storing material properties related to a specific material. The color can be a multitude of different formats, refer to https://matplotlib.org/api/colors_api.html and https://matplotlib.org/examples/color/named_colors.html for more information.

Parameters

- **name** (*string*) – Material name
- **elastic_modulus** (*float*) – Material modulus of elasticity
- **poissons_ratio** (*float*) – Material Poisson's ratio

- **yield_strength** (*float*) – Material yield strength
- **density** (*float*) – Material density (mass per unit volume)
- **color** (matplotlib.colors) – Material color for rendering

Variables

- **name** (*string*) – Material name
- **elastic_modulus** (*float*) – Material modulus of elasticity
- **poissons_ratio** (*float*) – Material Poisson's ratio
- **shear_modulus** (*float*) – Material shear modulus, derived from the elastic modulus and Poisson's ratio assuming an isotropic material
- **density** (*float*) – Material density (mass per unit volume)
- **yield_strength** (*float*) – Material yield strength
- **color** (matplotlib.colors) – Material color for rendering

The following example creates materials for concrete, steel and timber:

```
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, density=2.4e-6,
        yield_strength=32, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, density=7.85e-6,
        yield_strength=500, color='grey'
)
timber = Material(
    name='Timber', elastic_modulus=8e3, poissons_ratio=0.35, density=6.5e-7,
        yield_strength=20, color='burlywood'
)
```

Each *Geometry* contains its own material definition, which is stored in the `.material` attribute. A geometry's material may be altered at any time by simply assigning a new *Material* to the `.material` attribute.

Warning: To understand how material properties affect the results reported by *sectionproperties*, see *How Material Properties Affect Results*.

3.2 Creating Section Geometries

In addition to creating geometries directly from `shapely.geometry.Polygon` and/or `shapely.geometry.MultiPolygon` objects, there are other ways to create geometries for analysis:

1. From lists of points, facets, hole regions, and control regions
2. From `.dxf` files
3. From `.3dm` files
4. From Rhino encodings

5. Using transformation methods on existing geometries and/or by applying set operations (e.g. |, +, -, &, ^)
6. From *sectionproperties*'s section library

For the first two approaches, an optional `.material` parameter can be passed containing a *Material* (or list of *Material* objects) to associate with the newly created geometry(ies). The material attribute can be altered afterward in a *Geometry* object at any time by simply assigning a different *Material* to the `.material` attribute.

3.2.1 Geometry from points, facets, holes, and control points

In sectionproperties `v1.x.x`, geometries were created by specifying lists of *points*, *facets*, *holes*, and *control_points*. This functionality has been preserved in `v2.0.0` by using the *from_points()* class method.

`sectionproperties.pre.geometry.Geometry.`**`from_points`**(*points: List[List[float]]*, *facets: List[List[int]]*, *control_points: List[List[float]]*, *holes: Optional[List[List[float]]] = None*, *material: Optional[*Material*] = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*)

An interface for the creation of Geometry objects through the definition of points, facets, and holes.

Variables

- **`points`** (*`list[list[float, float]]`*) – List of points *(x, y)* defining the vertices of the section geometry. If facets are not provided, it is a assumed the that the list of points are ordered around the perimeter, either clockwise or anti-clockwise.
- **`facets`** (*`list[list[int, int]]`*) – A list of *(start, end)* indexes of vertices defining the edges of the section geoemtry. Can be used to define both external and internal perimeters of holes. Facets are assumed to be described in the order of exterior perimeter, interior perimeter 1, interior perimeter 2, etc.
- **`control_points`** – An *(x, y)* coordinate that describes the distinct, contiguous, region of a single material within the geometry. Must be entered as a list of coordinates, e.g. [[0.5, 3.2]] Exactly one point is required for each geometry with a distinct material. If there are multiple distinct regions, then use CompoundGeometry.from_points()
- **`holes`** (*`list[list[float, float]]`*) – Optional. A list of points *(x, y)* that define interior regions as being holes or voids. The point can be located anywhere within the hole region. Only one point is required per hole region.
- **`material`** – Optional. A *Material* object that is to be assigned. If not given, then the `DEFAULT_MATERIAL` will be used.

For simple geometries (i.e. single-region shapes without holes), if the points are an ordered sequence of coordinates, only the *points* argument is required (*facets*, *holes*, and *control_points* are optional). If the geometry has holes, then all arguments are required.

If the geometry has multiple regions, then the `from_points()` class method must be used.

`sectionproperties.pre.geometry.CompoundGeometry.`**`from_points`**(*points: List[List[float]]*, *facets: List[List[int]]*, *control_points: List[List[float]]*, *holes: Optional[List[List[float]]] = None*, *materials: Optional[List[*Material*]] = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*)

An interface for the creation of CompoundGeometry objects through the definition of points, facets, holes, and control_points. Geometries created through this method are expected to be non-ambiguous meaning that no "overlapping" geometries exists and that nodal connectivity is maintained (e.g. there are no nodes "overlapping" with facets without nodal connectivity).

Variables

- **points** (*list[list[float]]*) – List of points *(x, y)* defining the vertices of the section geometry. If facets are not provided, it is a assumed the that the list of points are ordered around the perimeter, either clockwise or anti-clockwise
- **facets** (*list[list[int]]*) – A list of *(start, end)* indexes of vertices defining the edges of the section geoemtry. Can be used to define both external and internal perimeters of holes. Facets are assumed to be described in the order of exterior perimeter, interior perimeter 1, interior perimeter 2, etc.
- **control_points** (*list[list[float]]*) – Optional. A list of points *(x, y)* that define non-interior regions as being distinct, contiguous, and having one material. The point can be located anywhere within region. Only one point is permitted per region. The order of control_points must be given in the same order as the order that polygons are created by 'facets'. If not given, then points will be assigned automatically using shapely.geometry.Polygon.representative_point()
- **holes** (*list[list[float]]*) – Optional. A list of points *(x, y)* that define interior regions as being holes or voids. The point can be located anywhere within the hole region. Only one point is required per hole region.
- **materials** (list[*Material*]) – Optional. A list of *Material* objects that are to be assigned, in order, to the regions defined by the given control_points. If not given, then the `DEFAULT_MATERIAL` will be used for each region.

See *Creating Custom Geometry* for an example of this implementation.

3.2.2 Geometry from .dxf Files

Geometries can now be created from `.dxf` files using the *sectionproperties.pre.geometry.Geometry.from_dxf()* method. The returned geometry will either be a *Geometry* or *CompoundGeometry* object depending on the geometry in the file (i.e. the number of contiguous regions).

sectionproperties.pre.geometry.Geometry.**from_dxf**(*dxf_filepath: Union[str, Path]*) → Union[*Geometry*, *CompoundGeometry*]

An interface for the creation of Geometry objects from CAD .dxf files.

Variables

dxf_filepath (*Union[str, pathlib.Path]*) – A path-like object for the dxf file

sectionproperties.pre.geometry.CompoundGeometry.**from_dxf**(*dxf_filepath: Union[str, Path]*) → Union[*Geometry*, *CompoundGeometry*]

An interface for the creation of Geometry objects from CAD .dxf files.

Variables

dxf_filepath (*Union[str, pathlib.Path]*) – A path-like object for the dxf file

3.2.3 Geometry from Rhino

Geometries can now be created from .3dm files and BREP encodings. Various limitations and assumptions need to be acknowledged:

- sectional analysis is based in 2d and Rhino is a 3d environment.
- the recognised Rhino geometries are limited to planer-single-surfaced BREPs.
- Rhino uses NURBS for surface boundaries and *sectionproperties* uses piecewise linear boundaries.
- a search plane is defined.

See the keyword arguments below that are used to search and simplify the Rhino geometry.

Rhino files are read via the class methods *sectionproperties.pre.geometry.Geometry.from_3dm()* and `sectionproperties.pre.geometry.CompoundGeometry.from_3dm()`. Each class method returns the respective objects.

`sectionproperties.pre.geometry.Geometry.`**from_3dm**(*filepath: Union[str, Path]*, ***kwargs*) → *Geometry*

Class method to create a *Geometry* from the objects in a Rhino *.3dm* file.

Parameters

- **filepath** (*Union[str, pathlib.Path]*) – File path to the rhino *.3dm* file.
- **kwargs** – See below.

Raises

RuntimeError – A RuntimeError is raised if two or more polygons are found. This is dependent on the keyword arguments. Try adjusting the keyword arguments if this error is raised.

Returns

A Geometry object.

Return type

Geometry

Keyword Arguments

- ***refine_num*** (**int, optional**) –
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.
- ***vec1*** (**numpy.ndarray, optional**) –
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** (**numpy.ndarray, optional**) –
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** (**float, optional**) –
  The distance to the Shapely plane. Default is 0.
- ***project*** (**boolean, optional**) –
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.

- ***parallel*** **(boolean, optional) –**
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

sectionproperties.pre.geometry.CompoundGeometry.**from_3dm**(*filepath: Union[str, Path]*, ***kwargs*) → *CompoundGeometry*

Class method to create a *CompoundGeometry* from the objects in a Rhino *3dm* file.

Parameters

- **filepath** (*Union[str, pathlib.Path]*) – File path to the rhino *.3dm* file.
- **kwargs** – See below.

Returns
A *CompoundGeometry* object.

Return type
CompoundGeometry

Keyword Arguments

- ***refine_num*** **(int, optional) –**
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.
- ***vec1*** **(numpy.ndarray, optional) –**
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** **(numpy.ndarray, optional) –**
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** **(float, optional) –**
  The distance to the Shapely plane. Default is 0.
- ***project*** **(boolean, optional) –**
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.
- ***parallel*** **(boolean, optional) –**
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

Geometry objects can also be created from encodings of Rhino BREP.

sectionproperties.pre.geometry.Geometry.**from_rhino_encoding**(*r3dm_brep: str*, ***kwargs*) → *Geometry*

Load an encoded single surface planer brep.

Parameters

- **r3dm_brep** (*str*) – A Rhino3dm.Brep encoded as a string.
- **kwargs** – See below.

Returns
A Geometry object found in the encoded string.

Return type
Geometry

Keyword Arguments

- ***refine_num*** **(int, optional) –**
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.
- ***vec1*** **(numpy.ndarray, optional) –**
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** **(numpy.ndarray, optional) –**
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** **(float, optional) –**
  The distance to the Shapely plane. Default is 0.
- ***project*** **(boolean, optional) –**
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.
- ***parallel*** **(boolean, optional) –**
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

More advanced filtering can be achieved by working with the Shapely geometries directly. These can be accessed by *load_3dm()* and `load_rhino_brep_encoding()`.

`sectionproperties.pre.rhino.`**load_3dm**(*r3dm_filepath: Union[Path, str]*, ***kwargs*) → List[Polygon]

Load a Rhino *.3dm* file and import the single surface planer breps.

Parameters

- **r3dm_filepath** (*pathlib.Path or string*) – File path to the rhino *.3dm* file.
- **kwargs** – See below.

Raises
RuntimeError – A RuntimeError is raised if no polygons are found in the file. This is dependent on the keyword arguments. Try adjusting the keyword arguments if this error is raised.

Returns
List of Polygons found in the file.

Return type
List[shapely.geometry.Polygon]

Keyword Arguments

- ***refine_num*** **(int, optional) –**
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using

straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.

- ***vec1*** **(numpy.ndarray, optional) –**
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** **(numpy.ndarray, optional) –**
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** **(float, optional) –**
  The distance to the Shapely plane. Default is 0.
- ***project*** **(boolean, optional) –**
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.
- ***parallel*** **(boolean, optional) –**
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

sectionproperties.pre.rhino.**load_brep_encoding**(*brep: str*, ***kwargs*) → Polygon

Load an encoded single surface planer brep.

Parameters

- **brep** (*str*) – Rhino3dm.Brep encoded as a string.
- **kwargs** – See below.

Raises

RuntimeError – A RuntimeError is raised if no polygons are found in the encoding. This is dependent on the keyword arguments. Try adjusting the keyword arguments if this error is raised.

Returns

The Polygons found in the encoding string.

Return type

shapely.geometry.Polygon

Keyword Arguments

- ***refine_num*** **(int, optional) –**
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.
- ***vec1*** **(numpy.ndarray, optional) –**
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** **(numpy.ndarray, optional) –**
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].

- ***plane_distance*** (**float, optional**) –
  The distance to the Shapely plane. Default is 0.
- ***project*** (**boolean, optional**) –
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.
- ***parallel*** (**boolean, optional**) –
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

Note: Dependencies for importing files from rhino are not included by default. To obtain the required dependencies install *sectionproperties* with the rhino option:

```
pip install sectionproperties[rhino]
```

3.2.4 Combining Geometries Using Set Operations

Both *Geometry* and *CompoundGeometry* objects can be manipulated using Python's set operators:

- | Bitwise OR - Performs a union on the two geometries
- \- Bitwise DIFFERENCE - Performs a subtraction, subtracting the second geometry from the first
- & Bitwise AND - Performs an intersection operation, returning the regions of geometry common to both
- ^ Bitwise XOR - Performs a symmetric difference operation, returning the regions of geometry that are not overlapping
- \+ Addition - Combines two geometries into a *CompoundGeometry*

See *Advanced Geometry Creation* for an example using set operations.

3.2.5 *sectionproperties* Section Library

See *Creating Section Geometries from the Section Library*.

3.3 Manipulating Geometries

Each geometry instance is able to be manipulated in 2D space for the purpose of creating novel, custom section geometries that the user may require.

Note: Operations on geometries are **non-destructive**. For each operation, a new geometry object is returned.

This gives *sectionproperties* geoemtries a *fluent API* meaning that transformation methods can be chained together. Please see *Advanced Geometry Creation* for examples.

3.3.1 Aligning

sectionproperties.pre.geometry.Geometry.**align_center**(*self*, *align_to: Optional[Union[Geometry, Tuple[float, float]]] = None*)

Returns a new Geometry object, translated in both x and y, so that the the new object's centroid will be aligned with the centroid of the object in 'align_to'. If 'align_to' is an x, y coordinate, then the centroid will be aligned to the coordinate. If 'align_to' is None then the new object will be aligned with its centroid at the origin.

Parameters
align_to (Optional[Union[*Geometry*, Tuple[float, float]]]) – Another Geometry to align to or None (default is None)

Returns
Geometry object translated to new alignment

Return type
Geometry

sectionproperties.pre.geometry.Geometry.**align_to**(*self*, *other: Union[Geometry, Tuple[float, float]]*, *on: str*, *inner: bool = False*) → *Geometry*

Returns a new Geometry object, representing 'self' translated so that is aligned 'on' one of the outer bounding box edges of 'other'.

If 'other' is a tuple representing an *(x,y)* coordinate, then the new Geometry object will represent 'self' translated so that it is aligned 'on' that side of the point.

Parameters

- **other** (Union[*Geometry*, Tuple[float, float]]) – Either another Geometry or a tuple representing an *(x,y)* coordinate point that 'self' should align to.
- **on** – A str of either "left", "right", "bottom", or "top" indicating which side of 'other' that self should be aligned to.
- **inner** (*bool*) – Default False. If True, align 'self' to 'other' in such a way that 'self' is aligned to the "inside" of 'other'. In other words, align 'self' to 'other' on the specified edge so they overlap.

Returns
Geometry object translated to alignment location

Return type
Geometry

3.3.2 Mirroring

sectionproperties.pre.geometry.Geometry.**mirror_section**(*self*, *axis: str = 'x'*, *mirror_point: Union[List[float], str] = 'center'*)

Mirrors the geometry about a point on either the x or y-axis.

Parameters

- **axis** (*string*) – Axis about which to mirror the geometry, *'x'* or *'y'*

- **mirror_point** (*Union[list[float, float], str]*) – Point about which to mirror the geometry *(x, y)*. If no point is provided, mirrors the geometry about the centroid of the shape's bounding box. Default = 'center'.

Returns
New Geometry-object mirrored on 'axis' about 'mirror_point'

Return type
Geometry

The following example mirrors a 200PFC section about the y-axis and the point (0, 0):

```
import sectionproperties.pre.library.steel_sections as steel_sections

geometry = steel_sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12,
↪ n_r=8)
new_geometry = geometry.mirror_section(axis='y', mirror_point=[0, 0])
```

3.3.3 Rotating

sectionproperties.pre.geometry.Geometry.**rotate_section**(*self*, *angle: float*, *rot_point: Union[List[float], str] = 'center'*, *use_radians: bool = False*)

Rotates the geometry and specified angle about a point. If the rotation point is not provided, rotates the section about the center of the geometry's bounding box.

Parameters

- **angle** (*float*) – Angle (degrees by default) by which to rotate the section. A positive angle leads to a counter-clockwise rotation.
- **rot_point** (*list[float, float]*) – Optional. Point *(x, y)* about which to rotate the section. If not provided, will rotate about the center of the geometry's bounding box. Default = 'center'.
- **use_radians** – Boolean to indicate whether 'angle' is in degrees or radians. If True, 'angle' is interpreted as radians.

Returns
New Geometry-object rotated by 'angle' about 'rot_point'

Return type
Geometry

The following example rotates a 200UB25 section clockwise by 30 degrees:

```
import sectionproperties.pre.library.steel_sections as steel_sections

geometry = steel_sections.i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9,␣
↪n_r=8)
new_geometry = geometry.rotate_section(angle=-30)
```

3.3.4 Shifting

sectionproperties.pre.geometry.Geometry.**shift_points**(*self*, *point_idxs: Union[int, List[int]]*, *dx: float = 0*, *dy: float = 0*, *abs_x: Optional[float] = None*, *abs_y: Optional[float] = None*) → *Geometry*

Translates one (or many points) in the geometry by either a relative amount or to a new absolute location. Returns a new Geometry representing the original with the selected point(s) shifted to the new location.

Points are identified by their index, their relative location within the points list found in `self.points`. You can call `self.plot_geometry(labels="points")` to see a plot with the points labeled to find the appropriate point indexes.

Parameters

- **point_idxs** (*Union[int, List[int]]*) – An integer representing an index location or a list of integer index locations.
- **dx** (*float*) – The number of units in the x-direction to shift the point(s) by
- **dy** (*float*) – The number of units in the y-direction to shift the point(s) by
- **abs_x** (*Optional[float]*) – Absolute x-coordinate in coordinate system to shift the point(s) to. If abs_x is provided, dx is ignored. If providing a list to point_idxs, all points will be moved to this absolute location.
- **abs_y** (*Optional[float]*) – Absolute y-coordinate in coordinate system to shift the point(s) to. If abs_y is provided, dy is ignored. If providing a list to point_idxs, all points will be moved to this absolute location.

Returns

Geometry object with selected points translated to the new location.

Return type

Geometry

The following example expands the sides of a rectangle, one point at a time, to make it a square:

```
import sectionproperties.pre.library.primitive_sections as primitive_
↪sections

geometry = primitive_sections.rectangular_section(d=200, b=150)

# Using relative shifting
one_pt_shifted_geom = geometry.shift_points(point_idxs=1, dx=50)

# Using absolute relocation
both_pts_shift_geom = one_pt_shift_geom.shift_points(point_idxs=2, abs_
↪x=200)
```

sectionproperties.pre.geometry.Geometry.**shift_section**(*self*, *x_offset=0.0*, *y_offset=0.0*)

Returns a new Geometry object translated by 'x_offset' and 'y_offset'.

Parameters

- **x_offset** (*float*) – Distance in x-direction by which to shift the geometry.
- **y_offset** (*float*) – Distance in y-direction by which to shift the geometry.

Returns
New Geometry-object shifted by 'x_offset' and 'y_offset'

Return type
Geometry

3.3.5 Splitting

sectionproperties.pre.geometry.Geometry.**split_section**(*self*, *point_i: Tuple[float, float]*, *point_j: Optional[Tuple[float, float]] = None*, *vector: Union[Tuple[float, float], None, ndarray] = None*) → Tuple[List[*Geometry*], List[*Geometry*]]

Splits, or bisects, the geometry about a line, as defined by two points on the line or by one point on the line and a vector. Either `point_j` or `vector` must be given. If `point_j` is given, `vector` is ignored.

Returns a tuple of two lists each containing new Geometry instances representing the "top" and "bottom" portions, respectively, of the bisected geometry.

If the line is a vertical line then the "right" and "left" portions, respectively, are returned.

Parameters

- **point_i** (*Tuple[float, float]*) – A tuple of *(x, y)* coordinates to define a first point on the line
- **point_j** (*Tuple[float, float]*) – Optional. A tuple of *(x, y)* coordinates to define a second point on the line
- **vector** (*Union[Tuple[float, float], numpy.ndarray]*) – Optional. A tuple or numpy ndarray of *(x, y)* components to define the line direction.

Returns
A tuple of lists containing Geometry objects that are bisected about the line defined by the two given points. The first item in the tuple represents the geometries on the "top" of the line (or to the "right" of the line, if vertical) and the second item represents the geometries to the "bottom" of the line (or to the "left" of the line, if vertical).

Return type
Tuple[List[*Geometry*], List[*Geometry*]]

The following example splits a 200PFC section about the y-axis:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from shapely import LineString

geometry = steel_sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12,
↪ n_r=8)
right_geom, left_geom = geometry.split_section((0, 0), (0, 1))
```

3.3.6 Offsetting the Perimeter

sectionproperties.pre.geometry.Geometry.**offset_perimeter**(*self*, *amount: float = 0*, *where: str = 'exterior'*, *resolution: float = 12*)

Dilates or erodes the section perimeter by a discrete amount.

Parameters

- **amount** (*float*) – Distance to offset the section by. A -ve value "erodes" the section. A +ve value "dilates" the section.
- **where** (*str*) – One of either "exterior", "interior", or "all" to specify which edges of the geometry to offset. If geometry has no interiors, then this parameter has no effect. Default is "exterior".
- **resolution** (*float*) – Number of segments used to approximate a quarter circle around a point

Returns
Geometry object translated to new alignment

Return type
Geometry

The following example erodes a 200PFC section by 2 mm:

```
import sectionproperties.pre.library.steel_sections as steel_sections

geometry = sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12, n_
↪r=8)
new_geometry = geometry.offset_perimeter(amount=-2)
```

sectionproperties.pre.geometry.CompoundGeometry.**offset_perimeter**(*self*, *amount: float = 0*, *where='exterior'*, *resolution: float = 12*)

Dilates or erodes the perimeter of a CompoundGeometry object by a discrete amount.

Parameters

- **amount** (*float*) – Distance to offset the section by. A -ve value "erodes" the section. A +ve value "dilates" the section.
- **where** (*str*) – One of either "exterior", "interior", or "all" to specify which edges of the geometry to offset. If geometry has no interiors, then this parameter has no effect. Default is "exterior".
- **resolution** (*float*) – Number of segments used to approximate a quarter circle around a point

Returns
Geometry object translated to new alignment

Return type
Geometry

The following example erodes a 200UB25 with a 12 plate stiffener section by 2 mm:

```
import sectionproperties.pre.library.steel_sections as steel_sections
import sectionproperties.pre.library.primitive_sections as primitive_
↪sections

geometry_1 = steel_sections.i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.
↪9, n_r=8)
geometry_2 = primitive_sections.rectangular_section(d=12, b=133)
compound = geometry_2.align_center(geometry_1).align_to(geometry_1, on="top
↪") + geometry_1
new_geometry = compound.offset_perimeter(amount=-2)
```

Note: If performing a positive offset on a CompoundGeometry with multiple materials, ensure that the materials propagate as desired by performing a .plot_mesh() prior to performing any analysis.

3.4 Visualising the Geometry

Visualisation of geometry objects is best performed in the Jupyter computing environment, however, most visualisation can also be done in any environment which supports display of matplotlib plots.

There are generally two ways to visualise geometry objects:

1. In the Jupyter computing environment, geometry objects utilise their underlying `shapely.geometry.Polygon` object's `_repr_svg_` method to show the geometry as it's own representation.
2. By using the *plot_geometry()* method

`Geometry.plot_geometry`(*labels=['control_points']*, *title='Cross-Section Geometry'*, *cp=True*, *legend=True*, ***kwargs*)

Plots the geometry defined by the input section.

Parameters

- **labels** (*list[str]*) – A list of str which indicate which labels to plot. Can be one or a combination of "points", "facets", "control_points", or an empty list to indicate no labels. Default is ["control_points"]
- **title** (*string*) – Plot title
- **cp** (*bool*) – If set to True, plots the control points
- **legend** (*bool*) – If set to True, plots the legend
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

`matplotlib.axes`

The following example creates a CHS discretised with 64 points, with a diameter of 48 and thickness of 3.2, and plots the geometry:

```
import sectionproperties.pre.library.steel_sections as steel_sections
```

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```
geometry = steel_sections.circular_hollow_section(d=48, t=3.2, n=64)
geometry.plot_geometry()
```

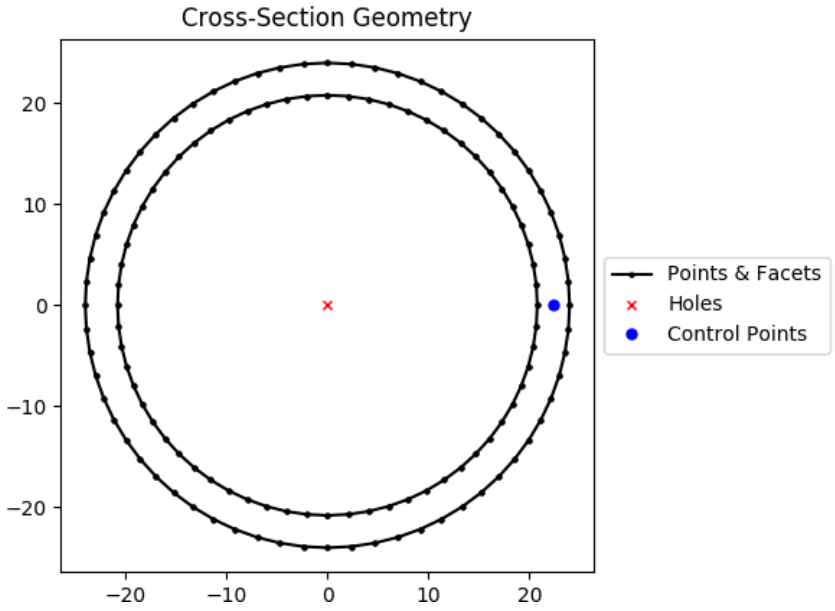

Fig. 1: Geometry generated by the above example.

Note: You can also use `.plot_geometry()` with `CompoundGeometry` objects

3.5 Generating a Mesh

A finite element mesh is required to perform a cross-section analysis. After a geometry has been created, a finite element mesh can then be created for the geometry by using the *create_mesh()* method:

`Geometry.create_mesh`(*mesh_sizes: Union[float, List[float]]*, *coarse: bool = False*)

Creates a quadratic triangular mesh from the Geometry object.

Parameters

- **mesh_sizes** (*Union[float, List[float]]*) – A float describing the maximum mesh element area to be used within the Geometry-object finite-element mesh.
- **coarse** (*bool*) – If set to True, will create a coarse mesh (no area or quality constraints)

Returns

Geometry-object with mesh data stored in .mesh attribute. Returned Geometry-object is self, not a new instance.

Return type

Geometry

The following example creates a circular cross-section with a diameter of 50 with 64 points, and generates a mesh with a maximum triangular area of 2.5:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections

geometry = primitive_sections.circular_section(d=50, n=64)
geometry = geometry.create_mesh(mesh_sizes=2.5)
```

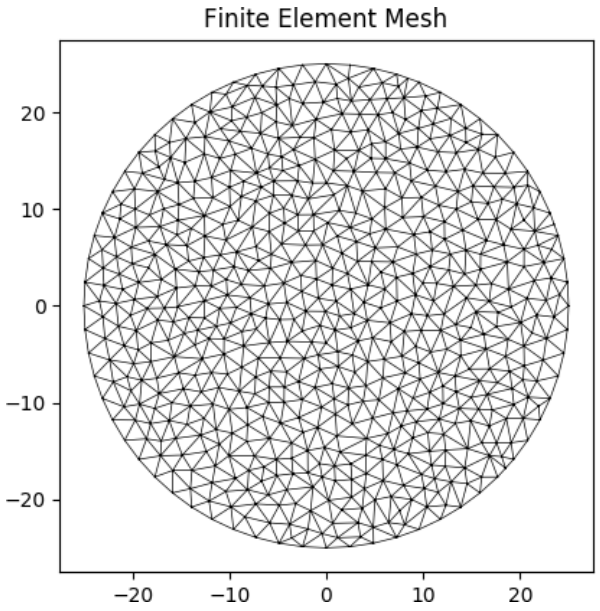

Fig. 2: Mesh generated from the above geometry.

Warning: The length of `mesh_sizes` must match the number of regions in the geometry object.

Once the mesh has been created, it is stored within the geometry object and the geometry object can then be passed to *`Section`* for analysis.

Please see *Running an Analysis* for further information on performing analyses.

CHAPTER

FOUR

CREATING SECTION GEOMETRIES FROM THE SECTION LIBRARY

In order to make your life easier, there are a number of built-in functions that generate typical structural cross-sections, resulting in *Geometry* objects. These typical cross-sections reside in the `sectionproperties.pre.library` module.

4.1 Primitive Sections Library

4.1.1 Rectangular Section

sectionproperties.pre.library.primitive_sections.**rectangular_section**(*b: float*, *d: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a rectangular section with the bottom left corner at the origin *(0, 0)*, with depth *d* and width *b*.

Parameters

- **d** (*float*) – Depth (y) of the rectangle
- **b** (*float*) – Width (x) of the rectangle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a rectangular cross-section with a depth of 100 and width of 50, and generates a mesh with a maximum triangular area of 5:

```
from sectionproperties.pre.library.primitive_sections import rectangular_section

geometry = rectangular_section(d=100, b=50)
geometry.create_mesh(mesh_sizes=[5])
```

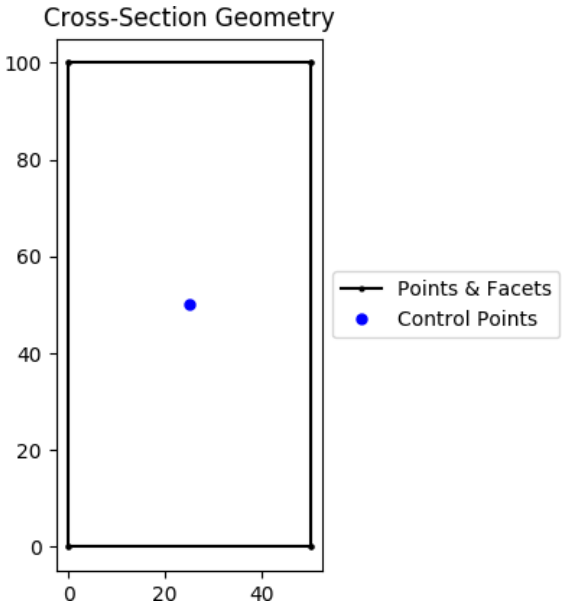

Fig. 1: Rectangular section geometry.

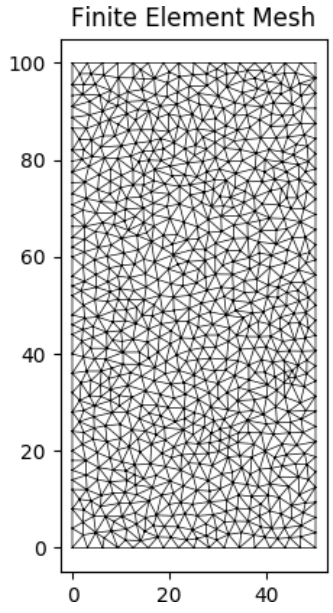

Fig. 2: Mesh generated from the above geometry.

4.1.2 Circular Section

sectionproperties.pre.library.primitive_sections.**circular_section**(*d: float*, *n: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a solid circle centered at the origin *(0, 0)* with diameter *d* and using *n* points to construct the circle.

Parameters

- **d** (*float*) – Diameter of the circle
- **n** (*int*) – Number of points discretising the circle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a circular geometry with a diameter of 50 with 64 points, and generates a mesh with a maximum triangular area of 2.5:

```
from sectionproperties.pre.library.primitive_sections import circular_section

geometry = circular_section(d=50, n=64)
geometry.create_mesh(mesh_sizes=[2.5])
```

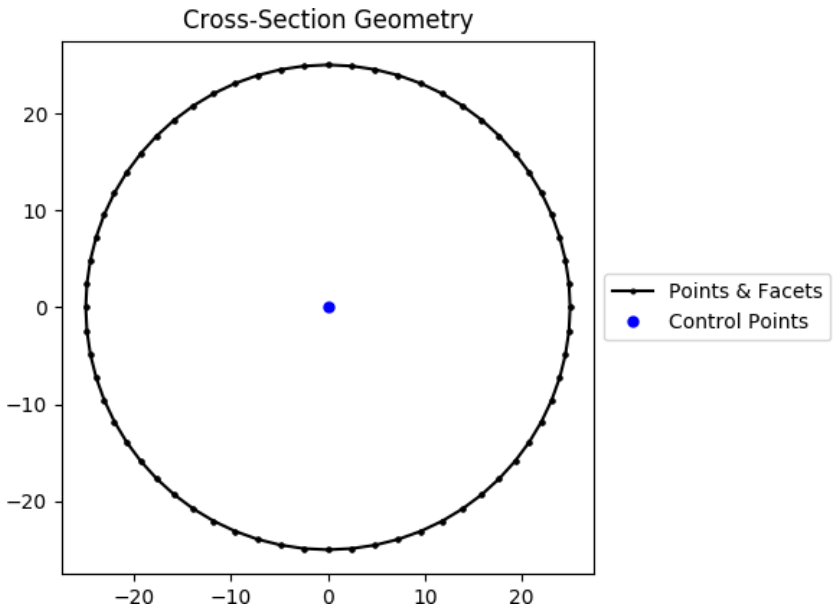

Fig. 3: Circular section geometry.

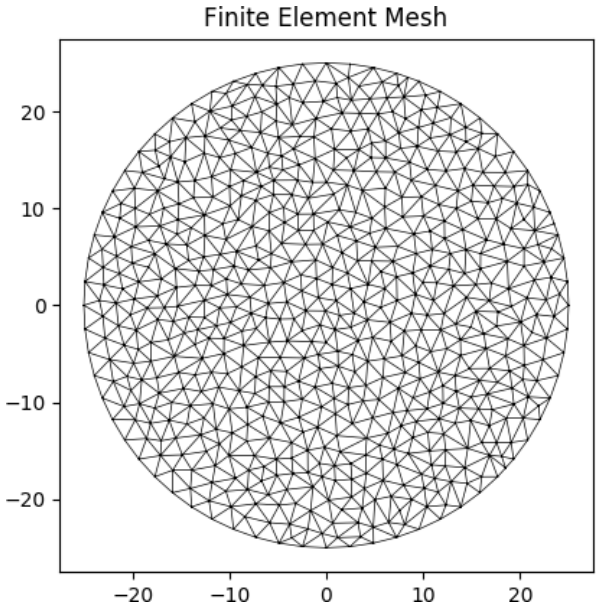

Fig. 4: Mesh generated from the above geometry.

4.1.3 Circular Section By Area

sectionproperties.pre.library.primitive_sections.**circular_section_by_area**(*area: float*, *n: int*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a solid circle centered at the origin *(0, 0)* defined by its *area*, using *n* points to construct the circle.

Parameters

- **area** (*float*) – Area of the circle
- **n** (*int*) – Number of points discretising the circle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a circular geometry with an area of 200 with 32 points, and generates a mesh with a maximum triangular area of 5:

```
from sectionproperties.pre.library.primitive_sections import circular_section_by_
↪area

geometry = circular_section_by_area(area=310, n=32)
geometry.create_mesh(mesh_sizes=[5])
```

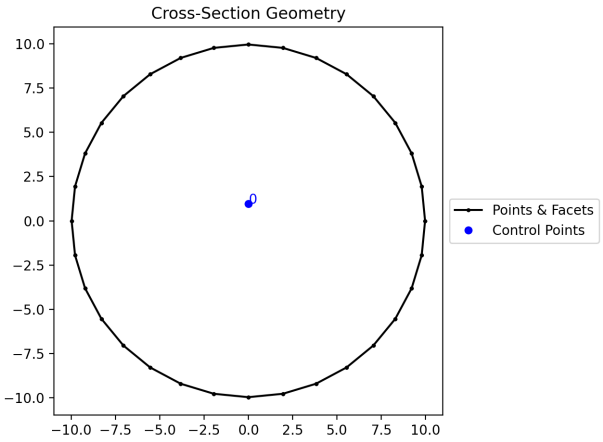

Fig. 5: Circular section by area geometry.

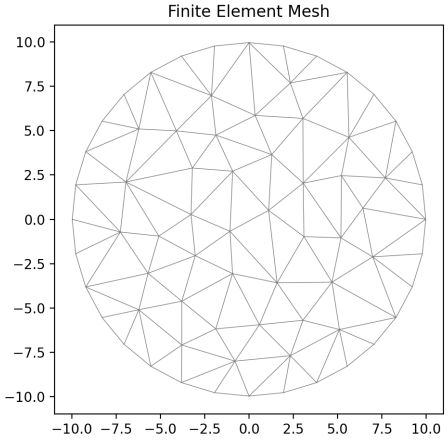

Fig. 6: Mesh generated from the above geometry.

4.1.4 Elliptical Section

sectionproperties.pre.library.primitive_sections.**elliptical_section**(*d_y: float*, *d_x: float*, *n: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a solid ellipse centered at the origin *(0, 0)* with vertical diameter *d_y* and horizontal diameter *d_x*, using *n* points to construct the ellipse.

Parameters

- **d_y** (*float*) – Diameter of the ellipse in the y-dimension
- **d_x** (*float*) – Diameter of the ellipse in the x-dimension
- **n** (*int*) – Number of points discretising the ellipse
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an elliptical cross-section with a vertical diameter of 25 and horizontal diameter of 50, with 40 points, and generates a mesh with a maximum triangular area of 1.0:

```
from sectionproperties.pre.library.primitive_sections import elliptical_section

geometry = elliptical_section(d_y=25, d_x=50, n=40)
geometry.create_mesh(mesh_sizes=[1.0])
```

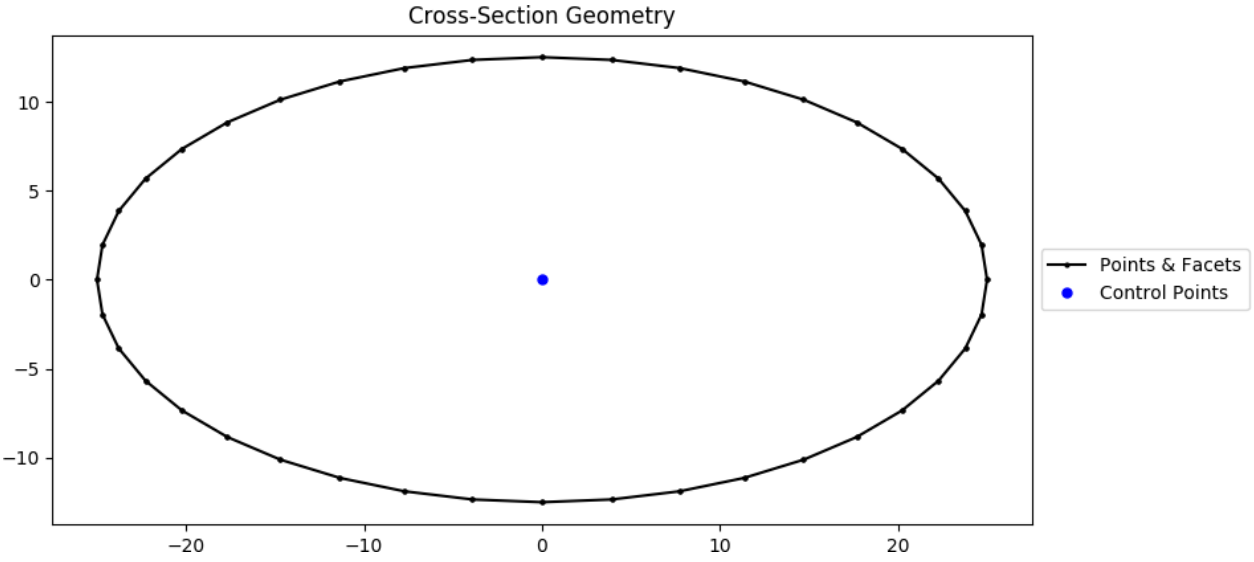

Fig. 7: Elliptical section geometry.

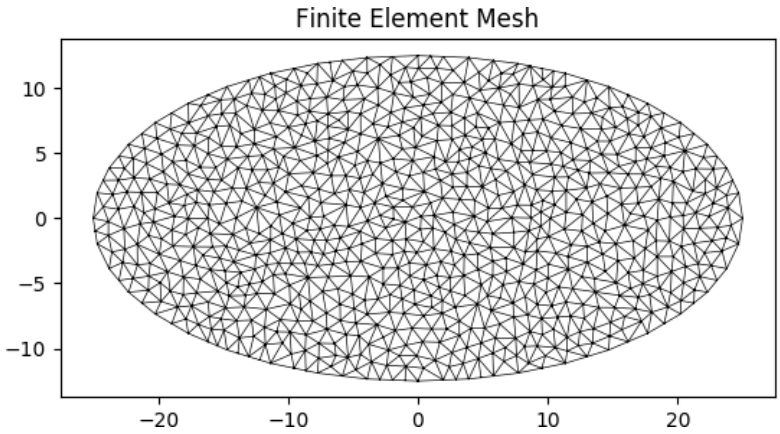

Fig. 8: Mesh generated from the above geometry.

4.1.5 Triangular Section

sectionproperties.pre.library.primitive_sections.**triangular_section**(*b: float*, *h: float*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a right angled triangle with points *(0, 0)*, *(b, 0)*, *(0, h)*.

Parameters

- **b** (*float*) – Base length of triangle
- **h** (*float*) – Height of triangle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a triangular cross-section with a base width of 10 and height of 10, and generates a mesh with a maximum triangular area of 0.5:

```
from sectionproperties.pre.library.primitive_sections import triangular_section

geometry = triangular_section(b=10, h=10)
geometry.create_mesh(mesh_sizes=[0.5])
```

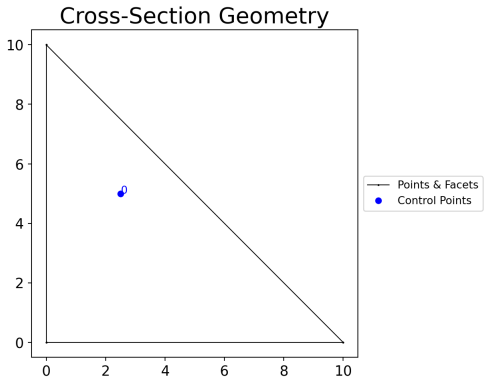

Fig. 9: Triangular section geometry.

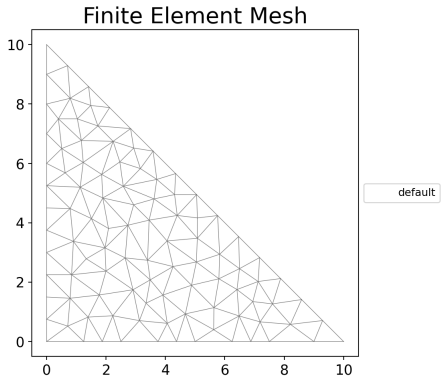

Fig. 10: Mesh generated from the above geometry.

4.1.6 Triangular Radius Section

sectionproperties.pre.library.primitive_sections.**triangular_radius_section**(*b: float*, *n_r: float*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a right angled isosceles triangle with points *(0, 0)*, *(b, 0)*, *(0, h)* and a concave radius on the hypotenuse.

Parameters

- **b** (*float*) – Base length of triangle
- **n_r** (*int*) – Number of points discretising the radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a triangular radius cross-section with a base width of 6, using *n_r* points to construct the radius, and generates a mesh with a maximum triangular area of 0.5:

```
from sectionproperties.pre.library.primitive_sections import triangular_radius_
↪section

geometry = triangular_radius_section(b=6, n_r=16)
geometry.create_mesh(mesh_sizes=[0.5])
```

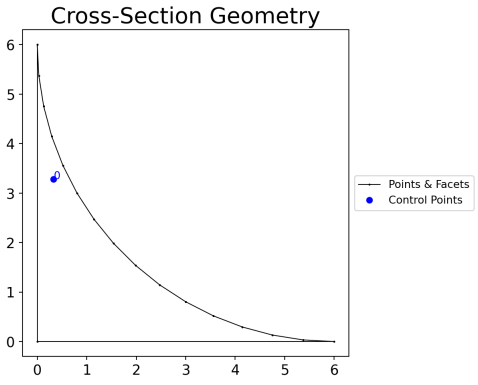

Fig. 11: Triangular radius section geometry.

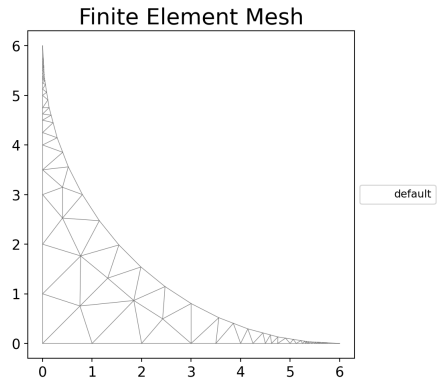

Fig. 12: Mesh generated from the above geometry.

4.1.7 Cruciform Section

sectionproperties.pre.library.primitive_sections.**cruciform_section**(*d: float*, *b: float*, *t: float*, *r: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a cruciform section centered at the origin *(0, 0)*, with depth *d*, width *b*, thickness *t* and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the cruciform section
- **b** (*float*) – Width of the cruciform section
- **t** (*float*) – Thickness of the cruciform section
- **r** (*float*) – Root radius of the cruciform section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a cruciform section with a depth of 250, a width of 175, a thickness of 12 and a root radius of 16, using 16 points to discretise the radius. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.primitive_sections import cruciform_
↪section
```

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```
geometry = cruciform_section(d=250, b=175, t=12, r=16, n_r=16)
geometry.create_mesh(mesh_sizes=[5.0])
```

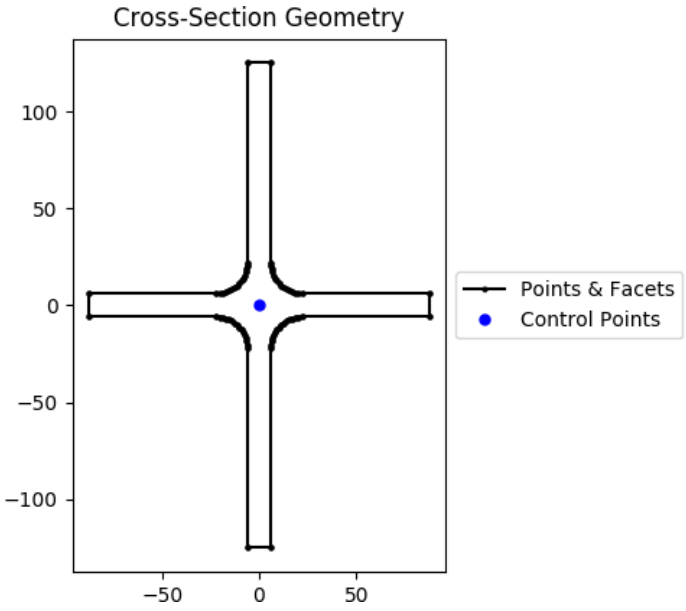


Fig. 13: Cruciform section geometry.

4.2 Steel Sections Library

4.2.1 Circular Hollow Section (CHS)

sectionproperties.pre.library.steel_sections.**circular_hollow_section**(*d: float*, *t: float*, *n: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a circular hollow section (CHS) centered at the origin *(0, 0)*, with diameter *d* and thickness *t*, using *n* points to construct the inner and outer circles.

Parameters

- **d** (*float*) – Outer diameter of the CHS
- **t** (*float*) – Thickness of the CHS
- **n** (*int*) – Number of points discretising the inner and outer circles
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a CHS discretised with 64 points, with a diameter of 48 and thickness of 3.2, and generates a mesh with a maximum triangular area of 1.0:

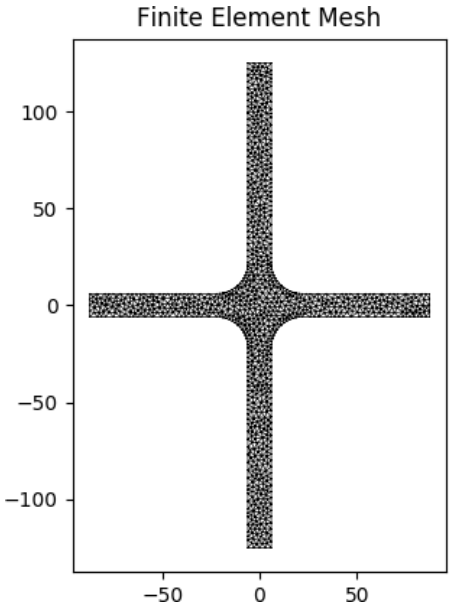


Fig. 14: Mesh generated from the above geometry.

```
from sectionproperties.pre.library.steel_sections import circular_hollow_section

geometry = circular_hollow_section(d=48, t=3.2, n=64)
geometry.create_mesh(mesh_sizes=[1.0])
```

4.2.2 Elliptical Hollow Section (EHS)

sectionproperties.pre.library.steel_sections.**elliptical_hollow_section**(*d_y: float*, *d_x: float*, *t: float*, *n: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs an elliptical hollow section (EHS) centered at the origin *(0, 0)*, with outer vertical diameter *d_y*, outer horizontal diameter *d_x*, and thickness *t*, using *n* points to construct the inner and outer ellipses.

Parameters

- **d_y** (*float*) – Diameter of the ellipse in the y-dimension
- **d_x** (*float*) – Diameter of the ellipse in the x-dimension
- **t** (*float*) – Thickness of the EHS
- **n** (*int*) – Number of points discretising the inner and outer ellipses
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

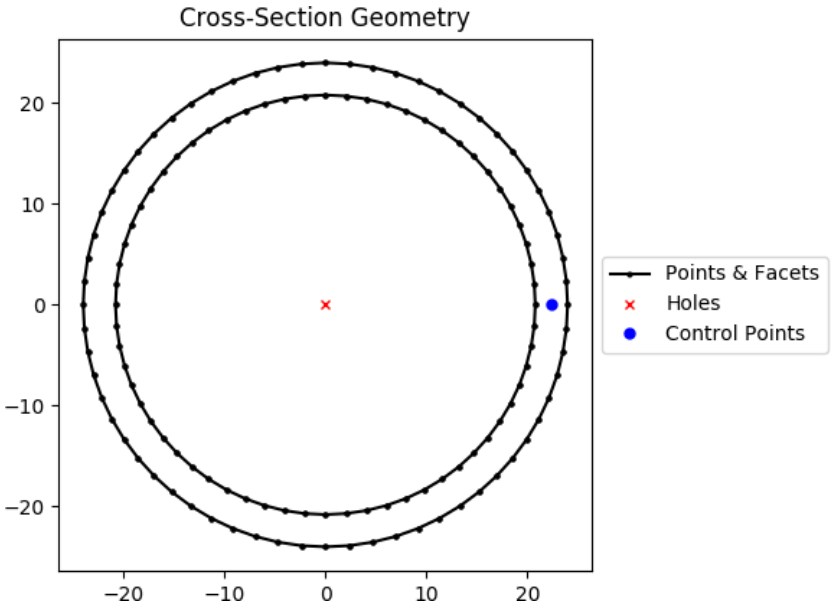


Fig. 15: CHS geometry.

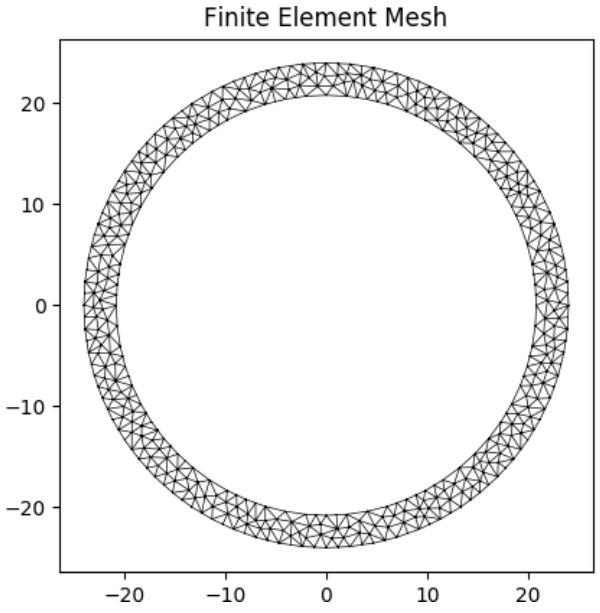


Fig. 16: Mesh generated from the above geometry.

The following example creates a EHS discretised with 30 points, with a outer vertical diameter of 25, outer horizontal diameter of 50, and thickness of 2.0, and generates a mesh with a maximum triangular area of 0.5:

```
from sectionproperties.pre.library.steel_sections import elliptical_hollow_section

geometry = elliptical_hollow_section(d_y=25, d_x=50, t=2.0, n=64)
geometry.create_mesh(mesh_sizes=[0.5])
```

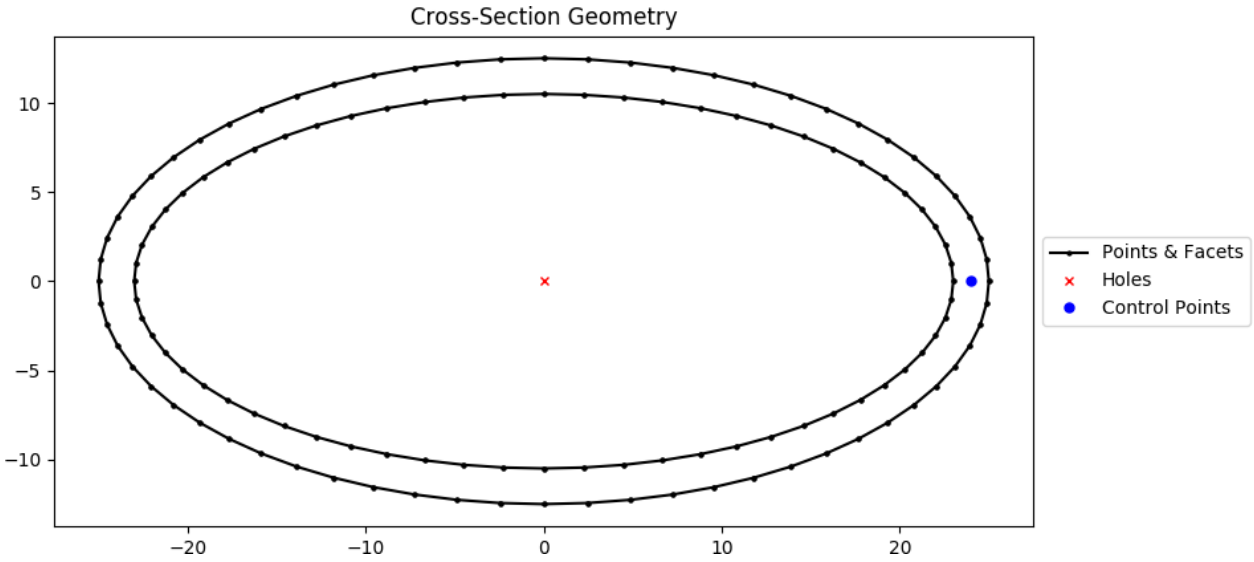


Fig. 17: EHS geometry.

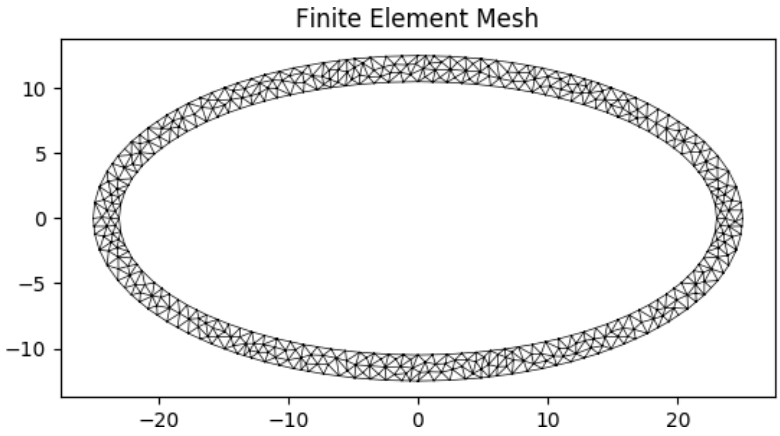


Fig. 18: Mesh generated from the above geometry.

4.2.3 Rectangular Hollow Section (RHS)

sectionproperties.pre.library.steel_sections.**rectangular_hollow_section**(*b: float*, *d: float*, *t: float*, *r_out: float*, *n_r: int*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a rectangular hollow section (RHS) centered at *(b/2, d/2)*, with depth *d*, width *b*, thickness *t* and outer radius *r_out*, using *n_r* points to construct the inner and outer radii. If the outer radius is less than the thickness of the RHS, the inner radius is set to zero.

Parameters

- **d** (*float*) – Depth of the RHS
- **b** (*float*) – Width of the RHS
- **t** (*float*) – Thickness of the RHS
- **r_out** (*float*) – Outer radius of the RHS
- **n_r** (*int*) – Number of points discretising the inner and outer radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an RHS with a depth of 100, a width of 50, a thickness of 6 and an outer radius of 9, using 8 points to discretise the inner and outer radii. A mesh is generated with a maximum triangular area of 2.0:

```
from sectionproperties.pre.library.steel_sections import rectangular_hollow_section

geometry = rectangular_hollow_section(d=100, b=50, t=6, r_out=9, n_r=8)
geometry.create_mesh(mesh_sizes=[2.0])
```

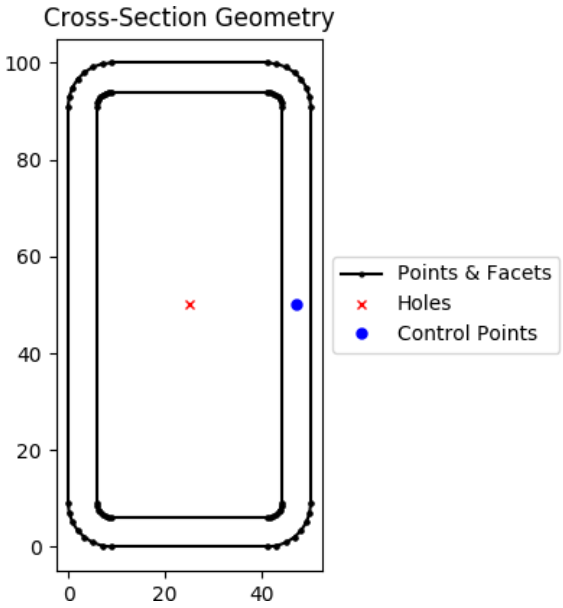


Fig. 19: RHS geometry.

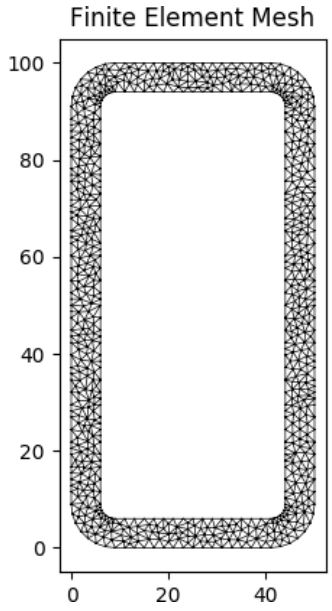


Fig. 20: Mesh generated from the above geometry.

4.2.4 Polygon Hollow Section

sectionproperties.pre.library.steel_sections.**polygon_hollow_section**(*d: float*, *t: float*, *n_sides: int*, *r_in: float = 0*, *n_r: int = 1*, *rot: float = 0*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a regular hollow polygon section centered at *(0, 0)*, with a pitch circle diameter of bounding polygon *d*, thickness *t*, number of sides *n_sides* and an optional inner radius *r_in*, using *n_r* points to construct the inner and outer radii (if radii is specified).

Parameters

- **d** (*float*) – Pitch circle diameter of the outer bounding polygon (i.e. diameter of circle that passes through all vertices of the outer polygon)
- **t** (*float*) – Thickness of the polygon section wall
- **r_in** (*float*) – Inner radius of the polygon corners. By default, if not specified, a polygon with no corner radii is generated.
- **n_r** (*int*) – Number of points discretising the inner and outer radii, ignored if no inner radii is specified
- **rot** (*float*) – Initial counterclockwise rotation in degrees. By default bottom face is aligned with x axis.
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

Raises

Exception – Number of sides in polygon must be greater than or equal to 3

The following example creates an Octagonal section (8 sides) with a diameter of 200, a thickness of 6 and an inner radius of 20, using 12 points to discretise the inner and outer radii. A mesh is generated with a maximum triangular area of 5:

```
from sectionproperties.pre.library.steel_sections import polygon_hollow_
↪section

geometry = polygon_hollow_section(d=200, t=6, n_sides=8, r_in=20, n_r=12)
geometry.create_mesh(mesh_sizes=[5])
```

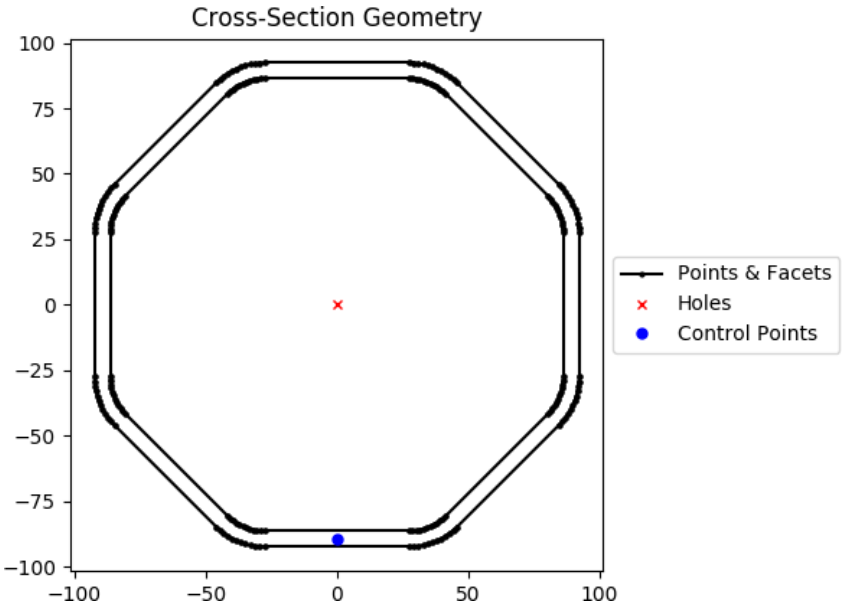


Fig. 21: Octagonal section geometry.

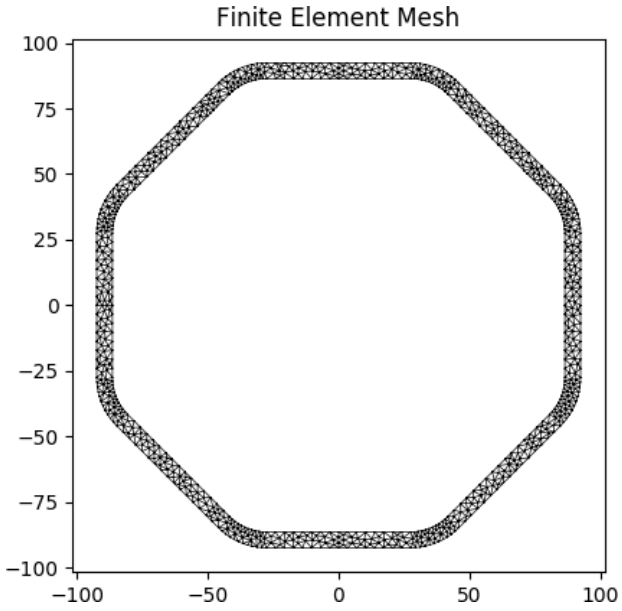


Fig. 22: Mesh generated from the above geometry.

4.2.5 I Section

sectionproperties.pre.library.steel_sections.**i_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs an I Section centered at *(b/2, d/2)*, with depth *d*, width *b*, flange thickness *t_f*, web thickness *t_w*, and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the I Section
- **b** (*float*) – Width of the I Section
- **t_f** (*float*) – Flange thickness of the I Section
- **t_w** (*float*) – Web thickness of the I Section
- **r** (*float*) – Root radius of the I Section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an I Section with a depth of 203, a width of 133, a flange thickness of 7.8, a web thickness of 5.8 and a root radius of 8.9, using 16 points to discretise the root radius. A mesh is generated with a maximum triangular area of 3.0:

```
from sectionproperties.pre.library.steel_sections import i_section

geometry = i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=16)
geometry.create_mesh(mesh_sizes=[3.0])
```

4.2.6 Monosymmetric I Section

sectionproperties.pre.library.steel_sections.**mono_i_section**(*d: float*, *b_t: float*, *b_b: float*, *t_ft: float*, *t_fb: float*, *t_w: float*, *r: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a monosymmetric I Section centered at *(max(b_t, b_b)/2, d/2)*, with depth *d*, top flange width *b_t*, bottom flange width *b_b*, top flange thickness *t_ft*, top flange thickness *t_fb*, web thickness *t_w*, and root radius *r*, using *n_r* points to construct the root radius.

Parameters

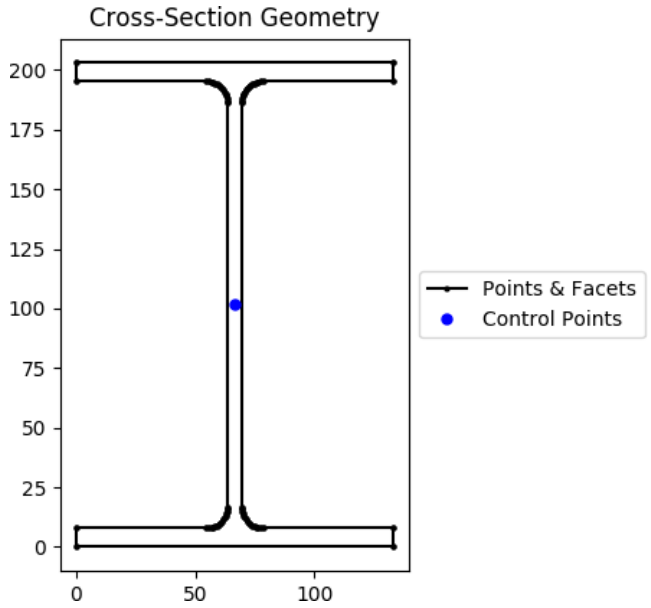


Fig. 23: I Section geometry.

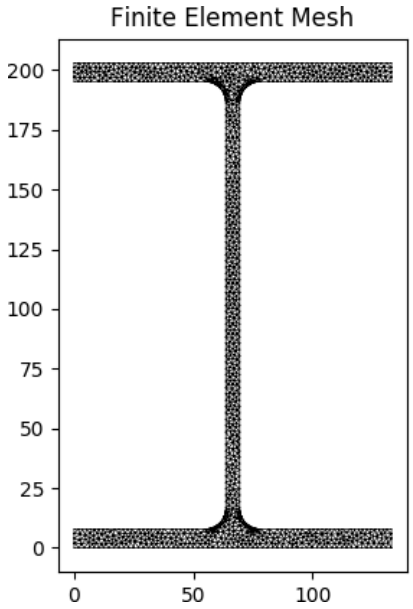


Fig. 24: Mesh generated from the above geometry.

- **d** (*float*) – Depth of the I Section
- **b_t** (*float*) – Top flange width
- **b_b** (*float*) – Bottom flange width
- **t_ft** (*float*) – Top flange thickness of the I Section
- **t_fb** (*float*) – Bottom flange thickness of the I Section
- **t_w** (*float*) – Web thickness of the I Section
- **r** (*float*) – Root radius of the I Section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a monosymmetric I Section with a depth of 200, a top flange width of 50, a top flange thickness of 12, a bottom flange width of 130, a bottom flange thickness of 8, a web thickness of 6 and a root radius of 8, using 16 points to discretise the root radius. A mesh is generated with a maximum triangular area of 3.0:

```
from sectionproperties.pre.library.steel_sections import mono_i_section

geometry = mono_i_section(
    d=200, b_t=50, b_b=130, t_ft=12, t_fb=8, t_w=6, r=8, n_r=16
)
geometry.create_mesh(mesh_sizes=[3.0])
```

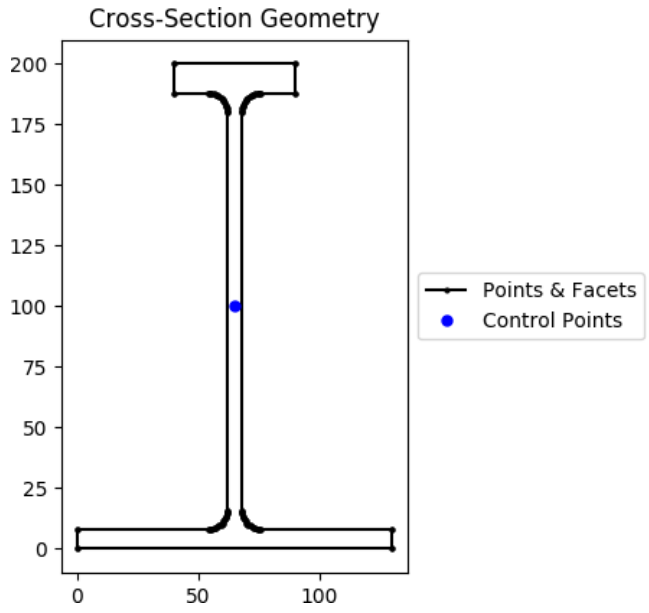


Fig. 25: I Section geometry.

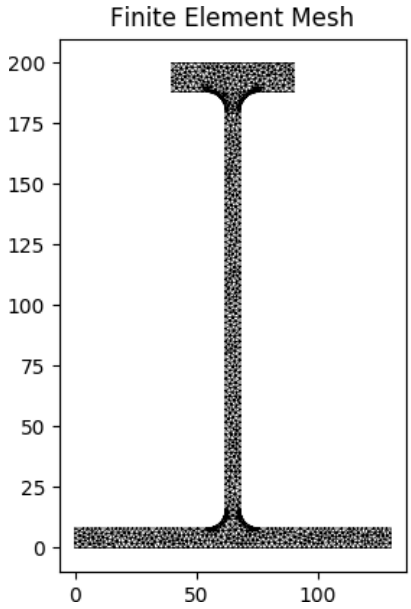


Fig. 26: Mesh generated from the above geometry.

4.2.7 Tapered Flange I Section

sectionproperties.pre.library.steel_sections.**tapered_flange_i_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r_r: float*, *r_f: float*, *alpha: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Tapered Flange I Section centered at *(b/2, d/2)*, with depth *d*, width *b*, mid-flange thickness *t_f*, web thickness *t_w*, root radius *r_r*, flange radius *r_f* and flange angle *alpha*, using *n_r* points to construct the radii.

Parameters

- **d** (*float*) – Depth of the Tapered Flange I Section
- **b** (*float*) – Width of the Tapered Flange I Section
- **t_f** (*float*) – Mid-flange thickness of the Tapered Flange I Section (measured at

the point equidistant from the face of the web to the edge of the flange)

- **t_w** (*float*) – Web thickness of the Tapered Flange I Section
- **r_r** (*float*) – Root radius of the Tapered Flange I Section
- **r_f** (*float*) – Flange radius of the Tapered Flange I Section
- **alpha** (*float*) – Flange angle of the Tapered Flange I Section (degrees)
- **n_r** (*int*) – Number of points discretising the radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a Tapered Flange I Section with a depth of 588, a width of 191, a mid-flange thickness of 27.2, a web thickness of 15.2, a root radius of 17.8, a flange radius of 8.9 and a flange angle of 8°, using 16 points to discretise the radii. A mesh is generated with a maximum triangular area of 20.0:

```
from sectionproperties.pre.library.steel_sections import tapered_flange_i_
↪section

geometry = tapered_flange_i_section(
    d=588, b=191, t_f=27.2, t_w=15.2, r_r=17.8, r_f=8.9, alpha=8, n_r=16
)
geometry.create_mesh(mesh_sizes=[20.0])
```

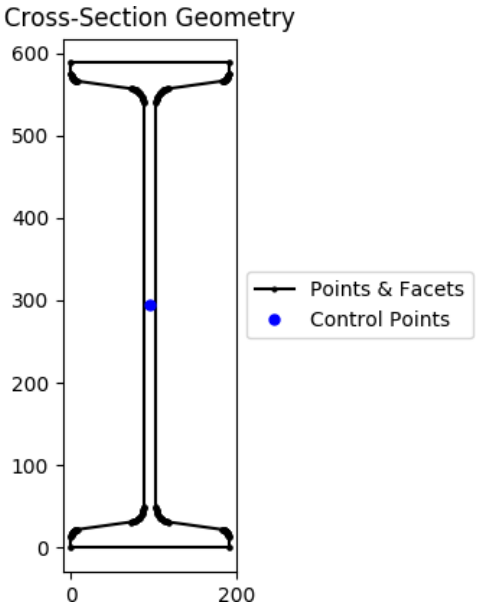


Fig. 27: I Section geometry.

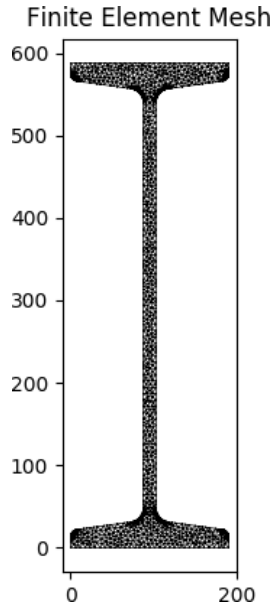


Fig. 28: Mesh generated from the above geometry.

4.2.8 Parallel Flange Channel (PFC) Section

sectionproperties.pre.library.steel_sections.**channel_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a parallel-flange channel (PFC) section with the bottom left corner at the origin *(0, 0)*, with depth *d*, width *b*, flange thickness *t_f*, web thickness *t_w* and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the PFC section
- **b** (*float*) – Width of the PFC section
- **t_f** (*float*) – Flange thickness of the PFC section
- **t_w** (*float*) – Web thickness of the PFC section
- **r** (*float*) – Root radius of the PFC section
- **n_r** (*int*) – Number of points discretising the root radius
- **shift** (*list[float, float]*) – Vector that shifts the cross-section by *(x, y)*
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a PFC section with a depth of 250, a width of 90, a flange thickness of 15, a web thickness of 8 and a root radius of 12, using 8 points to discretise the root radius. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.steel_sections import channel_section

geometry = channel_section(d=250, b=90, t_f=15, t_w=8, r=12, n_r=8)
geometry.create_mesh(mesh_sizes=[5.0])
```

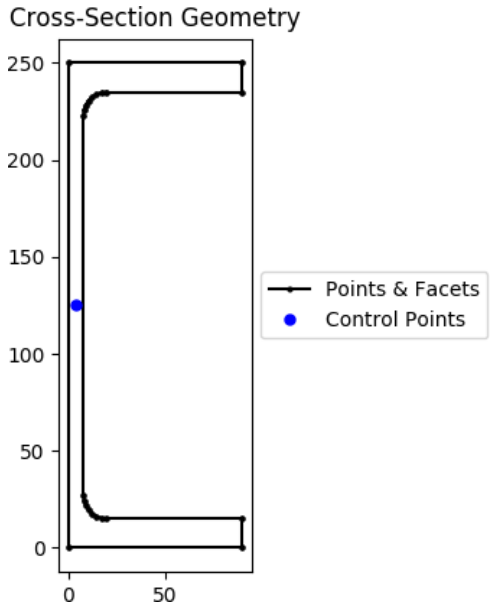


Fig. 29: PFC geometry.

4.2.9 Tapered Flange Channel Section

sectionproperties.pre.library.steel_sections.**tapered_flange_channel**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r_r: float*, *r_f: float*, *alpha: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Tapered Flange Channel section with the bottom left corner at the origin *(0, 0)*, with

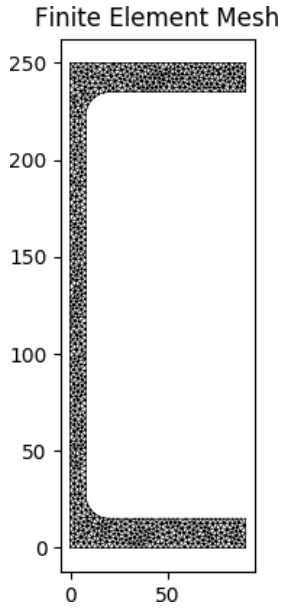


Fig. 30: Mesh generated from the above geometry.

depth *d*, width *b*, mid-flange thickness *t_f*, web thickness *t_w*, root radius *r_r*, flange radius *r_f* and flange angle *alpha*, using *n_r* points to construct the radii.

Parameters

- **d** (*float*) – Depth of the Tapered Flange Channel section
- **b** (*float*) – Width of the Tapered Flange Channel section
- **t_f** (*float*) – Mid-flange thickness of the Tapered Flange Channel section (measured at the point equidistant from the face of the web to the edge of the flange)
- **t_w** (*float*) – Web thickness of the Tapered Flange Channel section
- **r_r** (*float*) – Root radius of the Tapered Flange Channel section
- **r_f** (*float*) – Flange radius of the Tapered Flange Channel section
- **alpha** (*float*) – Flange angle of the Tapered Flange Channel section (degrees)
- **n_r** (*int*) – Number of points discretising the radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a Tapered Flange Channel section with a depth of 10, a width of 3.5, a mid-flange thickness of 0.575, a web thickness of 0.475, a root radius of 0.575, a flange radius of 0.4 and a flange angle of 8°, using 16 points to discretise the radii. A mesh is generated with a maximum triangular area of 0.02:

```
from sectionproperties.pre.library.steel_sections import tapered_flange_
↪channel

geometry = tapered_flange_channel(
    d=10, b=3.5, t_f=0.575, t_w=0.475, r_r=0.575, r_f=0.4, alpha=8, n_r=16
)
geometry.create_mesh(mesh_sizes=[0.02])
```

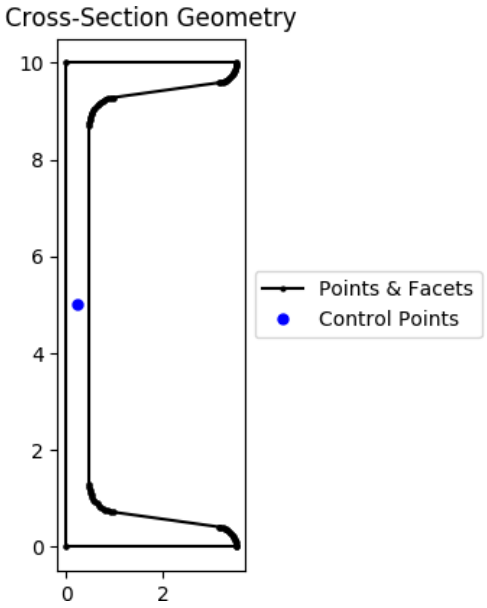


Fig. 31: Tapered flange channel geometry.

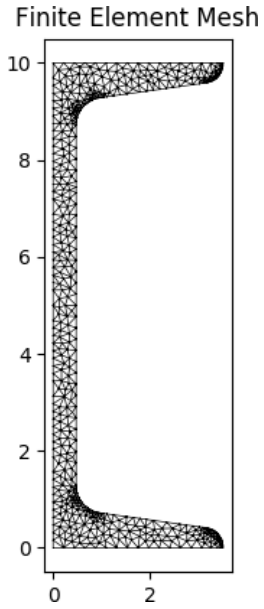


Fig. 32: Mesh generated from the above geometry.

4.2.10 Tee Section

sectionproperties.pre.library.steel_sections.**tee_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Tee section with the top left corner at *(0, d)*, with depth *d*, width *b*, flange thickness *t_f*, web thickness *t_w* and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the Tee section
- **b** (*float*) – Width of the Tee section
- **t_f** (*float*) – Flange thickness of the Tee section
- **t_w** (*float*) – Web thickness of the Tee section
- **r** (*float*) – Root radius of the Tee section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a Tee section with a depth of 200, a width of 100, a flange thickness of 12, a web thickness of 6 and a root radius of 8, using 8 points to discretise the root radius. A mesh is generated with a maximum triangular area of 3.0:

```
from sectionproperties.pre.library.steel_sections import tee_section

geometry = tee_section(d=200, b=100, t_f=12, t_w=6, r=8, n_r=8)
geometry.create_mesh(mesh_sizes=[3.0])
```

4.2.11 Angle Section

sectionproperties.pre.library.steel_sections.**angle_section**(*d: float*, *b: float*, *t: float*, *r_r: float*, *r_t: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs an angle section with the bottom left corner at the origin *(0, 0)*, with depth *d*, width *b*, thickness *t*, root radius *r_r* and toe radius *r_t*, using *n_r* points to construct the radii.

Parameters

- **d** (*float*) – Depth of the angle section
- **b** (*float*) – Width of the angle section

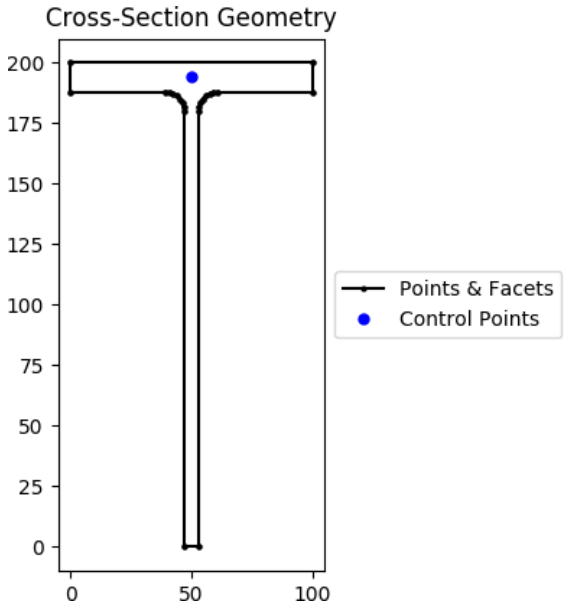


Fig. 33: Tee section geometry.

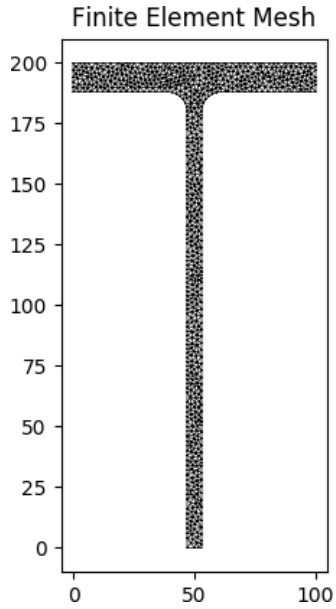


Fig. 34: Mesh generated from the above geometry.

- **t** (*float*) – Thickness of the angle section
- **r_r** (*float*) – Root radius of the angle section
- **r_t** (*float*) – Toe radius of the angle section
- **n_r** (*int*) – Number of points discretising the radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an angle section with a depth of 150, a width of 100, a thickness of 8, a root radius of 12 and a toe radius of 5, using 16 points to discretise the radii. A mesh is generated with a maximum triangular area of 2.0:

```
from sectionproperties.pre.library.steel_sections import angle_section

geometry = angle_section(d=150, b=100, t=8, r_r=12, r_t=5, n_r=16)
geometry.create_mesh(mesh_sizes=[2.0])
```

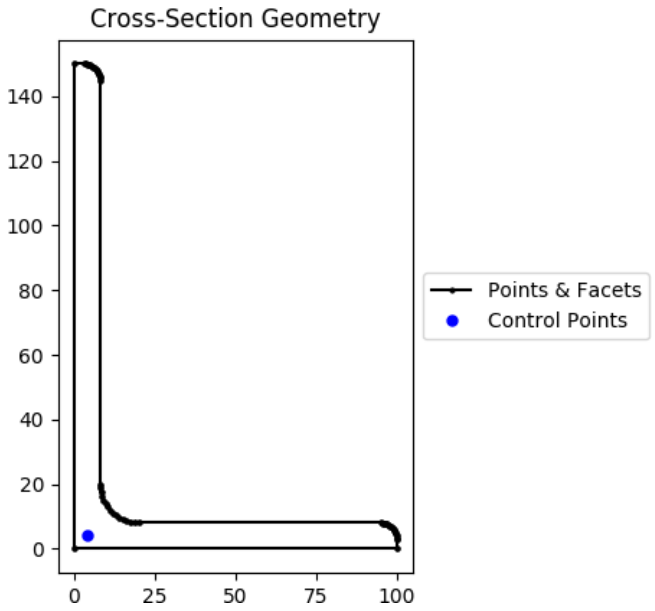
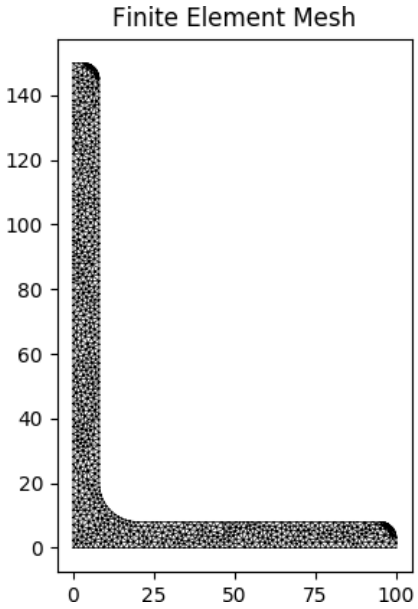


Fig. 35: Angle section geometry.

4.2.12 Cee Section

sectionproperties.pre.library.steel_sections.**cee_section**(*d: float*, *b: float*, *l: float*, *t: float*, *r_out: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Cee section (typical of cold-formed steel) with the bottom left corner at the origin *(0, 0)*, with depth *d*, width *b*, lip *l*, thickness *t* and outer radius *r_out*, using *n_r* points to construct the radius. If the outer radius is less than the thickness of the Cee Section, the inner radius is set to zero.



Parameters

- **d** (*float*) – Depth of the Cee section
- **b** (*float*) – Width of the Cee section
- **l** (*float*) – Lip of the Cee section
- **t** (*float*) – Thickness of the Cee section
- **r_out** (*float*) – Outer radius of the Cee section
- **n_r** (*int*) – Number of points discretising the outer radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

Raises

Exception – Lip length must be greater than the outer radius

The following example creates a Cee section with a depth of 125, a width of 50, a lip of 30, a thickness of 1.5 and an outer radius of 6, using 8 points to discretise the radius. A mesh is generated with a maximum triangular area of 0.25:

```
from sectionproperties.pre.library.steel_sections import cee_section

geometry = cee_section(d=125, b=50, l=30, t=1.5, r_out=6, n_r=8)
geometry.create_mesh(mesh_sizes=[0.25])
```

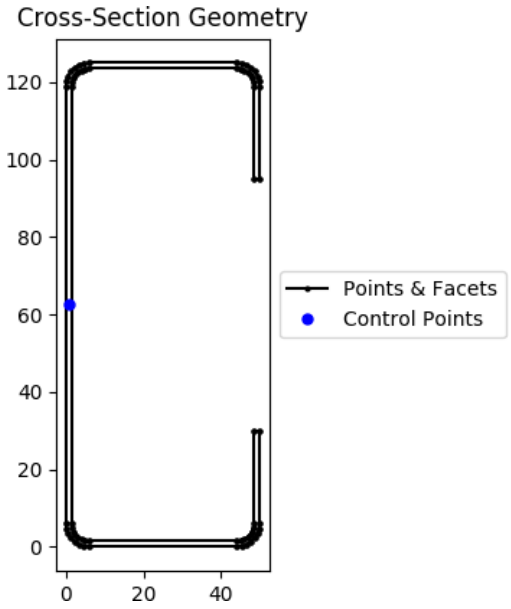
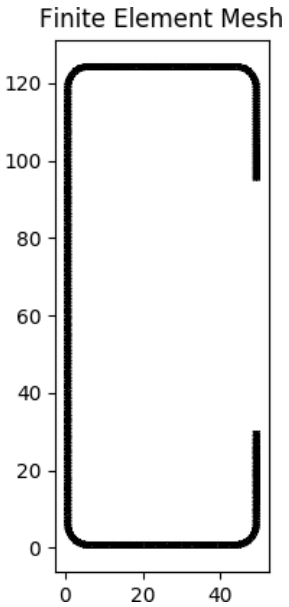


Fig. 36: Cee section geometry.



4.2.13 Zed Section

sectionproperties.pre.library.steel_sections.**zed_section**(*d: float*, *b_l: float*, *b_r: float*, *l: float*, *t: float*, *r_out: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a zed section with the bottom left corner at the origin *(0, 0)*, with depth *d*, left flange width *b_l*, right flange width *b_r*, lip *l*, thickness *t* and outer radius *r_out*, using *n_r* points to construct the radius. If the outer radius is less than the thickness of the Zed Section, the inner radius is set to zero.

Parameters

- **d** (*float*) – Depth of the zed section
- **b_l** (*float*) – Left flange width of the Zed section
- **b_r** (*float*) – Right flange width of the Zed section
- **l** (*float*) – Lip of the Zed section
- **t** (*float*) – Thickness of the Zed section
- **r_out** (*float*) – Outer radius of the Zed section
- **n_r** (*int*) – Number of points discretising the outer radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a zed section with a depth of 100, a left flange width of 40, a right flange width of 50, a lip of 20, a thickness of 1.2 and an outer radius of 5, using 8 points to discretise the radius. A mesh is generated with a maximum triangular area of 0.15:

```
from sectionproperties.pre.library.steel_sections import zed_section

geometry = zed_section(d=100, b_l=40, b_r=50, l=20, t=1.2, r_out=5, n_r=8)
geometry.create_mesh(mesh_sizes=[0.15])
```

4.2.14 Box Girder Section

sectionproperties.pre.library.steel_sections.**box_girder_section**(*d: float*, *b_t: float*, *b_b: float*, *t_ft: float*, *t_fb: float*, *t_w: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*)

Constructs a box girder section centered at at *(max(b_t, b_b)/2, d/2)*, with depth *d*, top width *b_t*, bottom width *b_b*, top flange thickness *t_ft*, bottom flange thickness *t_fb* and web thickness *t_w*.

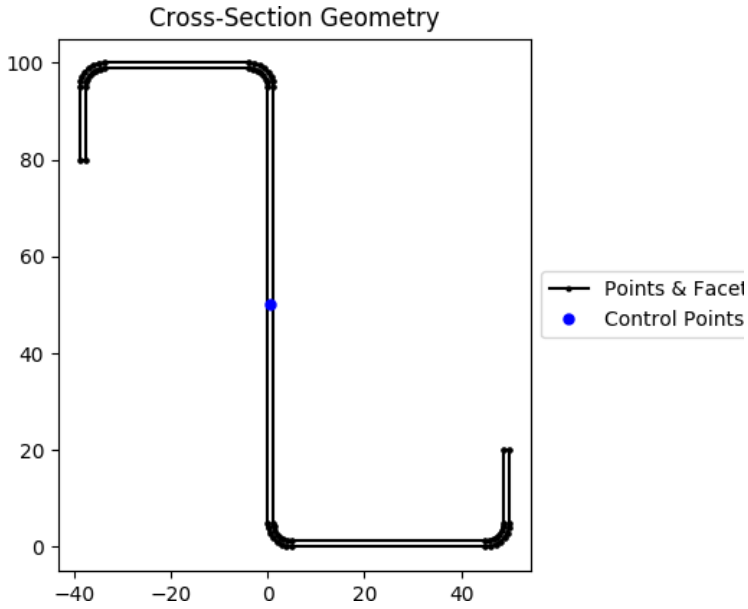
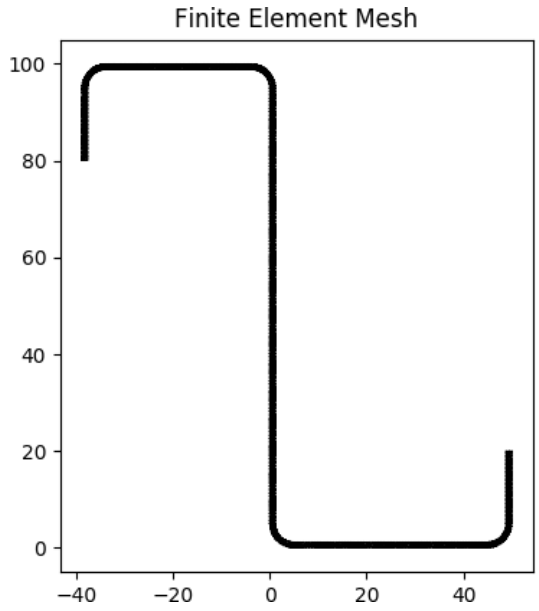


Fig. 37: zed section geometry.



Parameters

- **d** (*float*) – Depth of the Box Girder section
- **b_t** (*float*) – Top width of the Box Girder section
- **b_b** (*float*) – Bottom width of the Box Girder section
- **t_ft** (*float*) – Top flange thickness of the Box Girder section
- **t_fb** (*float*) – Bottom flange thickness of the Box Girder section
- **t_w** (*float*) – Web thickness of the Box Girder section

The following example creates a Box Girder section with a depth of 1200, a top width of 1200, a bottom width of 400, a top flange thickness of 16, a bottom flange thickness of 12 and a web thickness of 8. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.steel_sections import box_girder_section

geometry = box_girder_section(d=1200, b_t=1200, b_b=400, t_ft=100, t_fb=80,
↪ t_w=50)
geometry.create_mesh(mesh_sizes=[200.0])
```

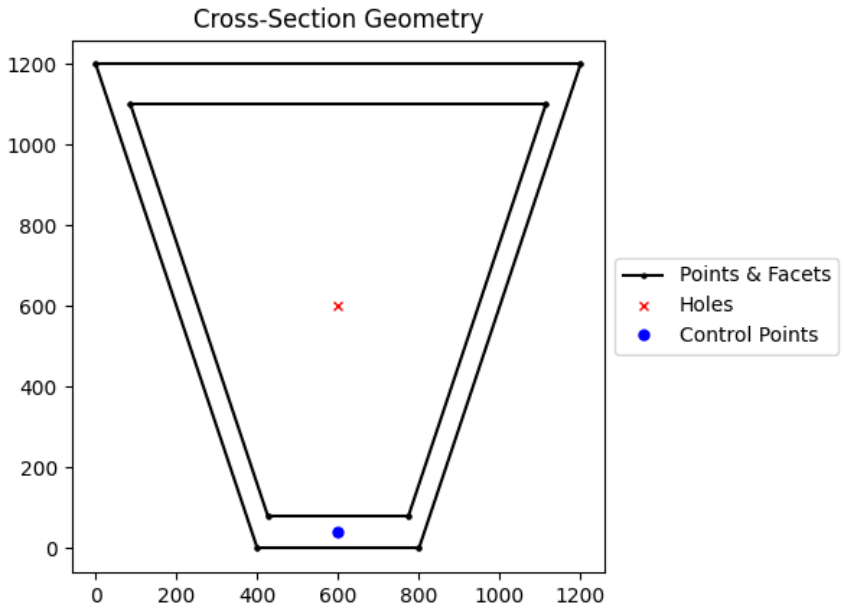


Fig. 38: Box Girder geometry.

4.2.15 Bulb Section

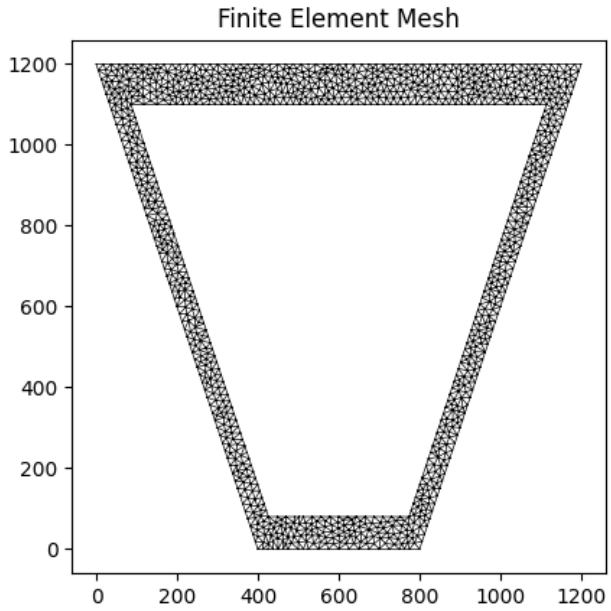


Fig. 39: Mesh generated from the above geometry.

sectionproperties.pre.library.steel_sections.**bulb_section**(*d: float*, *b: float*, *t: float*, *r: float*, *n_r: int*, *d_b: Optional[float] = None*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a bulb section with the bottom left corner at the point *(-t / 2, 0)*, with depth *d*, bulb depth *d_b*, bulb width *b*, web thickness *t* and radius *r*, using *n_r* points to construct the radius.

Parameters

- **d** (*float*) – Depth of the section
- **b** (*float*) – Bulb width
- **t** (*float*) – Web thickness
- **r** (*float*) – Bulb radius
- **d_b** (*float*) – Depth of the bulb (automatically calculated for standard sections, if provided the section may have sharp edges)
- **n_r** (*int*) – Number of points discretising the radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a bulb section with a depth of 240, a width of 34, a web thickness of 12 and a bulb radius of 16, using 16 points to discretise the radius. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.steel_sections import bulb_section

geometry = bulb_section(d=240, b=34, t=12, r=10, n_r=16)
geometry.create_mesh(mesh_sizes=[5.0])
```

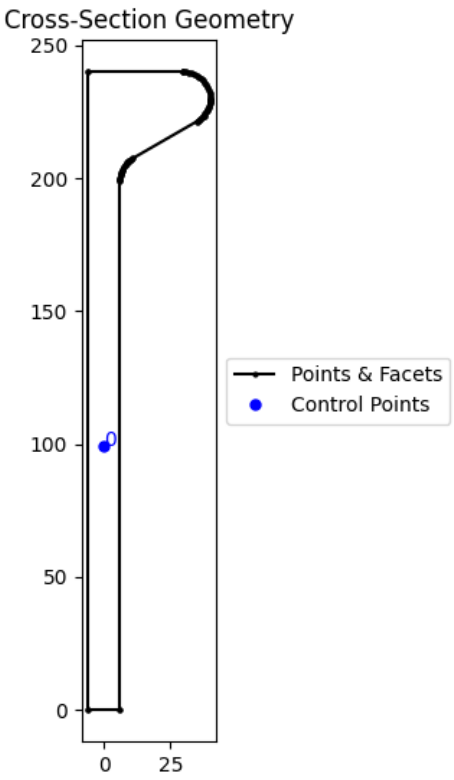


Fig. 40: Bulb section geometry.

4.3 Concrete Sections Library

4.3.1 Concrete Rectangular Section

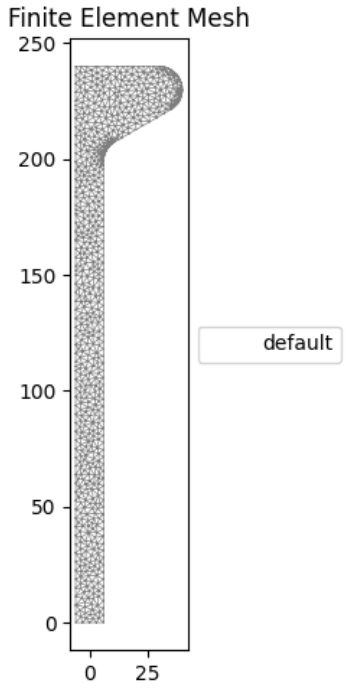


Fig. 41: Mesh generated from the above geometry.

sectionproperties.pre.library.concrete_sections.**concrete_rectangular_section**(*b: float*, *d: float*, *dia_top: float*, *n_top: int*, *dia_bot: float*, *n_bot: int*, *n_circle: int*, *cover: float*, *dia_side: Optional[float] = None*, *n_side: int = 0*, *area_top: Optional[float] = None*, *area_bot: Optional[float] = None*, *area_side: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Mate-*

Constructs a concrete rectangular section of width *b* and depth *d*, with *n_top* top steel bars of diameter *dia_top*, *n_bot* bottom steel bars of diameter *dia_bot*, *n_side* left & right side steel bars of diameter *dia_side* discretised with *n_circle* points with equal side and top/bottom *cover* to the steel.

Parameters

- **b** (*float*) – Concrete section width
- **d** (*float*) – Concrete section depth
- **dia_top** (*float*) – Diameter of the top steel reinforcing bars
- **n_top** (*int*) – Number of top steel reinforcing bars
- **dia_bot** (*float*) – Diameter of the bottom steel reinforcing bars
- **n_bot** (*int*) – Number of bottom steel reinforcing bars
- **n_circle** (*int*) – Number of points discretising the steel reinforcing bars
- **cover** (*float*) – Side and bottom cover to the steel reinforcing bars
- **dia_side** (*float*) – If provided, diameter of the side steel reinforcing bars
- **n_side** (*int*) – If provided, number of side bars either side of the section
- **area_top** (*float*) – If provided, constructs top reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **area_bot** (*float*) – If provided, constructs bottom reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **area_side** (*float*) – If provided, constructs side reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **conc_mat** – Material to associate with the concrete
- **steel_mat** – Material to associate with the steel

Raises

ValueError – If the number of bars is not greater than or equal to 2 in an active layer

The following example creates a 600D x 300W concrete beam with 3N20 bottom steel reinforcing bars and 30 mm cover:

```
from sectionproperties.pre.library.concrete_sections import concrete_rectangular_
↪section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_rectangular_section(
    b=300, d=600, dia_top=20, n_top=0, dia_bot=20, n_bot=3, n_circle=24, cover=30,
    conc_mat=concrete, steel_mat=steel
```

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```
)
geometry.create_mesh(mesh_sizes=[500])
```

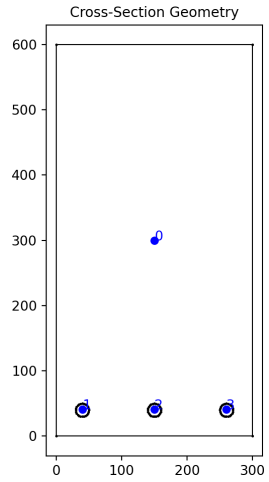

Fig. 42: Concrete rectangular section geometry.

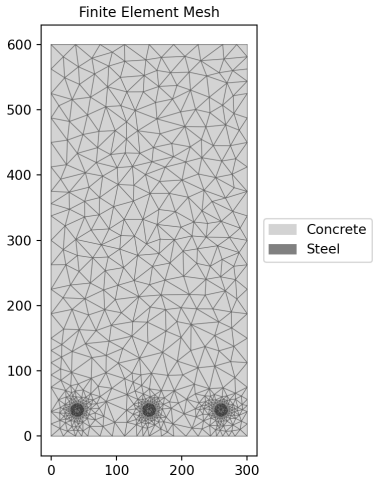

Fig. 43: Mesh generated from the above geometry.

4.3.2 Concrete Column Section

sectionproperties.pre.library.concrete_sections.**concrete_column_section**(*b: float*, *d: float*, *cover: float*, *n_bars_b: int*, *n_bars_d: int*, *dia_bar: float*, *bar_area: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *filled: bool = False*, *n_circle: int = 4*) → *CompoundGeometry*

Constructs a concrete rectangular section of width *b* and depth *d*, with steel bar reinforcing organized as an *n_bars_b* by *n_bars_d* array, discretised with *n_circle* points with equal sides and top/bottom *cover* to the steel which is taken as the clear cover (edge of bar to edge of concrete).

Parameters

- **b** (*float*) – Concrete section width, parallel to the x-axis
- **d** (*float*) – Concrete section depth, parallel to the y-axis
- **cover** (*float*) – Clear cover, calculated as distance from edge of reinforcing bar to edge of section.
- **n_bars_b** (*int*) – Number of bars placed across the width of the section, minimum 2.
- **n_bars_d** (*int*) – Number of bars placed across the depth of the section, minimum 2.
- **dia_bar** (*float*) – Diameter of reinforcing bars. Used for calculating bar placement and, optionally, for calculating the bar area for section capacity calculations.
- **bar_area** (*float*) – Area of reinforcing bars. Used for section capacity calculations. If not provided, then dia_bar will be used to calculate the bar area.
- **conc_mat** (sectionproperties.pre.pre.Material) – Material to associate with the concrete
- **steel_mat** (sectionproperties.pre.pre.Material) – Material to associate with the reinforcing steel
- **filled** (*bool*) – When True, will populate the concrete section with an equally spaced 2D array of reinforcing bars numbering 'n_bars_b' by 'n_bars_d'. When False, only the bars around the perimeter of the array will be present.

- **n_circle** (*int*) – The number of points used to discretize the circle of the reinforcing bars. The bars themselves will have an exact area of 'bar_area' regardless of the number of points used in the circle. Useful for making the reinforcing bars look more circular when plotting the concrete section.

Raises

ValueError – If the number of bars in either 'n_bars_b' or 'n_bars_d' is not greater than or equal to 2.

The following example creates a 600D x 300W concrete column with 25 mm diameter reinforcing bars each with 500 mm**2 area and 35 mm cover in a 3x6 array without the interior bars being filled:

```
from sectionproperties.pre.library.concrete_sections import concrete_column_section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_column_section(
    b=300, d=600, dia_bar=25, bar_area=500, cover=35, n_bars_b=3, n_bars_d=6,
    conc_mat=concrete, steel_mat=steel, filled=False, n_circle=4
)
geometry.create_mesh(mesh_sizes=[500])
```

4.3.3 Concrete Tee Section

sectionproperties.pre.library.concrete_sections.**concrete_tee_section**(*b: float*, *d: float*, *b_f: float*, *d_f: float*, *dia_top: float*, *n_top: int*, *dia_bot: float*, *n_bot: int*, *n_circle: int*, *cover: float*, *area_top: Optional[float] = None*, *area_bot: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *CompoundGeometry*

Constructs a concrete tee section of width *b*, depth *d*, flange width *b_f* and flange depth *d_f*, with *n_top* top steel bars of diameter *dia_top*, *n_bot* bottom steel bars of diameter *dia_bot*, discretised with *n_circle* points with equal side and top/bottom *cover* to the steel.

Parameters

- **b** (*float*) – Concrete section width
- **d** (*float*) – Concrete section depth
- **b_f** (*float*) – Concrete section flange width
- **d_f** (*float*) – Concrete section flange depth
- **dia_top** (*float*) – Diameter of the top steel reinforcing bars
- **n_top** (*int*) – Number of top steel reinforcing bars
- **dia_bot** (*float*) – Diameter of the bottom steel reinforcing bars
- **n_bot** (*int*) – Number of bottom steel reinforcing bars
- **n_circle** (*int*) – Number of points discretising the steel reinforcing bars
- **cover** (*float*) – Side and bottom cover to the steel reinforcing bars
- **area_top** (*float*) – If provided, constructs top reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **area_bot** (*float*) – If provided, constructs bottom reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **conc_mat** – Material to associatewith the concrete
- **steel_mat** – Material toassociate with the steel

Raises

ValueErorr – If the number of bars is not greater than or equal to 2 in an active layer

The following example creates a 900D x 450W concrete beam with a 1200W x 250D flange, with 5N24 steel reinforcing bars and 30 mm cover:

```
from sectionproperties.pre.library.concrete_sections import concrete_tee_section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_tee_section(
    b=450, d=900, b_f=1200, d_f=250, dia_top=24, n_top=0, dia_bot=24, n_bot=5,
    n_circle=24, cover=30, conc_mat=concrete, steel_mat=steel
)
geometry.create_mesh(mesh_sizes=[500])
```

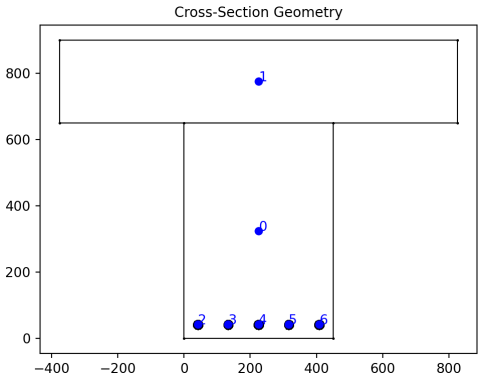

Fig. 44: Concrete tee section geometry.

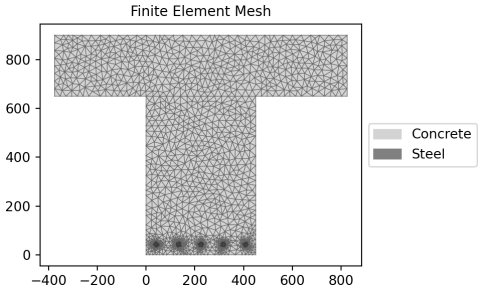

Fig. 45: Mesh generated from the above geometry.

4.3.4 Concrete Circular Section

sectionproperties.pre.library.concrete_sections.**concrete_circular_section**(*d: float*, *n: int*, *dia: float*, *n_bar: int*, *n_circle: int*, *cover: float*, *area_conc: Optional[float] = None*, *area_bar: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *CompoundGeometry*

Constructs a concrete circular section of diameter *d* discretised with *n* points, with *n_bar* steel bars of diameter *dia*, discretised with *n_circle* points with equal side and bottom *cover* to the steel.

Parameters

- **d** (*float*) – Concrete diameter
- **n** (*float*) – Number of points discretising the concrete section
- **dia** (*float*) – Diameter of the steel reinforcing bars
- **n_bar** (*int*) – Number of steel reinforcing bars
- **n_circle** (*int*) – Number of points discretising the steel reinforcing bars
- **cover** (*float*) – Side and bottom cover to the steel reinforcing bars
- **area_conc** (*float*) – If provided, constructs the concrete based on its area rather than diameter (prevents the underestimation of concrete area due to circle discretisation)
- **area_bar** (*float*) – If provided, constructs reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to
- **conc_mat** – Material to associate with the concrete
- **steel_mat** – Material to associate with the steel

Raises

ValueErorr – If the number of bars is not greater than or equal to 2

The following example creates a 450DIA concrete column with with 6N20 steel reinforcing bars and 45 mm cover:

```
from sectionproperties.pre.library.concrete_sections import concrete_circular_
↪section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_circular_section(
    d=450, n=64, dia=20, n_bar=6, n_circle=24, cover=45, conc_mat=concrete, steel_
↪mat=steel
)
geometry.create_mesh(mesh_sizes=[500])
```

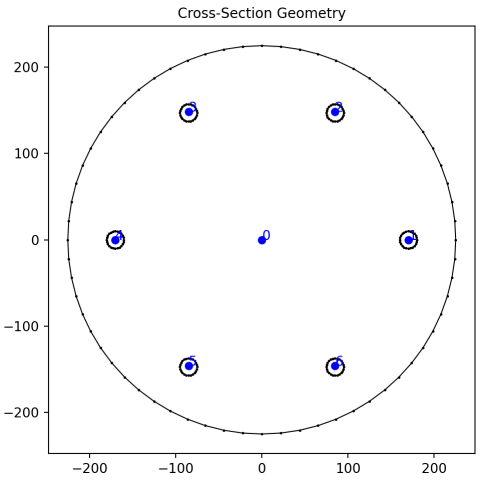

Fig. 46: Concrete circular section geometry.

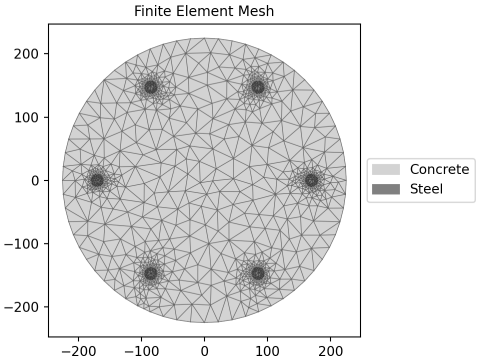

Fig. 47: Mesh generated from the above geometry.

4.3.5 Add Bar

sectionproperties.pre.library.concrete_sections.**add_bar**(*geometry: Union[Geometry, CompoundGeometry]*, *area: float*, *material: Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *x: float*, *y: float*, *n: int = 4*) → *CompoundGeometry*

Adds a reinforcing bar to a *sectionproperties* geometry.

Bars are discretised by four points by default.

Parameters

- **geometry** – Reinforced concrete geometry to which the new bar will be added
- **area** – Bar cross-sectional area
- **material** – Material object for the bar
- **x** – x-position of the bar
- **y** – y-position of the bar
- **n** – Number of points to discretise the bar circle

Returns

Reinforced concrete geometry with added bar

4.4 Bridge Sections Library

4.4.1 Super Tee Girder Section

sectionproperties.pre.library.bridge_sections.**super_t_girder_section**(*girder_type: int*, *girder_subtype: int = 2*, *w: float = 2100*, *t_w: Optional[float] = None*, *t_f: float = 75*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Super T Girder section to AS5100.5.

Parameters

- **girder_type** (*int*) – Type of Super T (1 to 5)
- **girder_subtype** (*int*) – Era Super T (1: pre-2001, 2:contemporary)
- **w** (*float*) – Overall width of top flange
- **t_w** (*float*) – Web thickness of the Super-T section (defaults to those of AS5100.5 Tb D3(B))

- **t_f** (*float*) – Thickness of top flange (VIC (default) = 75 mm; NSW = 90 mm)
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a T5 Super-T section with a 180 mm overlay slab and assigns the different material properties:

```
import sectionproperties.pre.library.bridge_sections as bridge_sections
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.pre.pre import Material
from sectionproperties.analysis.section import Section

Dslab, w, t_f = 180, 2100, 75

precast = Material(
    name="65 MPa",
    elastic_modulus=37.4e3,
    poissons_ratio=0.2,
    yield_strength=65,
    density=2.4e-6,
    color="grey",
)
insitu = Material(
    name="40 MPa",
    elastic_modulus=32.8e3,
    poissons_ratio=0.2,
    yield_strength=40,
    density=2.4e-6,
    color="lightgrey",
)

super_t = bridge_sections.super_t_girder_section(girder_type=5, w=w,␣
↪material=precast)
slab = primitive_sections.rectangular_section(
    d=Dslab, b=w, material=insitu
).shift_section(-w / 2, t_f)

geom = super_t + slab
geom.plot_geometry()
geom.create_mesh(mesh_sizes=[500])

sec = Section(geom)
sec.plot_mesh()

sec.calculate_geometric_properties()
sec.calculate_warping_properties()
sec.display_results(fmt=".3f")
```

Note that the properties are reported as `modulus weighted` properties (e.g. E.A) and can be normalized to the reference material by dividing by that elastic modulus:

```
A_65 = section.get_ea() / precast.elastic_modulus
```

The reported section centroids are already weighted.

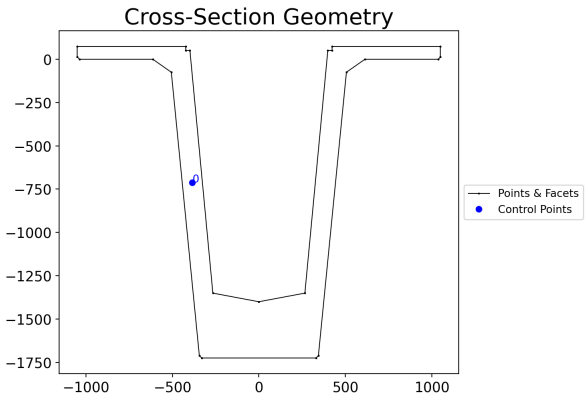

Fig. 48: Super Tee Girder.

4.4.2 I Girder Section

sectionproperties.pre.library.bridge_sections.**i_girder_section**(*girder_type: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a precast I girder section to AS5100.5.

Parameters

- **girder_type** (*int*) – Type of I Girder (1 to 4)
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

As an example, replicate the table shown in AS5100.5 Fig. D1(A):

```
import pandas as pd
import sectionproperties.pre.library.bridge_sections as bridge_sections
from sectionproperties.analysis.section import Section

df = pd.DataFrame(columns=["Ag", "Zt", "Zb", "I", "dy", "th"])

for i in range(4):
    geom = bridge_sections.i_girder_section(girder_type=i + 1)
    dims = bridge_sections.get_i_girder_dims(girder_type=i + 1)
    d = sum(dims[-5:])
    geom.create_mesh(mesh_sizes=[200])
    geom.plot_geometry()
    sec = Section(geom)
    sec.plot_mesh()
    sec.calculate_geometric_properties()
    sec.calculate_warping_properties()

    A = sec.get_area()
```

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```
    th = A / (sec.get_perimeter() / 2)

    df.loc[i] = [
        A,
        *(sec.get_z()[:2]),
        sec.get_ic()[0],
        d + sec.get_c()[1],
        th,
    ]

print(df)
```

Note that the section depth is obtained by summing the heights from the section dictionary in `get_i_girder_dims()`.

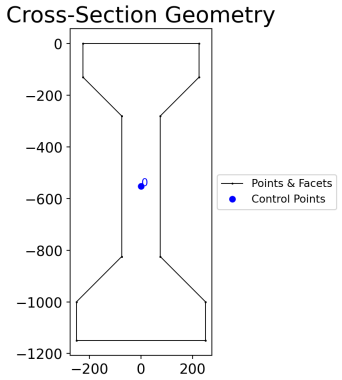


Fig. 49: I Girder.

4.5 Nastran Sections Library

See *nastran_sections Module*.

CHAPTER

FIVE

ADVANCED GEOMETRY CREATION

The below tutorial was created to demonstrate the creation of valid geometries for section analysis by combining multiple shapes.

Some key points to remember:

1. Geometries of two *different* materials should not overlap (can create unpredictable results)
2. If two geometries of the *same* materials are overlapping, then you should perform a union on the two sections
3. Two different section geometries that share a common edge (facet) should also share the same nodes (do not leave "floating" nodes along common edges)

These are general points to remember for any finite element analysis.

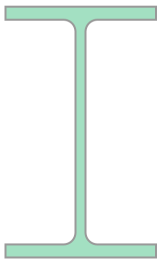
Note: *sectionproperties* will not prevent the creation of these ambiguous sections. The flexibility of the new pre-processing engine (shapely) allows for a wide variety of intermediate modelling steps but the user must ensure that the final model is one that is appropriate for analysis.

5.1 Creating Merged Sections

For this example, we will create a custom section out of two similar "I" sections:

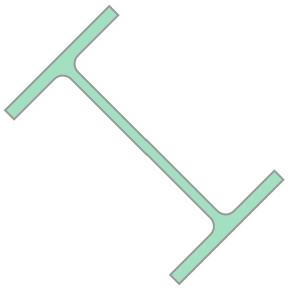
```
import sectionproperties.pre.library.steel_sections as steel_sections
import sectionproperties.analysis.section as cross_section

i_sec1 = steel_sections.i_section(d=250, b=150, t_f=13, t_w=10, r=12, n_r=12)
i_sec2 = i_sec1.rotate_section(45)
```

```
sectionproperties.pre.sections.Geometry
object at: 0x1612aa2c488
Material: default
```



Assign a unique material to each geometry:

```
sectionproperties.pre.sections.Geometry
object at: 0x1612dbc4448
Material: default
```

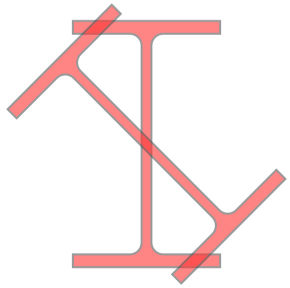


```
from sectionproperties.pre.pre import Material

mat1 = Material("Material_1", 200e3, 0.3, 100, 400, "red")
mat2 = Material("Material_2", 150e3, 0.2, 100, 200, "blue")  # Just some differing␣
↪properties

i_sec1.material = mat1
i_sec2.material = mat2
```

Now, we can use the + operator to naively combine these two sections into a *CompoundGeometry*. Note, the two different materials:

```
i_sec1 + i_sec2
```

```
sectionproperties.pre.sections.CompoundGeometry
object at: 0x1612dac0b88
Materials incl.: ['Material_2', 'Material_1']
```

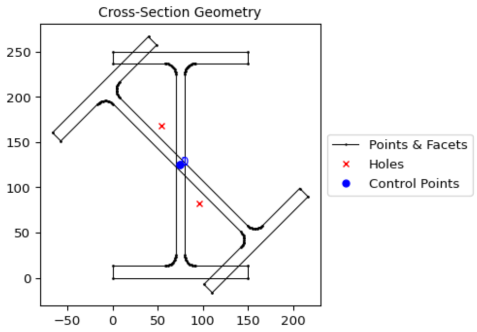


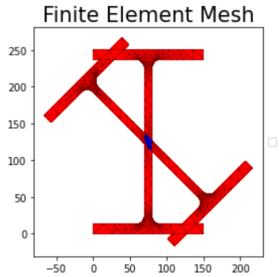
When we plot the geometry, we will see that even though we have two materials, we only have one control point for both geometries:

```
(i_sec1 + i_sec2).plot_geometry()
```

If we went a few steps further by creating a mesh and then plotting that mesh as part of an analysis section, we would see the unpredictable result of the mesh:

```
cross_section.Section((i_sec1 + i_sec2).create_mesh([10])).plot_mesh()
```

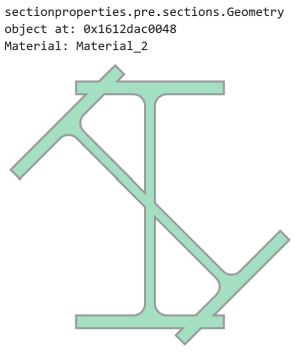




5.2 Preventing Ambiguity

To prevent ambiguity between geometries and their analytical regions, there are a few options we can take. We can perform a simple union operation but that will lose the material information for one of our sections: whichever section comes first in the operation will have it's information preserved. In this example, we will use | (union) with `i_sec2` taking precedence by being the first object in the operation:

```
i_sec2 | i_sec1
```



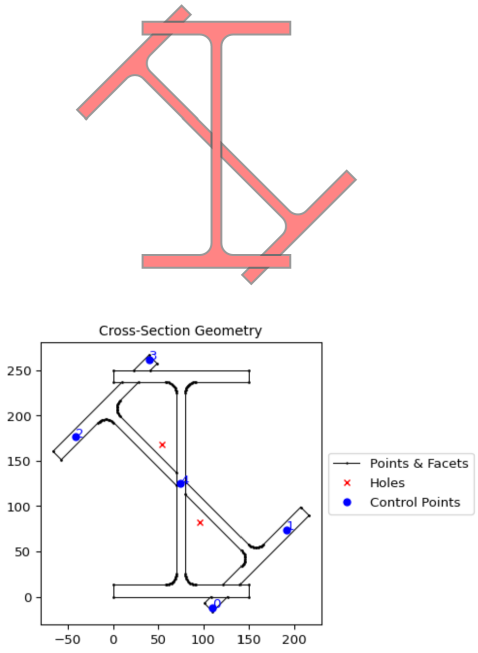
However, this is unsatisfactory as a solution. We want this section to more aptly represent a real section that might be created by cutting and welding two sections together.

Lets say we want the upright "I" section to be our main section and the diagonal section will be added on to it.

It is sometimes possible to do this in a quick operation, one which does not create nodes in common at the intersection points. Here, we will simply "slice" `i_sec2` with `i_sec1` and add it to `i_sec1`. This will create "floating nodes" along the common edges of `i_sec2` and `i_sec1` because the nodes are not a part of `i_sec1`:

```
(i_sec2 - i_sec1) + i_sec1
```

```
sectionproperties.pre.sections.CompoundGeometry
object at: 0x1612e2b0248
Materials incl.: ['Material_1', 'Material_2']
```



Sometimes, we can get away with this as in this example. We can see in the plot that there are five distinct regions indicated with five control points.

When we are "unlucky", sometimes gaps can be created (due to floating point errors) where the two sections meet and a proper hole might not be detected, resulting in an incorrect section.

5.3 Creating Nodes in Common

It is best practice to *first* create nodes in common on both sections and *then* combine them. For this, an extra step is required:

```
cut_2_from_1 = (i_sec1 - i_sec2)  # locates intersection nodes
sec_1_nodes_added = cut_2_from_1 | i_sec1

# This can also be done in one line
sec_1_nodes_added = (i_sec1 - i_sec2) | i_sec1
```

Now, when we use `.plot_geometry()`, we can see the additional nodes added to "section 1":

```
sec_1_nodes_added.plot_geometry()
```

At this point, we can use our "section 1 with additional nodes" to create our complete geometry:

```
analysis_geom = (i_sec2 - i_sec1) + sec_1_nodes_added
analysis_geom.plot_geometry()
```

And when we create our mesh and analysis section:

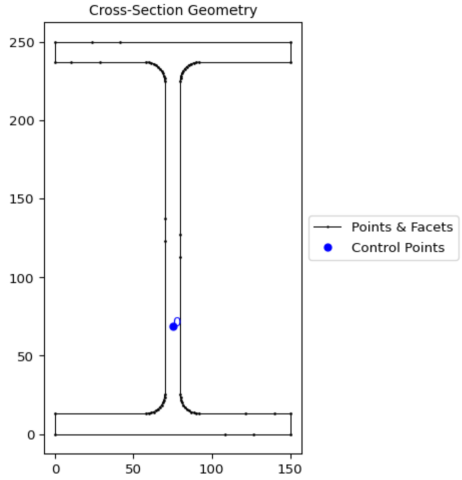
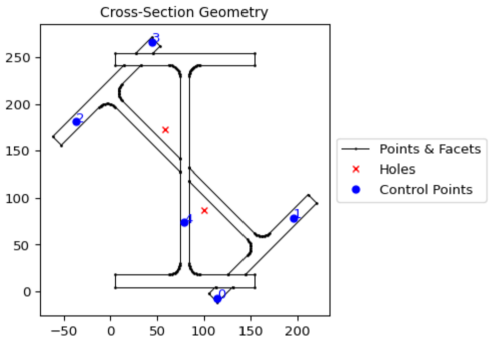
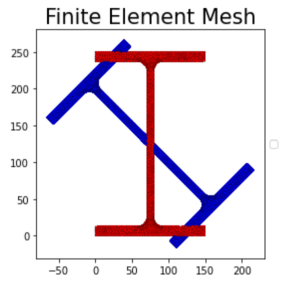


Fig. 1: The additional nodes from the cut portion are now merged as part of the "section 1" geometry.



```
analysis_geom.create_mesh([10])
analysis_sec = cross_section.Section(analysis_geom)
analysis_sec.plot_mesh()
```



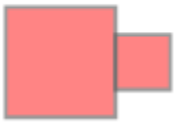
We can see that the mesh represents how we expect the section to be.

5.4 Another example

Here, we will simply combine two squares with the default material:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.analysis.section import Section

s1 = primitive_sections.rectangular_section(1,1)
s2 = primitive_sections.rectangular_section(0.5,0.5).shift_section(1,0.25)
geometry = s1 + s2
geometry
```

```
sectionproperties.pre.sections.CompoundGeometry
object at: 0x1d34df86b80
Materials incl.: ['default']
```



From the shapely vector representation, we can see that the squares are shaded red. This indicates an "invalid" geometry from shapely's perspective because there are two polygons that share an edge. For this geometry, the intention is to have two squares that are connected on one side and so the red shading provided by the shapely representation tells us that we are getting what we expect.

Now, say this is not our final geometry and we actually want to have it rotated by 30 degrees:

```
geometry = geometry.rotate_section(30)
```

Here, we can see that the shapely representation is now showing as green indicating a "valid" shapely geometry. Even though it is now valid for shapely, because it is green we know that these two polygons no longer share an edge because there is a miniscule separation between them as a result of a floating point error.

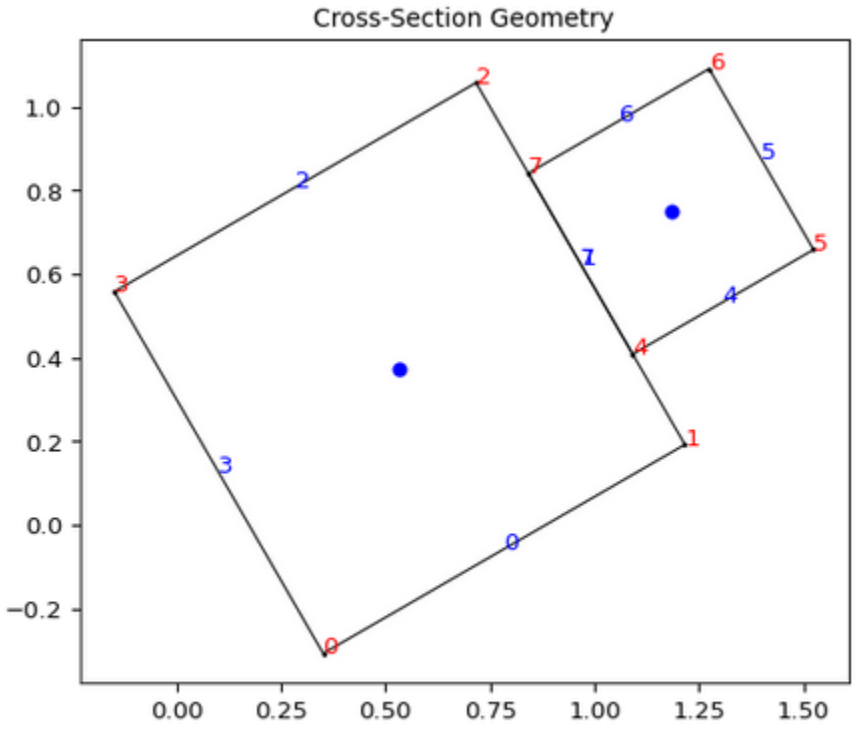
When we try to mesh this geometry, we will actually cause a crash with triangle, the meshing tool used behind-the-scenes by sectionproperties:

```
sectionproperties.pre.sections.CompoundGeometry
object at: 0x23ce9966a00
Materials incl.: ['default']
```

```
geometry.create_mesh(mesh_sizes=[0.2, 0.1]) # This may crash the kernel
```

The crash occurs because the distance between the two polygons is so small, even though they are separated and the space between them will not be meshed. The same crash would occur if the polygons were overlapping by this same small distance.

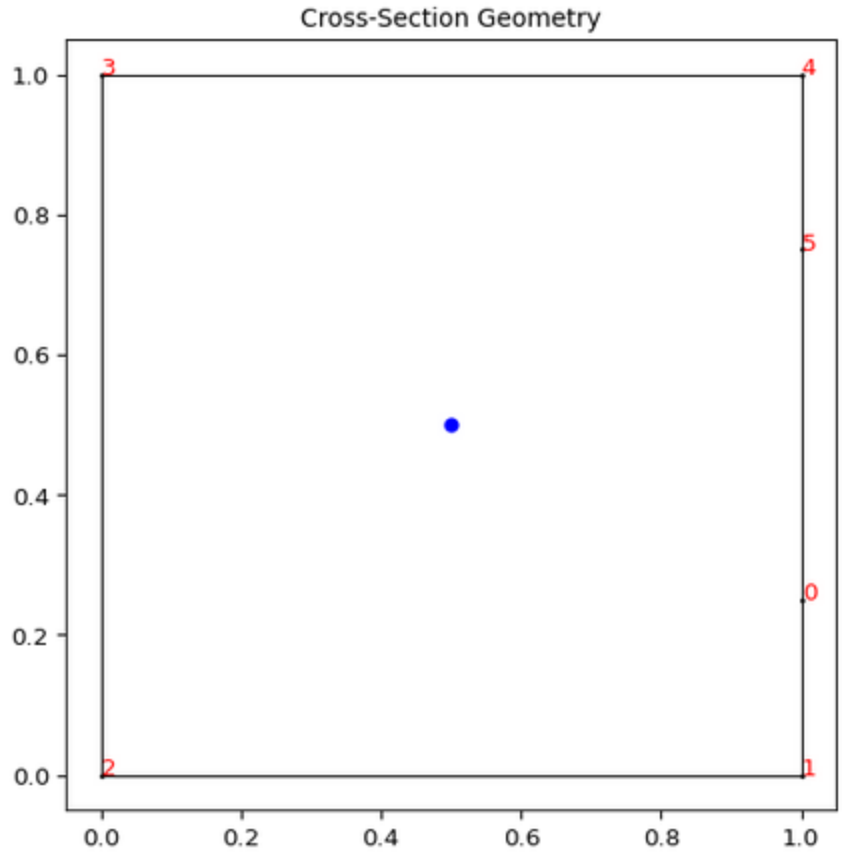
If we plot the geometry, you can see that each of the two squares has only four nodes and four facets and their relationship is only incidental. If their edges happen to perfectly align, they will be considered as one continuous section. If their edges do not perfectly align, they will be considered as discontinuous.




```
(<Figure size 500x500 with 1 Axes>,
 <AxesSubplot:title={'center':'Cross-Section Geometry'}>)
```

To remedy this, take the same approach as in the preceding example by creating intermediate nodes where the two polygons intersect by using set operations. If we subtract `s2` from `s1` then we will have the larger square with intermediate nodes created:

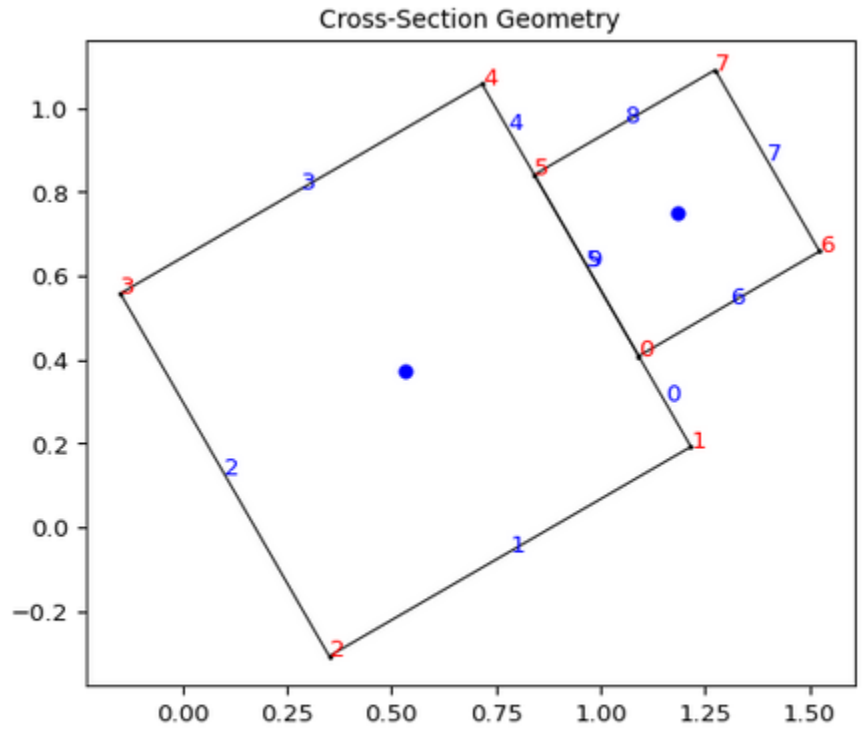
```
(s1 - s2).plot_geometry(labels=['points'])
```




```
(<Figure size 500x500 with 1 Axes>,
 <AxesSubplot:title={'center':'Cross-Section Geometry'}>)
```

Now, if we build the compound geometry up from this larger square with the intermediate points, then our section will work.:

```
geometry_fixed = (s1 - s2) + s2
geometry_fixed_rotated = geometry_fixed.rotate_section(angle=30)
geometry_rot.create_mesh(mesh_sizes=[0.2, 0.1])
geometry_rot.plot_geometry(labels=["points", "facets"])
section = Section(geometry_rot)
section.display_mesh_info()
```




```
Mesh Statistics:
--52 nodes
--19 elements
--2 regions
```

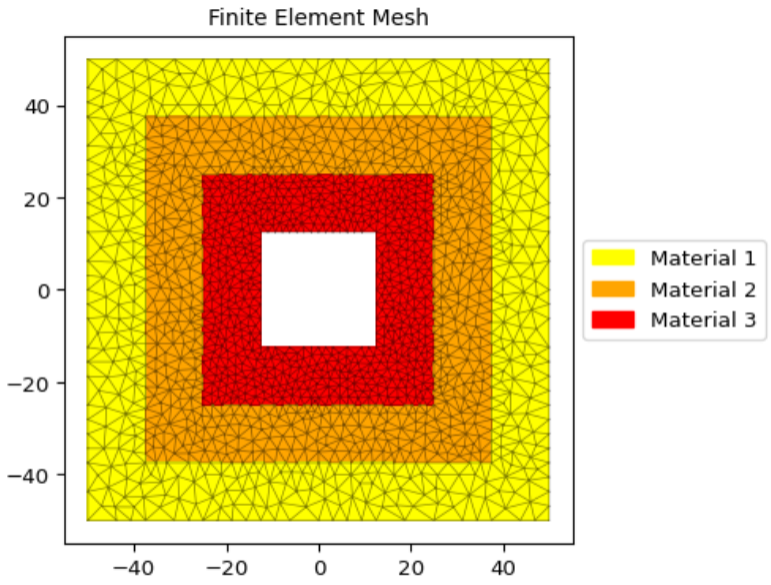
5.5 Another example (but with nested geometries)

This example demonstrates creating nested geometries using two different approaches. These approaches reflect the differences between how shapely (geometry pre-processor) "perceives" geometry and how Triangle (meshing tool) "perceives" geometry and how the modeller might adapt their input style depending on the situation.

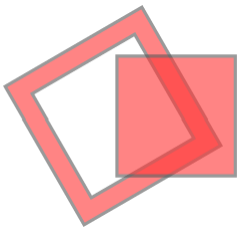
The nested geometry we are trying to create looks as follows:

In creating this geometry consider the following:

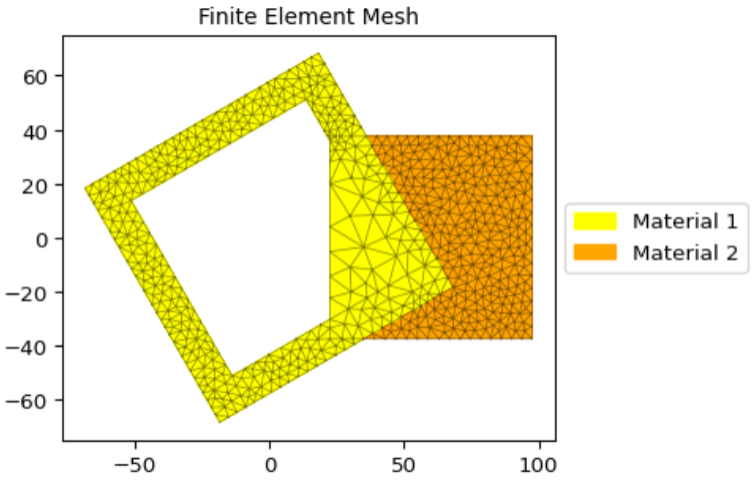
- shapely has a concept of "z-ordering" where it is possible for one geometry to be "over" another geometry and for an overlap section to exist. When a hole is created in a polygon, it is only local to that polygon.
- Triangle does not have a concept of "z-ordering" so there is only a single plane which may have regions of different materials (specified with control points). When a hole is created in the plane, it "punches" through




```
(<Figure size 500x500 with 1 Axes>,
 <AxesSubplot:title={'center':'Finite Element Mesh'}>)
```



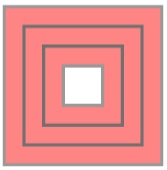
"all" polygons in the plane.



To create the nested geometry using shapely, the code would be as follows:

```
mat1 = Material(name="Material 1", elastic_modulus=100, poissons_ratio=0.3, yield_
↪strength=10, density=1e-6, color="yellow")
mat2 = Material(name="Material 2", elastic_modulus=100, poissons_ratio=0.3, yield_
↪strength=10, density=1e-6, color="orange")
mat3 = Material(name="Material 3", elastic_modulus=100, poissons_ratio=0.3, yield_
↪strength=10, density=1e-6, color="red")

sq1 = sections.rectangular_section(100, 100, material=mat1).align_center()
sq2 = sections.rectangular_section(75, 75, material=mat2).align_center()
sq3 = sections.rectangular_section(50, 50, material=mat3).align_center()
hole = sections.rectangular_section(25, 25).align_center()

compound = (
    (sq1 - sq2) # Create a big square with a medium hole in it and stack it over...
    + (sq2 - sq3) # ... a medium square with a medium-small hole in it and stack it over.
↪..
    + (sq3 - hole) # ...a medium-small square with a small hole in it.
)
compound
```

```
sectionproperties.pre.sections.CompoundGeometry
object at: 0x18628246b80
Materials incl.: ['Material 3', 'Material 1', 'Material 2']
```



To create the nested geometry using the Triangle interface, the code would be as follows:

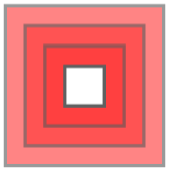
```
points = [ # Points for four squares are created
    [-50.0, 50.0], # Square 1
```

(continues on next page)

(continued from previous page)

```
    [50.0, 50.0],
    [50.0, -50.0],
    [-50.0, -50.0],
    [37.5, -37.5], # Square 2
    [37.5, 37.5],
    [-37.5, 37.5],
    [-37.5, -37.5],
    [25.0, -25.0], # Square 3
    [25.0, 25.0],
    [-25.0, 25.0],
    [-25.0, -25.0],
    [12.5, -12.5], # Square 4 (hole)
    [12.5, 12.5],
    [-12.5, 12.5],
    [-12.5, -12.5],
]

facets = [ # Facets trace each of the four squares
    [0, 1], # Square 1
    [1, 2],
    [2, 3],
    [3, 0],
    [4, 5], # Square 2
    [5, 6],
    [6, 7],
    [7, 4],
    [8, 9], # Square 3
    [9, 10],
    [10, 11],
    [11, 8],
    [12, 13], # Square 4 (hole)
    [13, 14],
    [14, 15],
    [15, 12],
]

control_points = [[-43.75, 0.0], [-31.25, 0.0], [-18.75, 0.0]] # Three squares
holes = [[0, 0]]

nested_compound = CompoundGeometry.from_points(
    points=points, facets=facets, control_points=control_points, holes=holes
)
nested_compound
```

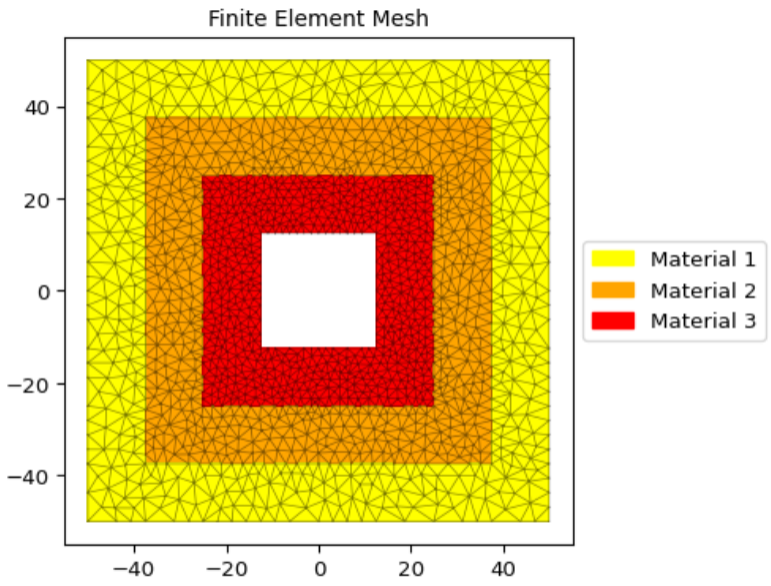
```
sectionproperties.pre.sections.CompoundGeometry
object at: 0x18626c77a90
Materials incl.: ['default']
```



Notice how the shapely representation shows the squares overlapping each other instead of the squares fitting into the "hole below".

Is one of these methods better than the other? Not necessarily. The shapely approach is suitable for manually creating the geometry whereas the Triangle approach is suitable for reading in serialized data from a file, for example.

And, for either case, when the compound geometry is meshed, we see this:




```
(<Figure size 500x500 with 1 Axes>,
 <AxesSubplot:title={'center':'Finite Element Mesh'}>)
```

CHAPTER

SIX

RUNNING AN ANALYSIS

The first step in running a section analysis is the creation of a *Section* object. This class stores the structural geometry and finite element mesh and provides methods to perform various types of sectional analyses.

class sectionproperties.analysis.section.**Section**(*geometry: Union[*Geometry, CompoundGeometry*]*, *time_info: bool = False*)

Class for structural cross-sections.

Stores the finite element geometry, mesh and material information and provides methods to compute the cross-section properties. The element type used in this program is the six-noded quadratic triangular element.

The constructor extracts information from the provided mesh object and creates and stores the corresponding Tri6 finite element objects.

Parameters

- **geometry** (*Geometry*) – Cross-section geometry object used to generate the mesh
- **time_info** (*bool*) – If set to True, a detailed description of the computation and the time cost is printed to the terminal for every computation performed.

The following example creates a *Section* object of a 100D x 50W rectangle using a mesh size of 5:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.analysis.section import Section

geometry = primitive_sections.rectangular_section(d=100, b=50)
geometry.create_mesh(mesh_sizes=[5])
section = Section(geometry)
```

Variables

- **elements** (list[*Tri6*]) – List of finite element objects describing the cross-section mesh
- **num_nodes** (*int*) – Number of nodes in the finite element mesh
- **geometry** (*Geometry*) – Cross-section geometry object used to generate the mesh
- **mesh** (*dict(mesh)*) – Mesh dict returned by triangle
- **mesh_nodes** (numpy.ndarray) – Array of node coordinates from the mesh
- **mesh_elements** (numpy.ndarray) – Array of connectivities from the mesh
- **mesh_attributes** (numpy.ndarray) – Array of attributes from the mesh
- **materials** – List of materials
- **material_groups** – List of objects containing the elements in each defined material

- **section_props** (*SectionProperties*) – Class to store calculated section properties

Raises

- **AssertionError** – If the number of materials does not equal the number of regions
- **ValueError** – If geometry does not contain a mesh

6.1 Checking the Mesh Quality

Before carrying out a section analysis it is a good idea to check the quality of the finite element mesh. Some useful methods are provided to display mesh statistics and to plot the finite element mesh:

Section.**display_mesh_info**()

Prints mesh statistics (number of nodes, elements and regions) to the command window.

The following example displays the mesh statistics for a Tee section merged from two rectangles:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.analysis.section import Section

rec1 = primitive_sections.rectangular_section(d=100, b=25)
rec2 = primitive_sections.rectangular_section(d=25, b=100)
rec1 = rec1.shift_section(x_offset=-12.5)
rec2 = rec2.shift_section(x_offset=-50, y_offset=100)

geometry = rec1 + rec2
geometry.create_mesh(mesh_sizes=[5, 2.5])
section = Section(geometry)
section.display_mesh_info()

>>>Mesh Statistics:
>>>--4920 nodes
>>>--2365 elements
>>>--2 regions
```

Section.**plot_mesh**(*alpha=0.5*, *materials=True*, *mask=None*, *title='Finite Element Mesh'*, ***kwargs*)

Plots the finite element mesh.

Parameters

- **alpha** (*float*) – Transparency of the mesh outlines: $0 \leq \alpha \leq 1$
- **materials** (*bool*) – If set to true shades the elements with the specified material colours
- **mask** (*list[bool]*) – Mask array, of length num_nodes, to mask out triangles
- **title** (*string*) – Plot title
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the mesh generated for the second example listed under the *Section* object definition:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.pre.pre import Material
from sectionproperties.analysis.section import Section

steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, density=7.85e-6,
    yield_strength=250, color='grey'
)
timber = Material(
    name='Timber', elastic_modulus=8e3, poissons_ratio=0.35, density=6.5e-7,
    yield_strength=20, color='burlywood'
)

geom_steel = primitive_sections.rectangular_section(d=50, b=50, material=steel)
geom_timber = primitive_sections.rectangular_section(d=50, b=50, material=timber)
geometry = geom_timber.align_to(geom_steel, on="right") + geom_steel

geometry.create_mesh(mesh_sizes=[10, 5])

section = Section(geometry)
section.plot_mesh(materials=True, alpha=0.5)
```

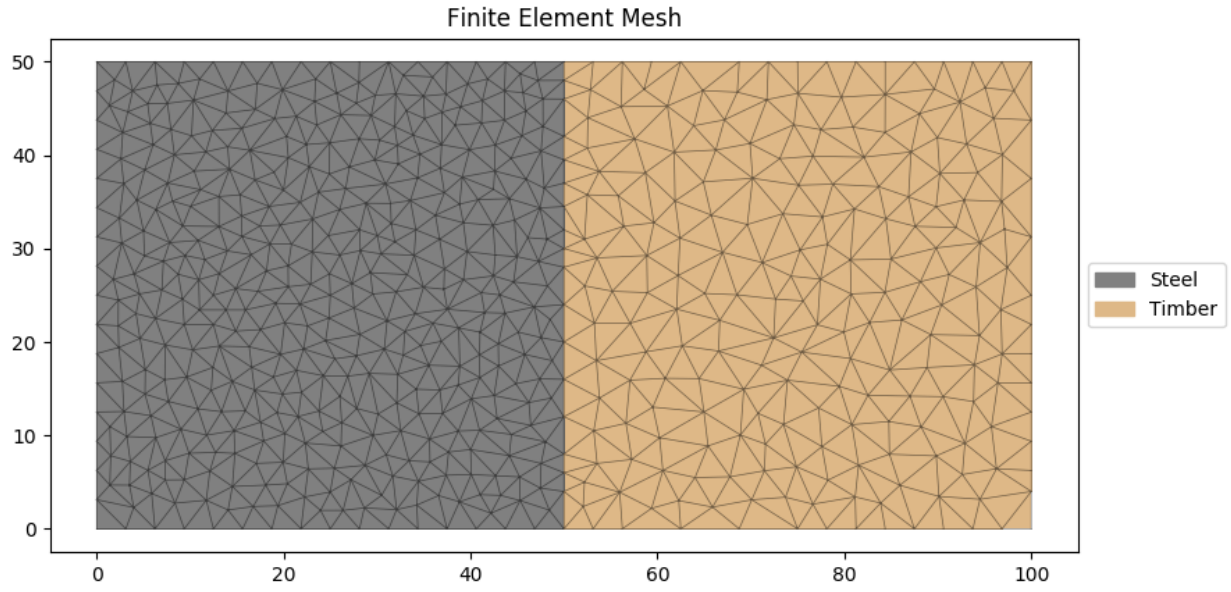


Fig. 1: Finite element mesh generated by the above example.

6.2 Geometric Analysis

A geometric analysis calculates the area properties of the section.

`Section.`**`calculate_geometric_properties`**`()`

Calculates the geometric properties of the cross-section and stores them in the *SectionProperties* object contained in the `section_props` class variable.

The following geometric section properties are calculated:

- Cross-sectional area
- Cross-sectional perimeter
- Cross-sectional mass
- Area weighted material properties, composite only E_{eff}, G_{eff}, nu_{eff}
- Modulus weighted area (axial rigidity)
- First moments of area
- Second moments of area about the global axis
- Second moments of area about the centroidal axis
- Elastic centroid
- Centroidal section moduli
- Radii of gyration
- Principal axis properties

If materials are specified for the cross-section, the moments of area and section moduli are elastic modulus weighted.

The following example demonstrates the use of this method:

```
section = Section(geometry)
section.calculate_geometric_properties()
```

6.3 Plastic Analysis

A plastic analysis calculates the plastic properties of the section.

Note: A geometric analysis must be performed on the Section object before a plastic analysis is carried out.

Warning: The plastic analysis in *sectionproperties* assumes all materials are able to reach their yield stress defined in the material properties. Care should be taken if analysing materials or cross-sections exhibiting non-linear behaviour, e.g. reinforced concrete or non-compact steel sections.

`Section.`**`calculate_plastic_properties`**`(`*verbose=False*`)`

Calculates the plastic properties of the cross-section and stores them in the *SectionProperties* object contained in the `section_props` class variable.

Parameters
verbose (*bool*) – If set to True, the number of iterations required for each plastic axis is printed to the terminal.

The following warping section properties are calculated:

- Plastic centroid for bending about the centroidal and principal axes
- Plastic section moduli for bending about the centroidal and principal axes
- Shape factors for bending about the centroidal and principal axes

If materials are specified for the cross-section, the plastic section moduli are displayed as plastic moments (i.e $M_p = f_y S$) and the shape factors are not calculated.

Note that the geometric properties must be calculated before the plastic properties are calculated:

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
```

Raises
RuntimeError – If the geometric properties have not been calculated prior to calling this method

6.4 Warping Analysis

A warping analysis calculates the torsion and shear properties of the section.

Note: A geometric analysis must be performed on the Section object before a warping analysis is carried out.

Warning: There must be connectivity between all elements of the mesh to perform a valid warping analysis. This is a limitation of the elastic theory that this implementation is based on, as there is no way to quantify the transfer of shear and warping between two unconnected regions.

Section.**calculate_warping_properties**(*solver_type='direct'*)

Calculates all the warping properties of the cross-section and stores them in the *SectionProperties* object contained in the `section_props` class variable.

Parameters
solver_type (*string*) – Solver used for solving systems of linear equations, either using the *'direct'* method or *'cgs'* iterative method

The following warping section properties are calculated:

- Torsion constant
- Shear centre
- Shear area
- Warping constant
- Monosymmetry constant

If materials are specified, the values calculated for the torsion constant, warping constant and shear area are elastic modulus weighted.

Note that the geometric properties must be calculated prior to the calculation of the warping properties:

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
```

Raises

RuntimeError – If the geometric properties have not been calculated prior to calling this method

6.5 Stress Analysis

A stress analysis calculates the section stresses arising from a set of forces and moments. Executing this method returns a *StressResult* object which stores the section stresses and provides stress plotting functions.

Note: A geometric analysis must be performed on the Section object before a stress analysis is carried out. Further, if the shear force or twisting moment is non-zero a warping analysis must also be performed.

Warning: The stress analysis in *sectionproperties* is linear-elastic and does not account for the yielding of materials or any non-linearities.

Section.**calculate_stress**(*N=0*, *Vx=0*, *Vy=0*, *Mxx=0*, *Myy=0*, *M11=0*, *M22=0*, *Mzz=0*)

Calculates the cross-section stress resulting from design actions and returns a *StressPost* object allowing post-processing of the stress results.

Parameters

- **N** (*float*) – Axial force
- **Vx** (*float*) – Shear force acting in the x-direction
- **Vy** (*float*) – Shear force acting in the y-direction
- **Mxx** (*float*) – Bending moment about the centroidal xx-axis
- **Myy** (*float*) – Bending moment about the centroidal yy-axis
- **M11** (*float*) – Bending moment about the centroidal 11-axis
- **M22** (*float*) – Bending moment about the centroidal 22-axis
- **Mzz** (*float*) – Torsion moment about the centroidal zz-axis

Returns

Object for post-processing cross-section stresses

Return type

StressPost

Note that a geometric analysis must be performed prior to performing a stress analysis. Further, if the shear force or torsion is non-zero a warping analysis must also be performed:

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(N=1e3, Vy=3e3, Mxx=1e6)
```

Raises

RuntimeError – If a geometric and warping analysis (if required) have not been performed prior to calling this method

6.6 Calculating Frame Properties

Calculates the section properties required for a 2D or 3D frame analysis.

Note: This method is significantly faster than performing a geometric and a warping analysis and has no prerequisites.

`Section.`**`calculate_frame_properties`**`(`*solver_type='direct'*`)`

Calculates and returns the properties required for a frame analysis. The properties are also stored in the *SectionProperties* object contained in the `section_props` class variable.

Parameters

solver_type (*string*) – Solver used for solving systems of linear equations, either using the *'direct'* method or *'cgs'* iterative method

Returns

Cross-section properties to be used for a frame analysis *(area, ixx, iyy, ixy, j, phi)*

Return type

tuple(float, float, float, float, float, float)

The following section properties are calculated:

- Cross-sectional area *(area)*
- Second moments of area about the centroidal axis *(ixx, iyy, ixy)*
- Torsion constant *(j)*
- Principal axis angle *(phi)*

If materials are specified for the cross-section, the area, second moments of area and torsion constant are elastic modulus weighted.

The following example demonstrates the use of this method:

```
section = Section(geometry)
(area, ixx, iyy, ixy, j, phi) = section.calculate_frame_properties()
```

CHAPTER

SEVEN

VIEWING THE RESULTS

7.1 Printing a List of the Section Properties

A list of section properties that have been calculated by various analyses can be printed to the terminal using the *display_results()* method that belongs to every *Section* object.

Section.**display_results**(*fmt='8.6e'*)

Prints the results that have been calculated to the terminal.

Parameters

fmt (*string*) – Number formatting string

The following example displays the geometric section properties for a 100D x 50W rectangle with three digits after the decimal point:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.analysis.section import Section

geometry = primitive_sections.rectangular_section(d=100, b=50)
geometry.create_mesh(mesh_sizes=[5])

section = Section(geometry)
section.calculate_geometric_properties()

section.display_results(fmt='.3f')
```

7.2 Getting Specific Section Properties

Alternatively, there are a number of methods that can be called on the *Section* object to return a specific section property:

7.2.1 Section Area

Section.**get_area**()

Returns
Cross-section area

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
area = section.get_area()
```

7.2.2 Section Perimeter

Section.**get_perimeter**()

Returns
Cross-section perimeter

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
perimeter = section.get_perimeter()
```

7.2.3 Section Mass

Section.**get_mass**()

Returns
Cross-section mass

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
perimeter = section.get_mass()
```

7.2.4 Axial Rigidity

If material properties have been specified, returns the axial rigidity of the section.

Section.**get_ea**()

Returns
Modulus weighted area (axial rigidity)

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
ea = section.get_ea()
```

7.2.5 First Moments of Area

Section.**get_q**()

Returns
First moments of area about the global axis *(qx, qy)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(qx, qy) = section.get_q()
```

7.2.6 Second Moments of Area

Section.**get_ig**()

Returns
Second moments of area about the global axis *(ixx_g, iyy_g, ixy_g)*

Return type
tuple(float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(ixx_g, iyy_g, ixy_g) = section.get_ig()
```

Section.**get_ic**()

Returns
Second moments of area centroidal axis *(ixx_c, iyy_c, ixy_c)*

Return type
tuple(float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(ixx_c, iyy_c, ixy_c) = section.get_ic()
```

Section.**get_ip**()

Returns
Second moments of area about the principal axis *(i11_c, i22_c)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(i11_c, i22_c) = section.get_ip()
```

7.2.7 Elastic Centroid

Section.**get_c**()

Returns
Elastic centroid *(cx, cy)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(cx, cy) = section.get_c()
```

7.2.8 Section Moduli

Section.**get_z**()

Returns
Elastic section moduli about the centroidal axis with respect to the top and bottom fibres *(zxx_plus, zxx_minus, zyy_plus, zyy_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(zxx_plus, zxx_minus, zyy_plus, zyy_minus) = section.get_z()
```

Section.**get_zp**()

Returns
Elastic section moduli about the principal axis with respect to the top and bottom fibres *(z11_plus, z11_minus, z22_plus, z22_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(z11_plus, z11_minus, z22_plus, z22_minus) = section.get_zp()
```

7.2.9 Radii of Gyration

Section.**get_rc**()

Returns
Radii of gyration about the centroidal axis *(rx, ry)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(rx, ry) = section.get_rc()
```

Section.**get_rp**()

Returns
Radii of gyration about the principal axis *(r11, r22)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(r11, r22) = section.get_rp()
```

7.2.10 Principal Axis Angle

Section.**get_phi**()

Returns
Principal bending axis angle

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
phi = section.get_phi()
```

7.2.11 Effective Material Properties

Section.**get_e_eff**()

Returns
Effective elastic modulus based on area

Return type
float

```
section = Section(geometry)
section.calculate_warping_properties()
e_eff = section.get_e_eff()
```

Section.**get_g_eff**()

Returns
Effective shear modulus based on area

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
g_eff = section.get_g_eff()
```

Section.**get_nu_eff**()

Returns
Effective Poisson's ratio

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
nu_eff = section.get_nu_eff()
```

7.2.12 Torsion Constant

Section.**get_j**()

Returns
St. Venant torsion constant

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
j = section.get_j()
```

7.2.13 Shear Centre

Section.**get_sc**()

Returns
Centroidal axis shear centre (elasticity approach) *(x_se, y_se)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(x_se, y_se) = section.get_sc()
```

Section.**get_sc_p**()

Returns
Principal axis shear centre (elasticity approach) *(x11_se, y22_se)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(x11_se, y22_se) = section.get_sc_p()
```

7.2.14 Trefftz's Shear Centre

Section.**get_sc_t**()

Returns
Centroidal axis shear centre (Trefftz's approach) *(x_st, y_st)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(x_st, y_st) = section.get_sc_t()
```

7.2.15 Warping Constant

Section.**get_gamma**()

Returns
Warping constant

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
gamma = section.get_gamma()
```

7.2.16 Shear Area

Section.**get_As**()

Returns
Shear area for loading about the centroidal axis *(A_sx, A_sy)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(A_sx, A_sy) = section.get_As()
```

Section.**get_As_p**()

Returns
Shear area for loading about the principal bending axis *(A_s11, A_s22)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(A_s11, A_s22) = section.get_As_p()
```

7.2.17 Monosymmetry Constants

Section.**get_beta**()

Returns
Monosymmetry constant for bending about both global axes *(beta_x_plus, beta_x_minus, beta_y_plus, beta_y_minus)*. The *plus* value relates to the top flange in compression and the *minus* value relates to the bottom flange in compression.

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(beta_x_plus, beta_x_minus, beta_y_plus, beta_y_minus) = section.get_beta()
```

Section.**get_beta_p**()

Returns
Monosymmetry constant for bending about both principal axes *(beta_11_plus, beta_11_minus, beta_22_plus, beta_22_minus)*. The *plus* value relates to the top flange in compression and the *minus* value relates to the bottom flange in compression.

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(beta_11_plus, beta_11_minus, beta_22_plus, beta_22_minus) = section.get_beta_p()
```

7.2.18 Plastic Centroid

Section.**get_pc**()

Returns
Centroidal axis plastic centroid *(x_pc, y_pc)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(x_pc, y_pc) = section.get_pc()
```

Section.**get_pc_p**()

Returns
Principal bending axis plastic centroid *(x11_pc, y22_pc)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(x11_pc, y22_pc) = section.get_pc_p()
```

7.2.19 Plastic Section Moduli

Section.**get_s**()

Returns
Plastic section moduli about the centroidal axis *(sxx, syy)*

Return type
tuple(float, float)

If material properties have been specified, returns the plastic moment $M_p = f_y S$.

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(sxx, syy) = section.get_s()
```

Section.**get_sp**()

Returns
Plastic section moduli about the principal bending axis *(s11, s22)*

Return type
tuple(float, float)

If material properties have been specified, returns the plastic moment $M_p = f_y S$.

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(s11, s22) = section.get_sp()
```

7.2.20 Shape Factors

Section.**get_sf**()

Returns
Centroidal axis shape factors with respect to the top and bottom fibres *(sf_xx_plus, sf_xx_minus, sf_yy_plus, sf_yy_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(sf_xx_plus, sf_xx_minus, sf_yy_plus, sf_yy_minus) = section.get_sf()
```

Section.**get_sf_p**()

Returns
Principal bending axis shape factors with respect to the top and bottom fibres *(sf_11_plus, sf_11_minus, sf_22_plus, sf_22_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(sf_11_plus, sf_11_minus, sf_22_plus, sf_22_minus) = section.get_sf_p()
```

7.3 How Material Properties Affect Results

If a *Geometry* containing a user defined `Material` is used to build a *Section*, *sectionproperties* will assume you are performing a **composite analysis** and this will affect the way some of the results are stored and presented.

In general, the calculation of gross composite section properties takes into account the elastic modulus, Poisson's ratio and yield strength of each material in the section. Unlike many design codes, *sectionproperties* is 'material property agnostic' and does not transform sections based on a defined material property, e.g. in reinforced concrete analysis it is commonplace to transform the reinforcing steel area based on the ratio between the elastic moduli, $n = E_{steel}/E_{conc}$. *sectionproperties* instead calculates the gross material weighted properties, which is analogous to transforming with respect to a material property with elastic modulus, $E = 1$.

Using the example of a reinforced concrete section, *sectionproperties* will calculate the gross section bending stiffness, $(EI)_g$, rather than an effective concrete second moment of area, $I_{c,eff}$:

$$(EI)_g = E_s \times I_s + E_c \times I_c$$

If the user wanted to obtain the effective concrete second moment of area for a code calculation, they could simply divide the gross bending stiffness by the elastic modulus for concrete:

$$I_{c,eff} = \frac{(EI)_g}{E_c}$$

With reference to the `get` methods described in *Printing a List of the Section Properties*, a **composite analysis** will modify the following properties:

- First moments of area `get_q()` - returns elastic modulus weighted first moments of area $E.Q$
- Second moments of area `get_ig()`, `get_ic()`, `get_ip()` - return elastic modulus weighted second moments of area $E.I$
- Section moduli `get_z()`, `get_zp()` - return elastic modulus weighted section moduli $E.Z$
- Torsion constant `get_j()` - returns elastic modulus weighted torsion constant $E.J$
- Warping constant `get_gamma()` - returns elastic modulus weighted warping constant $E.\Gamma$
- Shear areas `get_As()`, `get_As_p()` - return elastic modulus weighted shear areas $E.A_s$
- Plastic section moduli `get_s()`, `get_sp()` - return yield strength weighted plastic section moduli, i.e. plastic moments $M_p = f_y.S$

A **composite analysis** will also enable the user to retrieve effective gross section area-weighted material properties:

- Effective elastic modulus E_{eff} - `get_e_eff()`
- Effective shear modulus G_{eff} - `get_g_eff()`
- Effective Poisson's ratio ν_{eff} - `get_nu_eff()`

These values may be used to transform composite properties output by *sectionproperties* for practical use, e.g. to calculate torsional rigidity:

$$(GJ)_g = \frac{G_{eff}}{E_{eff}}(EJ)_g$$

For further information, see the theoretical background to the calculation of *Composite Cross-Sections*.

7.4 Section Property Centroids Plots

A plot of the centroids (elastic, plastic and shear centre) can be produced with the finite element mesh in the background:

`Section.`**`plot_centroids`**`(`*title='Centroids', **kwargs*`)`

Plots the elastic centroid, the shear centre, the plastic centroids and the principal axis, if they have been calculated, on top of the finite element mesh.

Parameters

- **`title`** (*`string`*) – Plot title
- **`kwargs`** – Passed to *`plotting_context()`*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example analyses a 200 PFC section and displays a plot of the centroids:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12, n_r=8)
geometry.create_mesh(mesh_sizes=[20])

section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties()

section.plot_centroids()
```

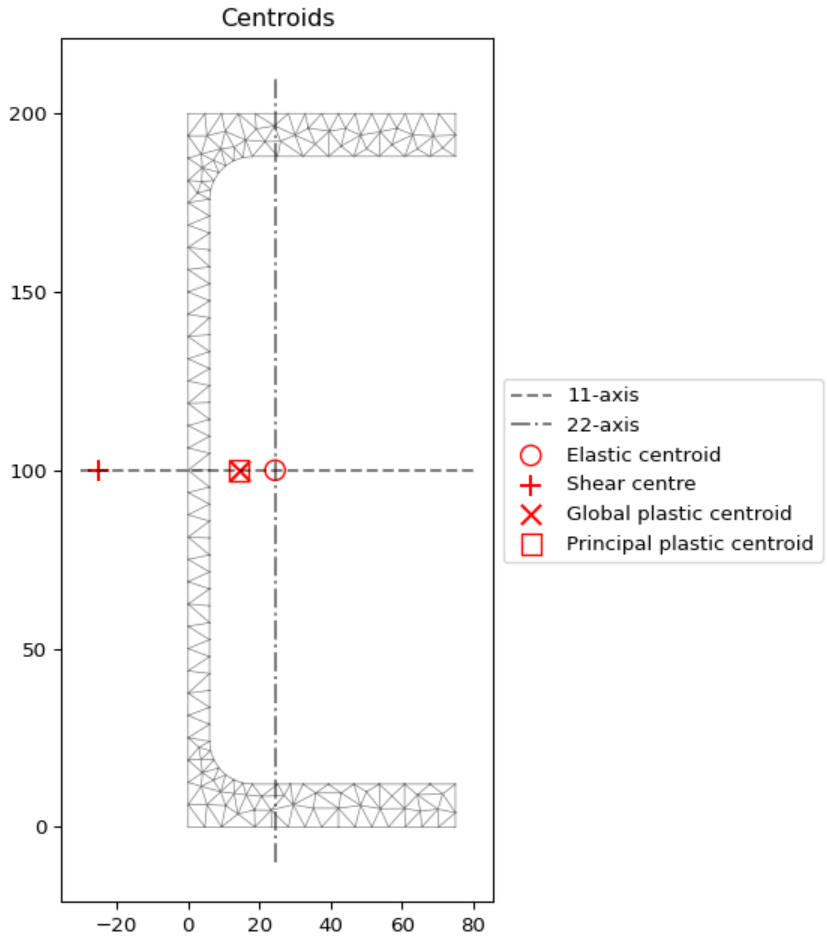


Fig. 1: Plot of the centroids generated by the above example.

The following example analyses a 150x90x12 UA section and displays a plot of the centroids:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
```

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```
geometry.create_mesh(mesh_sizes=[20])

section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties()

section.plot_centroids()
```

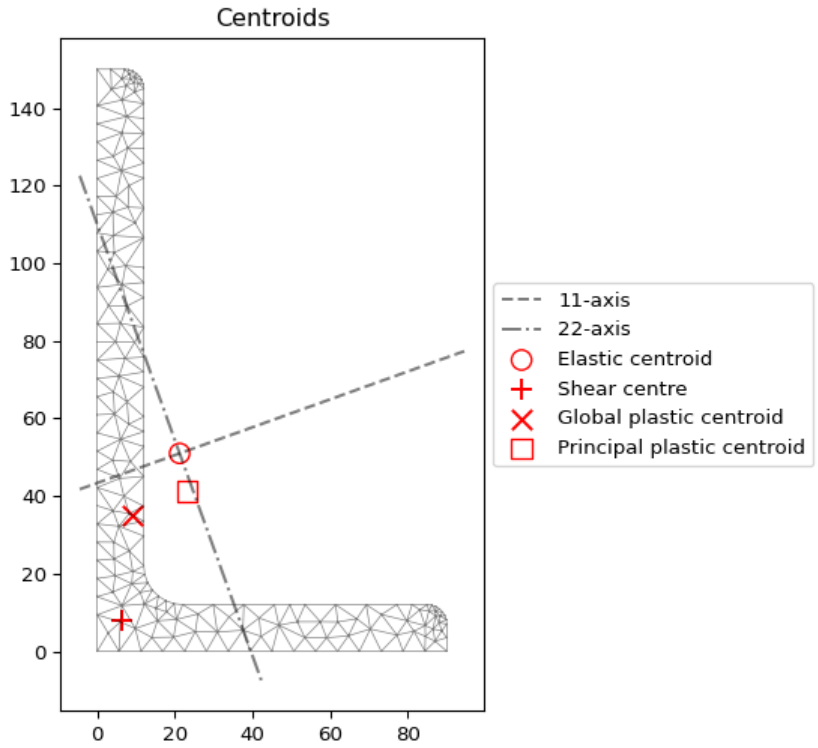

Fig. 2: Plot of the centroids generated by the above example.

7.5 Plotting Section Stresses

There are a number of methods that can be called from a `StressResult` object to plot the various cross-section stresses. These methods take the following form:

`StressResult`.plot_(*stress/vector*)_(*action*)_(*stresstype*)

where:

- *stress* denotes a contour plot and *vector* denotes a vector plot.
- *action* denotes the type of action causing the stress e.g. *mxx* for bending moment about the x-axis. Note that the action is omitted for stresses caused by the application of all actions.
- *stresstype* denotes the type of stress that is being plotted e.g. *zx* for the *x*-component of shear stress.

The examples shown in the methods below are performed on a 150x90x12 UA (unequal angle) section. The *Section* object is created below:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.AngleSection(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
mesh = geometry.create_mesh(mesh_sizes=[2.5])
section = Section(geometry, mesh)
```

7.5.1 Primary Stress Plots

Axial Stress ($\sigma_{zz,N}$)

StressPost.**plot_stress_n_zz**(*title='Stress Contour Plot - $\\sigma_{zz,N}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,N}$ resulting from the axial load N.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from an axial force of 10 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(N=10e3)

stress_post.plot_stress_n_zz()
```

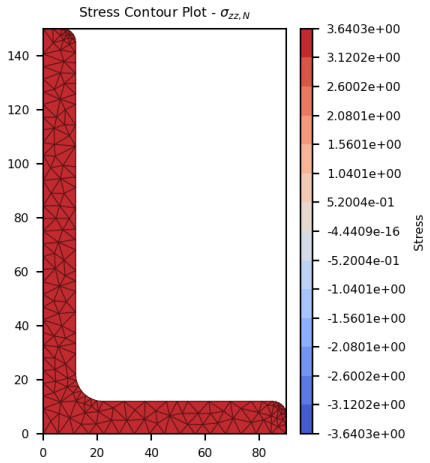

Fig. 3: Contour plot of the axial stress.

Bending Stress ($\sigma_{zz,Mxx}$)

StressPost.**plot_stress_mxx_zz**(*title='Stress Contour Plot - $\\sigma_{zz,Mxx}$', cmap='coolwarm', normalize=True, **kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,Mxx}$ resulting from the bending moment M_{xx}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the x-axis of 5 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mxx=5e6)

stress_post.plot_stress_mxx_zz()
```

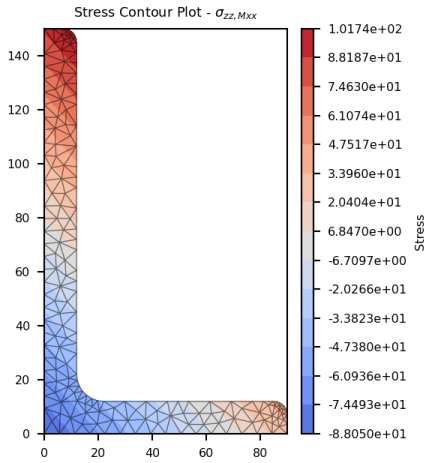

Fig. 4: Contour plot of the bending stress.

Bending Stress ($\sigma_{zz,Myy}$)

StressPost.**plot_stress_myy_zz**(*title='Stress Contour Plot - $\\sigma_{zz,Myy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,Myy}$ resulting from the bending moment M_{yy}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the y-axis of 2 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Myy=2e6)

stress_post.plot_stress_myy_zz()
```

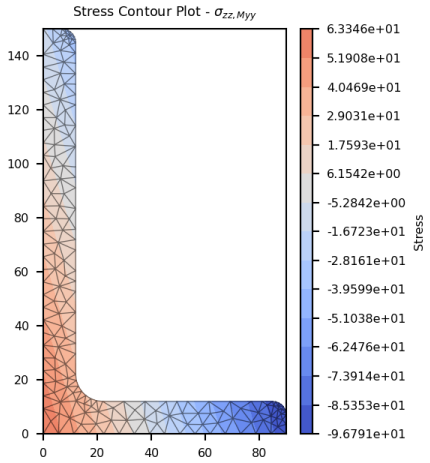

Fig. 5: Contour plot of the bending stress.

Bending Stress ($\sigma_{zz,M11}$)

StressPost.**plot_stress_m11_zz**(*title='Stress Contour Plot - $\\sigma_{zz,M11}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,M11}$ resulting from the bending moment M_{11}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the 11-axis of 5 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(M11=5e6)

stress_post.plot_stress_m11_zz()
```

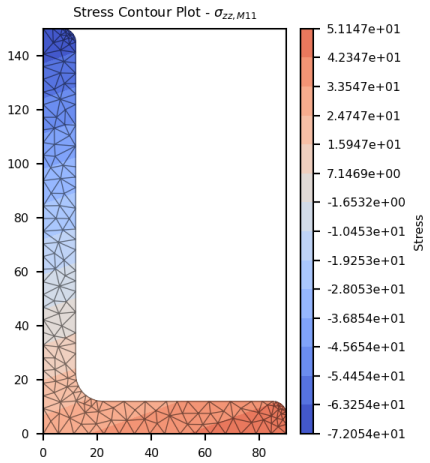

Fig. 6: Contour plot of the bending stress.

Bending Stress ($\sigma_{zz,M22}$)

StressPost.**plot_stress_m22_zz**(*title='Stress Contour Plot - $\\sigma_{zz,M22}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,M22}$ resulting from the bending moment M_{22}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the 22-axis of 2 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(M22=5e6)

stress_post.plot_stress_m22_zz()
```

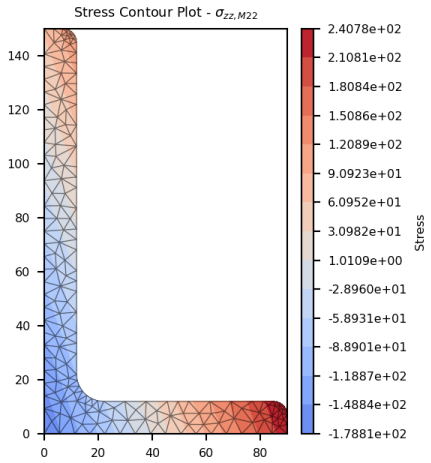

Fig. 7: Contour plot of the bending stress.

Bending Stress ($\sigma_{zz,\Sigma M}$)

StressPost.**plot_stress_m_zz**(*title='Stress Contour Plot - $\\sigma_{zz,\\Sigma M}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,\Sigma M}$ resulting from all bending moments $M_{xx} + M_{yy} + M_{11} + M_{22}$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the x-axis of 5 kN.m, a bending moment about the y-axis of 2 kN.m and a bending moment of 3 kN.m about the 11-axis:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mxx=5e6, Myy=2e6, M11=3e6)
```

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```
stress_post.plot_stress_m_zz()
```

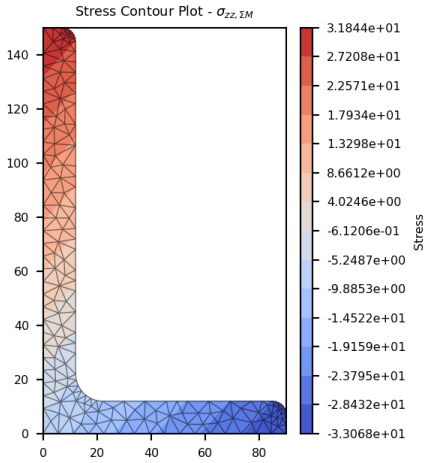

Fig. 8: Contour plot of the bending stress.

Torsion Stress ($\sigma_{zx,Mzz}$)

StressPost.**plot_stress_mzz_zx**(*title='Stress Contour Plot - $\\sigma_{zx,Mzz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)
```

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```
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_stress_mzz_zx()
```

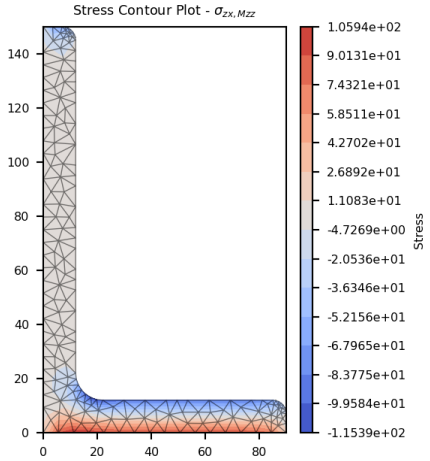

Fig. 9: Contour plot of the shear stress.

Torsion Stress ($\sigma_{zy,Mzz}$)

StressPost.**plot_stress_mzz_zy**(*title='Stress Contour Plot - $\\sigma_{zy,Mzz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress $\sigma_{zy,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
```

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```
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_stress_mzz_zy()
```

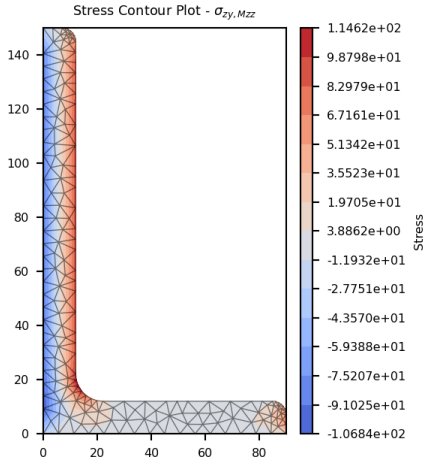

Fig. 10: Contour plot of the shear stress.

Torsion Stress ($\sigma_{zxy,Mzz}$)

StressPost.**plot_stress_mzz_zxy**(*title='Stress Contour Plot - $\\sigma_{zxy,Mzz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_stress_mzz_zxy()
```

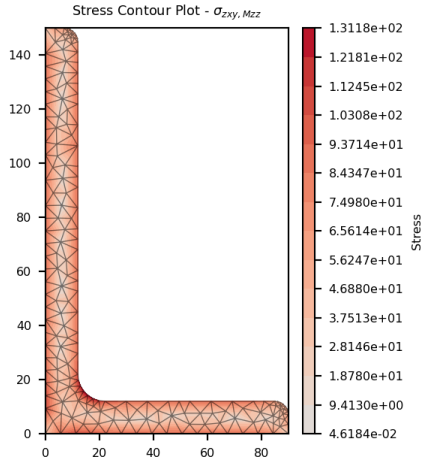

Fig. 11: Contour plot of the shear stress.

StressPost.**plot_vector_mzz_zxy**(*title='Stress Vector Plot - $\\sigma_{zxy,Mzz}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_vector_mzz_zxy()
```

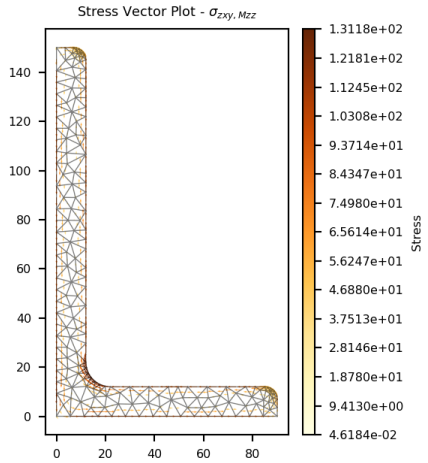

Fig. 12: Vector plot of the shear stress.

Shear Stress ($\sigma_{zx,Vx}$)

StressPost.**plot_stress_vx_zx**(*title='Stress Contour Plot - $\\sigma_{zx,Vx}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_stress_vx_zx()
```

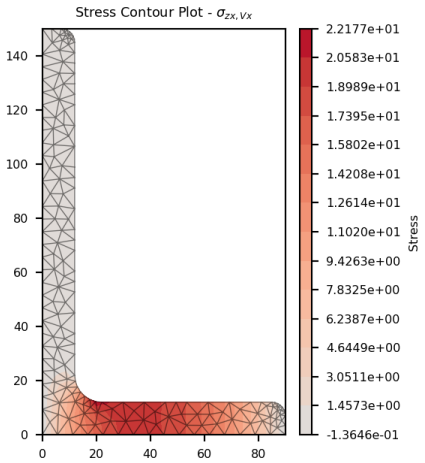

Fig. 13: Contour plot of the shear stress.

Shear Stress ($\sigma_{zy,Vx}$)

StressPost.**plot_stress_vx_zy**(*title='Stress Contour Plot - $\\sigma_{zy,Vx}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the *y*-component of the shear stress $\sigma_{zy,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_stress_vx_zy()
```

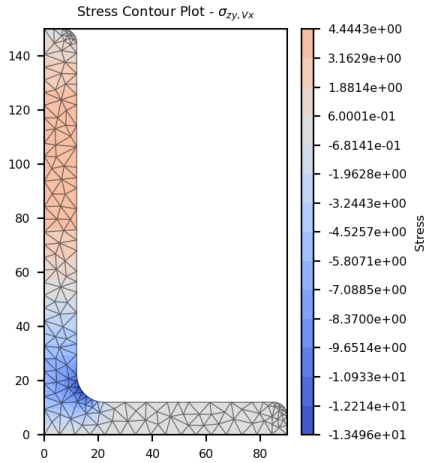

Fig. 14: Contour plot of the shear stress.

Shear Stress ($\sigma_{zxy,Vx}$)

StressPost.**plot_stress_vx_zxy**(*title='Stress Contour Plot - $\\sigma_{zz,Myy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_stress_vx_zxy()
```

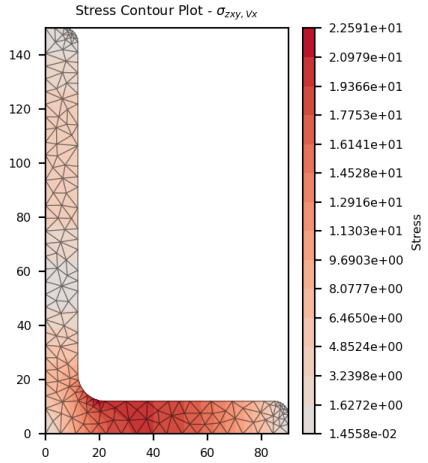

Fig. 15: Contour plot of the shear stress.

StressPost.**plot_vector_vx_zxy**(*title='Stress Vector Plot - $\\sigma_{zxy,Vx}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_vector_vx_zxy()
```

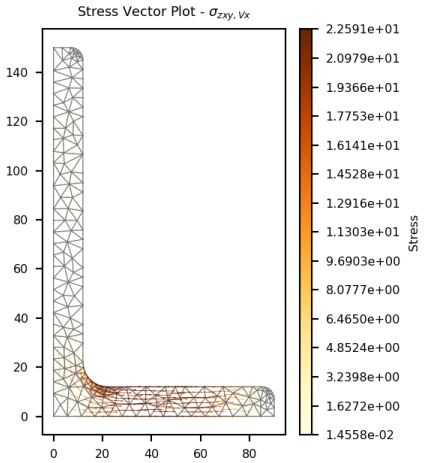

Fig. 16: Vector plot of the shear stress.

Shear Stress ($\sigma_{zx,Vy}$)

StressPost.**plot_stress_vy_zx**(*title='Stress Contour Plot - $\\sigma_{zx,Vy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_stress_vy_zx()
```

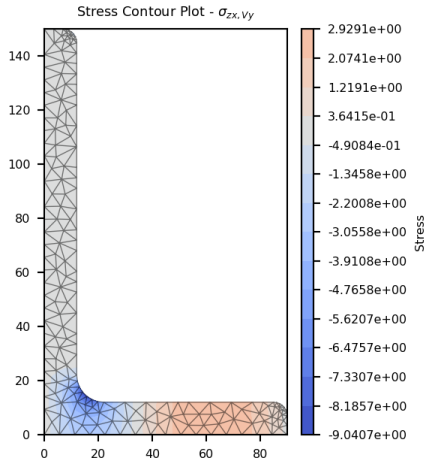

Fig. 17: Contour plot of the shear stress.

Shear Stress ($\sigma_{zy,Vy}$)

StressPost.**plot_stress_vy_zy**(*title='Stress Contour Plot - $\\sigma_{zy,Vy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress $\sigma_{zy,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_stress_vy_zy()
```

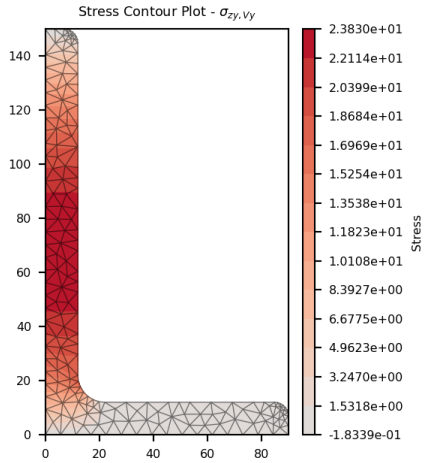

Fig. 18: Contour plot of the shear stress.

Shear Stress ($\sigma_{zxy,Vy}$)

StressPost.**plot_stress_vy_zxy**(*title='Stress Contour Plot - $\\sigma_{zxy,Vy}$', cmap='coolwarm', normalize=True, **kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_stress_vy_zxy()
```

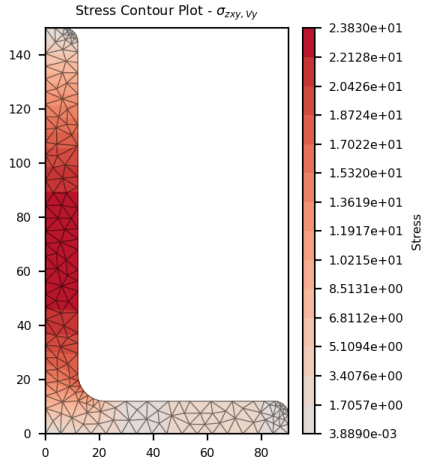

Fig. 19: Contour plot of the shear stress.

StressPost.**plot_vector_vy_zxy**(*title='Stress Vector Plot - $\\sigma_{zxy,Vy}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_vector_vy_zxy()
```

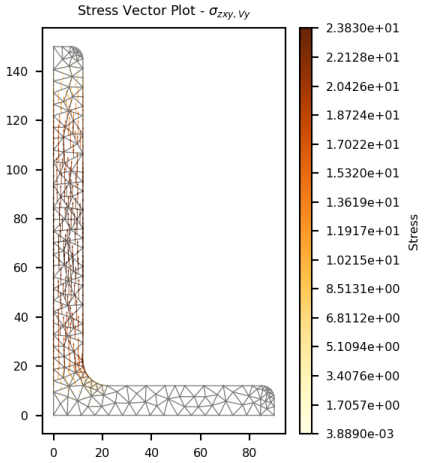

Fig. 20: Vector plot of the shear stress.

Shear Stress ($\sigma_{zx,\Sigma V}$)

StressPost.**plot_stress_v_zx**(*title='Stress Contour Plot - $\\sigma_{zx,\\Sigma V}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_stress_v_zx()
```

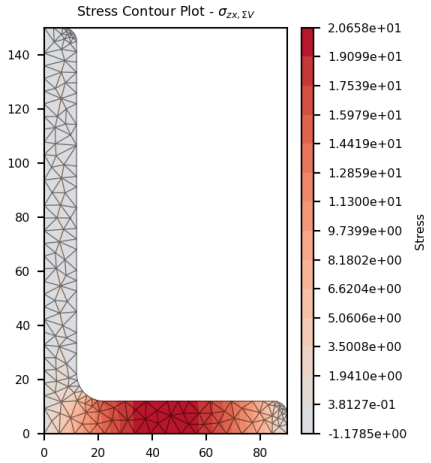

Fig. 21: Contour plot of the shear stress.

Shear Stress ($\sigma_{zy,\Sigma V}$)

StressPost.**plot_stress_v_zy**(*title='Stress Contour Plot - $\\sigma_{zy,\\Sigma V}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress $\sigma_{zy,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type
matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_stress_v_zy()
```

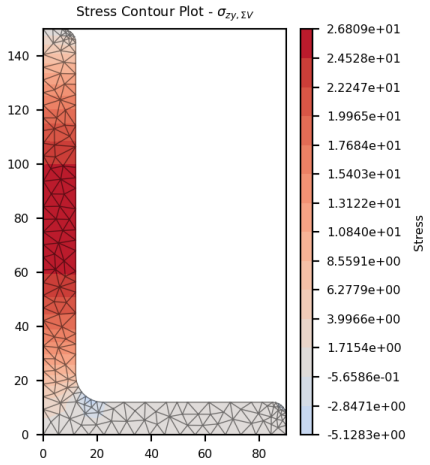

Fig. 22: Contour plot of the shear stress.

Shear Stress ($\sigma_{zxy,\Sigma V}$)

StressPost.**plot_stress_v_zxy**(*title='Stress Contour Plot - $\\sigma_{zxy,\\Sigma V}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_stress_v_zxy()
```

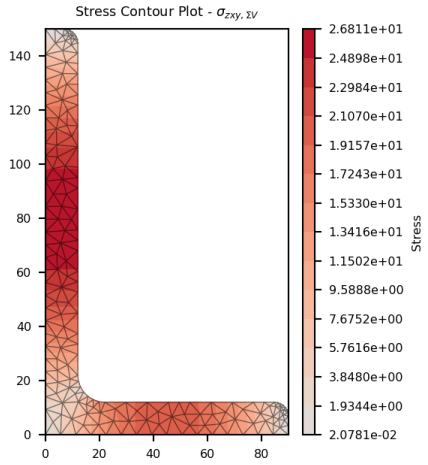

Fig. 23: Contour plot of the shear stress.

StressPost.**plot_vector_v_zxy**(*title='Stress Vector Plot - $\\sigma_{zxy,\\Sigma V}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_vector_v_zxy()
```

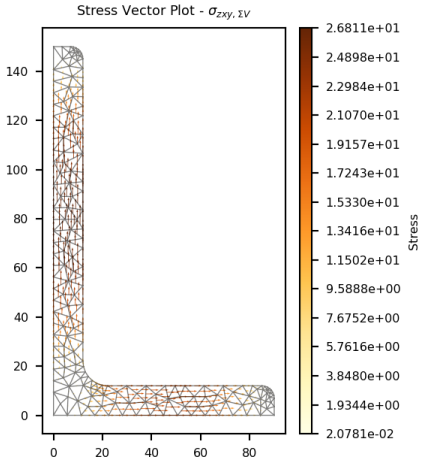

Fig. 24: Vector plot of the shear stress.

7.5.2 Combined Stress Plots

Normal Stress (σ_{zz})

StressPost.**plot_stress_zz**(*title='Stress Contour Plot - $\\sigma_{zz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the combined normal stress σ_{zz} resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots the normal stress within a 150x90x12 UA section resulting from an axial force of 100 kN, a bending moment about the x-axis of 5 kN.m and a bending moment about the y-axis of 2 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(N=100e3, Mxx=5e6, Myy=2e6)

stress_post.plot_stress_zz()
```

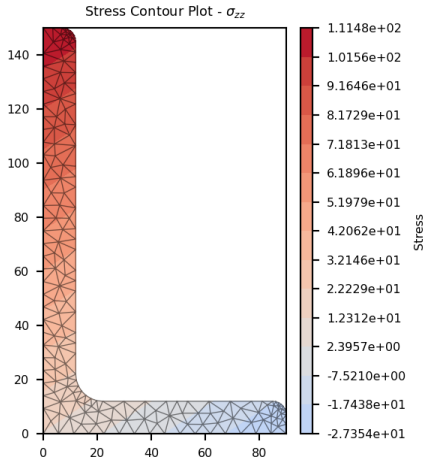

Fig. 25: Contour plot of the normal stress.

Shear Stress (σ_{zx})

`StressPost.`**`plot_stress_zx`**(*title='Stress Contour Plot - $\\sigma_{zx}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress σ_{zx} resulting from all actions.

Parameters

- **`title`** (*string*) – Plot title
- **`cmap`** (*string*) – Matplotlib color map.
- **`normalize`** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **`kwargs`** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_stress_zx()
```

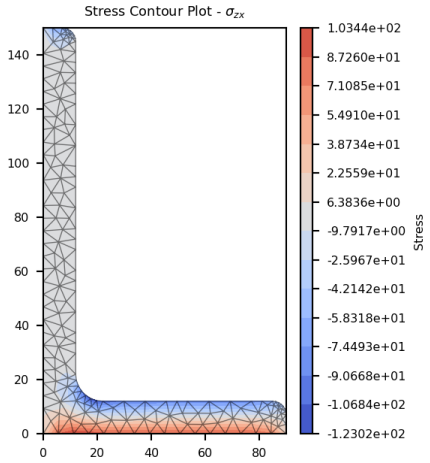

Fig. 26: Contour plot of the shear stress.

Shear Stress (σ_{zy})

StressPost.**plot_stress_zy**(*title='Stress Contour Plot - $\\sigma_{zy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress σ_{zy} resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_stress_zy()
```

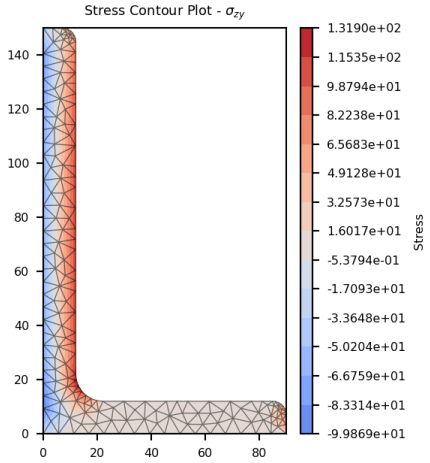

Fig. 27: Contour plot of the shear stress.

Shear Stress (σ_{zxy})

`StressPost.`**`plot_stress_zxy`**(*title='Stress Contour Plot - $\\sigma_{zxy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress σ_{zxy} resulting from all actions.

Parameters

- **`title`** (*string*) – Plot title
- **`cmap`** (*string*) – Matplotlib color map.
- **`normalize`** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **`kwargs`** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_stress_zxy()
```

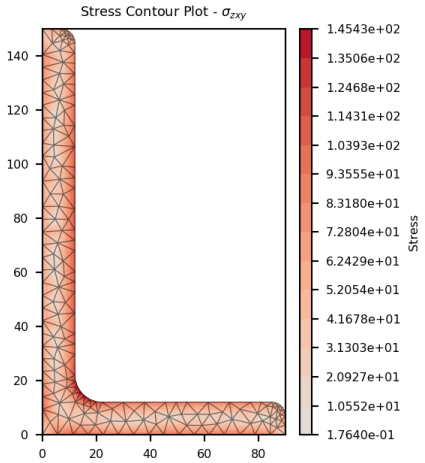

Fig. 28: Contour plot of the shear stress.

`StressPost.`**`plot_vector_zxy`**(*title='Stress Vector Plot - $\\sigma_{zxy}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress σ_{zxy} resulting from all actions.

Parameters

- **`title`** (*string*) – Plot title
- **`cmap`** (*string*) – Matplotlib color map.
- **`normalize`** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **`kwargs`** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_vector_zxy()
```

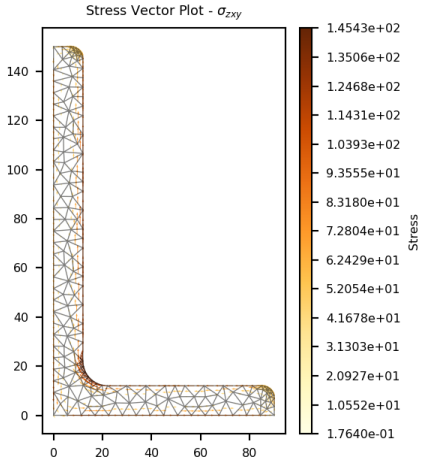

Fig. 29: Vector plot of the shear stress.

Major Principal Stress (σ_1)

StressPost.**plot_stress_1**(*title='Stress Contour Plot - $\\sigma_{1}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the major principal stress σ_1 resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots a contour of the major principal stress within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN
- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
mesh = geometry.create_mesh(mesh_sizes=[2.5])
section = CrossSection(geometry, mesh)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_stress_1()
```

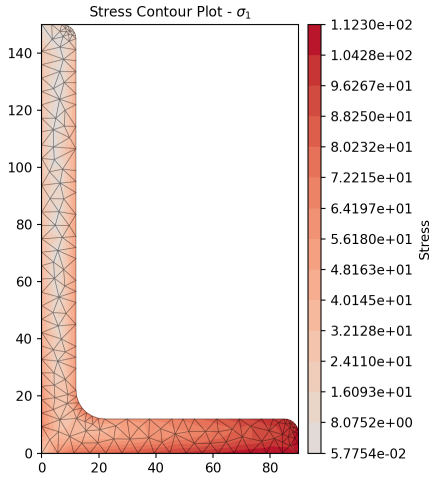

Fig. 30: Contour plot of the major principal stress.

Minor Principal Stress (σ_3)

StressPost.**plot_stress_3**(*title='Stress Contour Plot - $\\sigma_{3}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the Minor principal stress σ_3 resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots a contour of the Minor principal stress within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN
- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
mesh = geometry.create_mesh(mesh_sizes=[2.5])
section = CrossSection(geometry, mesh)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_stress_3()
```

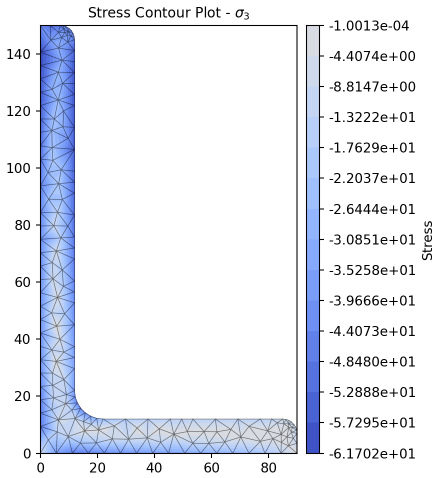

Fig. 31: Contour plot of the minor principal stress.

von Mises Stress (σ_{vM})

`StressPost.`**`plot_stress_vm`**(*title='Stress Contour Plot - $\\sigma_{vM}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the von Mises stress σ_{vM} resulting from all actions.

Parameters

- **`title`** (*`string`*) – Plot title
- **`cmap`** (*`string`*) – Matplotlib color map.
- **`normalize`** (*`bool`*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **`kwargs`** – Passed to *`plotting_context()`*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots a contour of the von Mises stress within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN
- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
```

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```
geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_stress_vm()
```

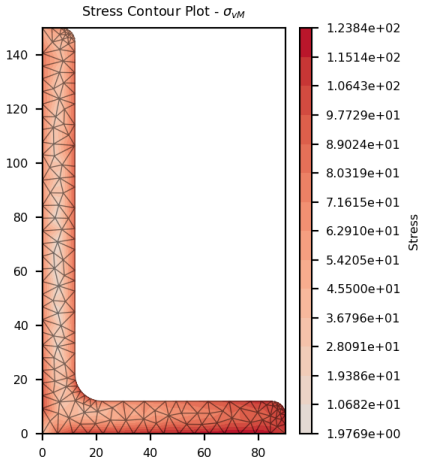

Fig. 32: Contour plot of the von Mises stress.

Mohr's Circles for Stresses at a Point

`StressPost.`**`plot_mohrs_circles`**(*x*, *y*, *title=None*, ***kwargs*)

Plots Mohr's Circles of the 3D stress state at position x, y

Parameters

- **x** (*float*) – x-coordinate of the point to draw Mohr's Circle
- **y** (*float*) – y-coordinate of the point to draw Mohr's Circle
- **title** (*string*) – Plot title
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

`matplotlib.axes`

The following example plots the Mohr's Circles for the 3D stress state within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN

- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

at the point (10, 88.9).

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
mesh = geometry.create_mesh(mesh_sizes=[2.5])
section = Section(geometry, mesh)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_mohrs_circles(10, 88.9)
```

7.6 Retrieving Section Stress

All cross-section stresses can be recovered using the *get_stress()* method that belongs to every *StressPost* object:

StressPost.**get_stress**()

Returns the stresses within each material belonging to the current *StressPost* object.

Returns
A list of dictionaries containing the cross-section stresses for each material.

Return type
list[dict]

A dictionary is returned for each material in the cross-section, containing the following keys and values:

- *'Material'*: Material name
- *'sig_zz_n'*: Normal stress $\sigma_{zz,N}$ resulting from the axial load N
- *'sig_zz_mxx'*: Normal stress $\sigma_{zz,Mxx}$ resulting from the bending moment M_{xx}
- *'sig_zz_myy'*: Normal stress $\sigma_{zz,Myy}$ resulting from the bending moment M_{yy}
- *'sig_zz_m11'*: Normal stress $\sigma_{zz,M11}$ resulting from the bending moment M_{11}
- *'sig_zz_m22'*: Normal stress $\sigma_{zz,M22}$ resulting from the bending moment M_{22}
- *'sig_zz_m'*: Normal stress $\sigma_{zz,\Sigma M}$ resulting from all bending moments
- *'sig_zx_mzz'*: x-component of the shear stress $\sigma_{zx,Mzz}$ resulting from the torsion moment
- *'sig_zy_mzz'*: y-component of the shear stress $\sigma_{zy,Mzz}$ resulting from the torsion moment
- *'sig_zxy_mzz'*: Resultant shear stress $\sigma_{zxy,Mzz}$ resulting from the torsion moment

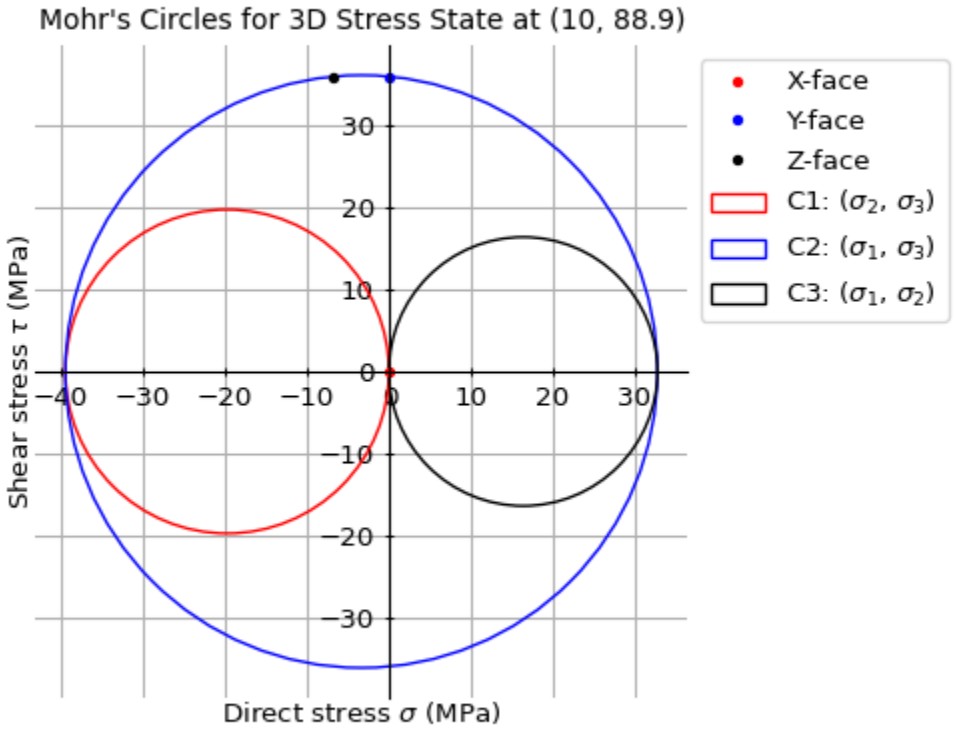

Fig. 33: Mohr's Circles of the 3D stress state at (10, 88.9).

- *'sig_zx_vx'*: x-component of the shear stress $\sigma_{zx,Vx}$ resulting from the shear force V_x
- *'sig_zy_vx'*: y-component of the shear stress $\sigma_{zy,Vx}$ resulting from the shear force V_x
- *'sig_zxy_vx'*: Resultant shear stress $\sigma_{zxy,Vx}$ resulting from the shear force V_x
- *'sig_zx_vy'*: x-component of the shear stress $\sigma_{zx,Vy}$ resulting from the shear force V_y
- *'sig_zy_vy'*: y-component of the shear stress $\sigma_{zy,Vy}$ resulting from the shear force V_y
- *'sig_zxy_vy'*: Resultant shear stress $\sigma_{zxy,Vy}$ resulting from the shear force V_y
- *'sig_zx_v'*: x-component of the shear stress $\sigma_{zx,\Sigma V}$ resulting from all shear forces
- *'sig_zy_v'*: y-component of the shear stress $\sigma_{zy,\Sigma V}$ resulting from all shear forces
- *'sig_zxy_v'*: Resultant shear stress $\sigma_{zxy,\Sigma V}$ resulting from all shear forces
- *'sig_zz'*: Combined normal stress σ_{zz} resulting from all actions
- *'sig_zx'*: x-component of the shear stress σ_{zx} resulting from all actions
- *'sig_zy'*: y-component of the shear stress σ_{zy} resulting from all actions
- *'sig_zxy'*: Resultant shear stress σ_{zxy} resulting from all actions
- *'sig_1'*: Major principal stress σ_1 resulting from all actions
- *'sig_3'*: Minor principal stress σ_3 resulting from all actions
- *'sig_vm'*: von Mises stress σ_{vM} resulting from all actions

The following example returns stresses for each material within a composite section, note that a result is generated for each node in the mesh for all materials irrespective of whether the materials exists at that point or not.

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)
stresses = stress_post.get_stress()

print("Number of nodes: {0}".format(section.num_nodes))

for stress in stresses:
    print('Material: {0}'.format(stress['Material']))
    print('List Size: {0}'.format(len(stress['sig_zz_n'])))
    print('Normal Stresses: {0}'.format(stress['sig_zz_n']))
    print('von Mises Stresses: {0}'.format(stress['sig_vm']))
```

```
$ Number of nodes: 2465

$ Material: Timber
$ List Size: 2465
```

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```
$ Normal Stresses: [0.76923077 0.76923077 0.76923077 ... 0.76923077 0.76923077 0.
↪76923077]
$ von Mises Stresses: [7.6394625  5.38571866 3.84784964 ... 3.09532948 3.66992556 2.
↪81976647]

$ Material: Steel
$ List Size: 2465
$ Normal Stresses: [19.23076923 0. 0. ... 0. 0. 0.]
$ von Mises Stresses: [134.78886419 0. 0. ... 0. 0. 0.]
```

CHAPTER

EIGHT

EXAMPLES GALLERY

Here is a gallery of examples demonstrating what *sectionproperties* can do!

8.1 Basic Examples

These are basic examples to calculate properties and results on a cross-section.

8.2 Advanced Examples

The following examples demonstrates how *sectionproperties* can be used for more academic purposes.

8.2.1 Basic Examples

These are basic examples to calculate properties and results on a cross-section.

Simple Example

Calculate section properties of a circle.

The following example calculates the geometric, warping and plastic properties of a 50 mm diameter circle. The circle is discretised with 64 points and a mesh size of 2.5 mm^2.

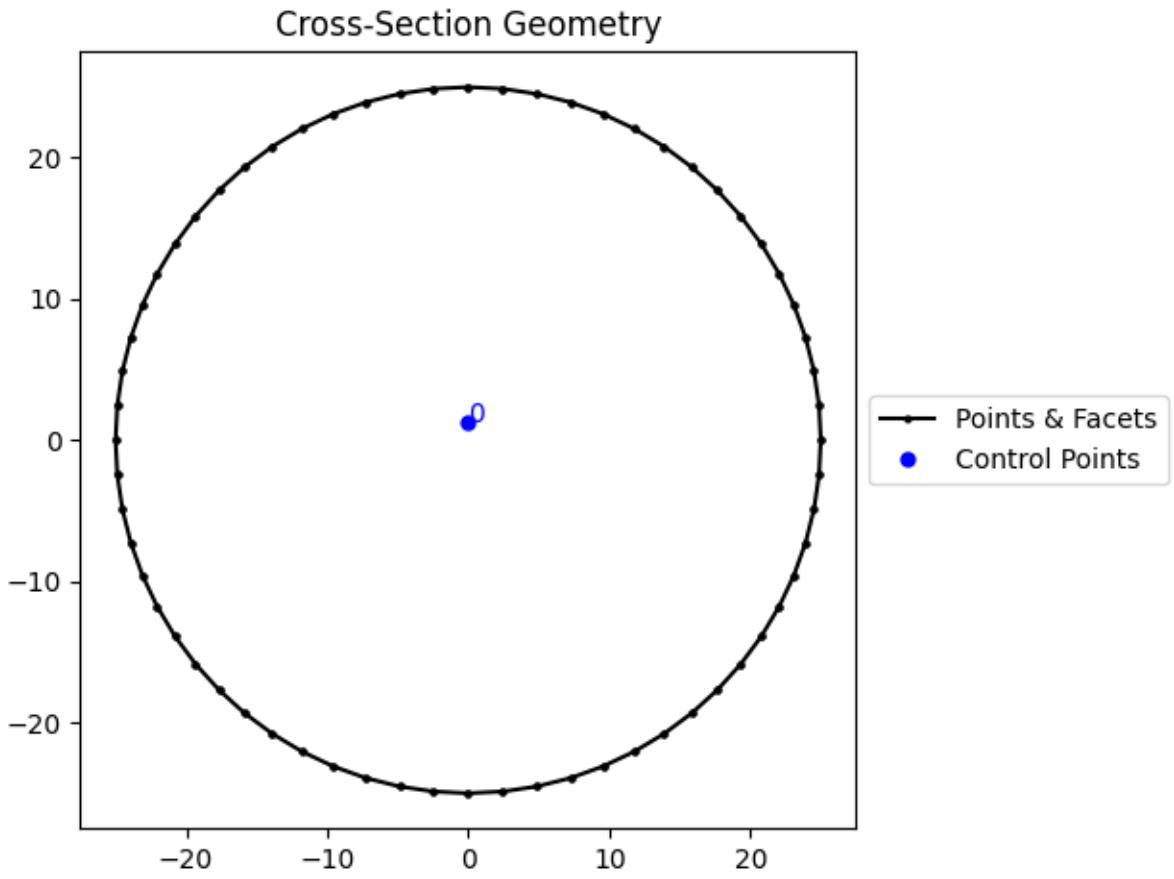
The geometry and mesh are plotted, and the mesh information printed to the terminal before the analysis is carried out. Detailed time information is printed to the terminal during the cross-section analysis stage. Once the analysis is complete, the cross-section properties are printed to the terminal. The centroidal axis second moments of area and torsion constant are saved to variables and it is shown that, for a circle, the torsion constant is equal to the sum of the second moments of area.

```
# sphinx_gallery_thumbnail_number = 1

import sectionproperties.pre.library.primitive_sections as sections
from sectionproperties.analysis.section import Section
```

Create a 50 diameter circle discretised by 64 points

```
geometry = sections.circular_section(d=50, n=64)
geometry.plot_geometry()
```

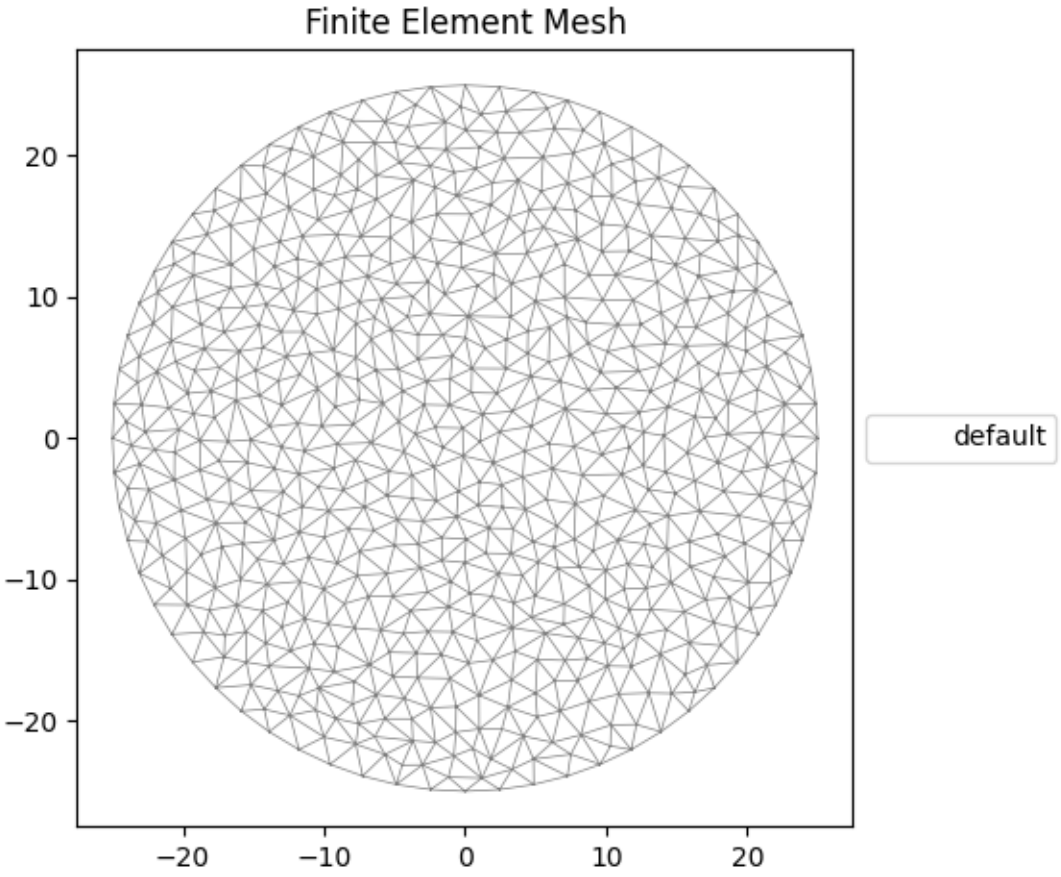



```
<AxesSubplot: title={'center': 'Cross-Section Geometry'}>
```

Create a mesh with a mesh size of 2.5 and display information about it

```
geometry.create_mesh(mesh_sizes=[2.5])

section = Section(geometry, time_info=True)
section.display_mesh_info()
section.plot_mesh()
```




```
Mesh Statistics:
- 2557 nodes
- 1246 elements
- 1 region


<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

perform a geometric, warping and plastic analysis, displaying the time info

```
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties()
```

```
──────────────────────────── Geometric Analysis ────────────────────────────
┌──────────────────────────────────────────────────────────────────────────┐
│  Geometric analysis         100% [ 1.3049 s ] │                          │
│ complete                                      │                          │
└──────────────────────────────────────────────────────────────────────────┘
──────────────────────────── Warping Analysis ─────────────────────────────
┌──────────────────────────────────────────────────────────────────────────┐
│  Warping analysis           100% [ 7.3832 s ] │
```

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```
   completed                                                                    │
    2557x2557 stiffness        100% [ 1.5734 s ] │
   matrix assembled                                                             │
    Warping function solved    100% [ 0.0976 s ] │
   (direct)                                                                     │
    Shear function vectors     100% [ 1.6762 s ] │
   assembled                                                                    │
    Shear functions solved     100% [ 0.1535 s ] │
   (direct)                                                                     │
    Shear and warping          100% [ 1.1067 s ] │
   integrals assembled                                                          │
    Shear deformation          100% [ 1.5808 s ] │
   coefficients assembled                                                       │
    Monosymmetry integrals     100% [ 1.1875 s ] │
   assembled                                                                    │
                                                                                │
──────────────────────────────────────────────────────────────────────────────────
──────────────────────────── Plastic Analysis ────────────────────────────────
                                                                                │
    Plastic analysis complete  100% [ 0.0332 s ] │
                                                                                │
──────────────────────────────────────────────────────────────────────────────────
```

Print the results to the terminal

```
section.display_results()
```

```
    Section Properties

 Property          Value

┌─────────┬──────────────┐
 A         1.960343e+03
 Perim.    1.570166e+02
 Qx       -8.764545e-12
 Qy        3.601119e-12
 cx        1.836985e-15
 cy       -4.470924e-15
 Ixx_g     3.058119e+05
 Iyy_g     3.058119e+05
 Ixy_g     8.526513e-13
 Ixx_c     3.058119e+05
 Iyy_c     3.058119e+05
 Ixy_c     8.526513e-13
 Zxx+      1.223248e+04
 Zxx-      1.223248e+04
 Zyy+      1.223248e+04
 Zyy-      1.223248e+04
 rx        1.248996e+01
 ry        1.248996e+01
 phi       0.000000e+00
 I11_c     3.058119e+05
 I22_c     3.058119e+05
```

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```
| Z11+      |  1.223248e+04 |
| Z11-      |  1.223248e+04 |
| Z22+      |  1.223248e+04 |
| Z22-      |  1.223248e+04 |
| r11       |  1.248996e+01 |
| r22       |  1.248996e+01 |
| J         |  6.116238e+05 |
| Iw        |  1.588407e-21 |
| x_se      | -6.613922e-16 |
| y_se      |  1.426706e-15 |
| x_st      | -6.613922e-16 |
| y_st      |  1.426706e-15 |
| x1_se     | -2.498377e-15 |
| y2_se     |  5.897630e-15 |
| A_sx      |  1.680296e+03 |
| A_sy      |  1.680296e+03 |
| A_s11     |  1.680296e+03 |
| A_s22     |  1.680296e+03 |
| betax+    | -8.344890e-15 |
| betax-    |  8.344890e-15 |
| betay+    |  1.526682e-14 |
| betay-    | -1.526682e-14 |
| beta11+   | -8.344890e-15 |
| beta11-   |  8.344890e-15 |
| beta22+   |  1.526682e-14 |
| beta22-   | -1.526682e-14 |
| x_pc      |  1.836985e-15 |
| y_pc      | -4.470924e-15 |
| Sxx       |  2.078317e+04 |
| Syy       |  2.078317e+04 |
| SF_xx+    |  1.699016e+00 |
| SF_xx-    |  1.699016e+00 |
| SF_yy+    |  1.699016e+00 |
| SF_yy-    |  1.699016e+00 |
| x11_pc    |  1.836985e-15 |
| y22_pc    | -4.470924e-15 |
| S11       |  2.078317e+04 |
| S22       |  2.078317e+04 |
| SF_11+    |  1.699016e+00 |
| SF_11-    |  1.699016e+00 |
| SF_22+    |  1.699016e+00 |
| SF_22-    |  1.699016e+00 |
+-----------+---------------+
```

Get and print the second moments of area and the torsion constant

```
(ixx_c, iyy_c, ixy_c) = section.get_ic()
j = section.get_j()
print("Ixx + Iyy = {0:.3f}".format(ixx_c + iyy_c))
print("J = {0:.3f}".format(j))
```

```
Ixx + Iyy = 611623.837
```

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```
J = 611623.837
```

Total running time of the script: (0 minutes 9.971 seconds)

Creating a Nastran Section

Calculate section properties of Nastran HAT1 section.

The following example demonstrates how to create a cross-section defined in a Nastran-based finite element analysis program. The following creates a HAT1 cross-section and calculates the geometric, warping and plastic properties. The HAT1 cross-section is meshed with a maximum elemental area of 0.005.

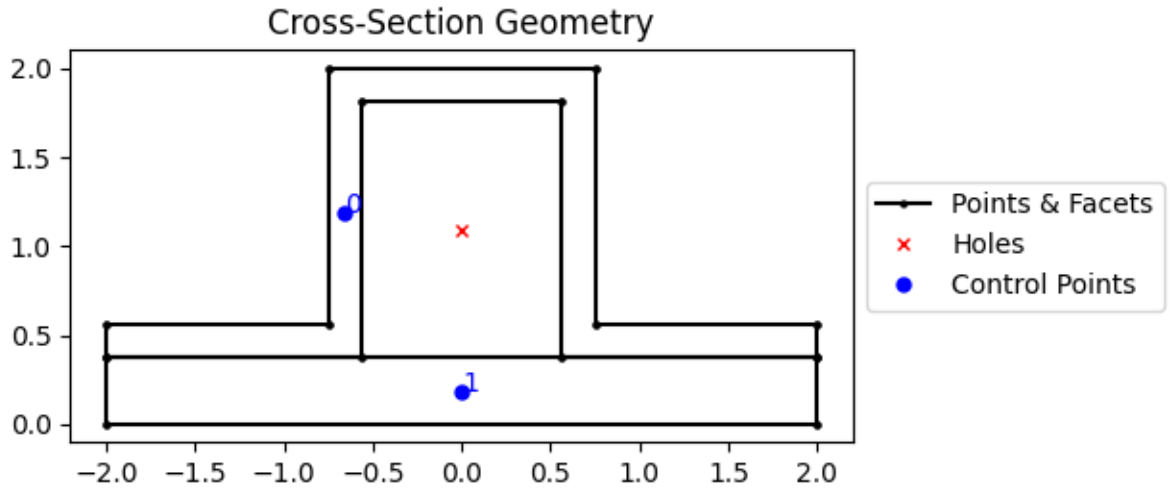
The geometry and mesh are plotted, and the mesh information printed to the terminal before the analysis is carried out. Detailed time information is printed to the terminal during the cross-section analysis stage. Once the analysis is complete, the cross-section properties are printed to the terminal. The centroidal axis second moments of area and torsion constant are saved to variables and it is shown that, for non-circular sections, the torsion constant is not equal to the sum of the second moments of area.

```
# sphinx_gallery_thumbnail_number = 1

from typing import get_origin
import sectionproperties.pre.library.nastran_sections as nsections
from sectionproperties.analysis.section import Section
```

Create a HAT1 section

```
geometry = nsections.nastran_hat1(DIM1=4.0, DIM2=2.0, DIM3=1.5, DIM4=0.1875, DIM5=0.375)
geometry.plot_geometry()  # plot the geometry
print(geometry.geom)
```

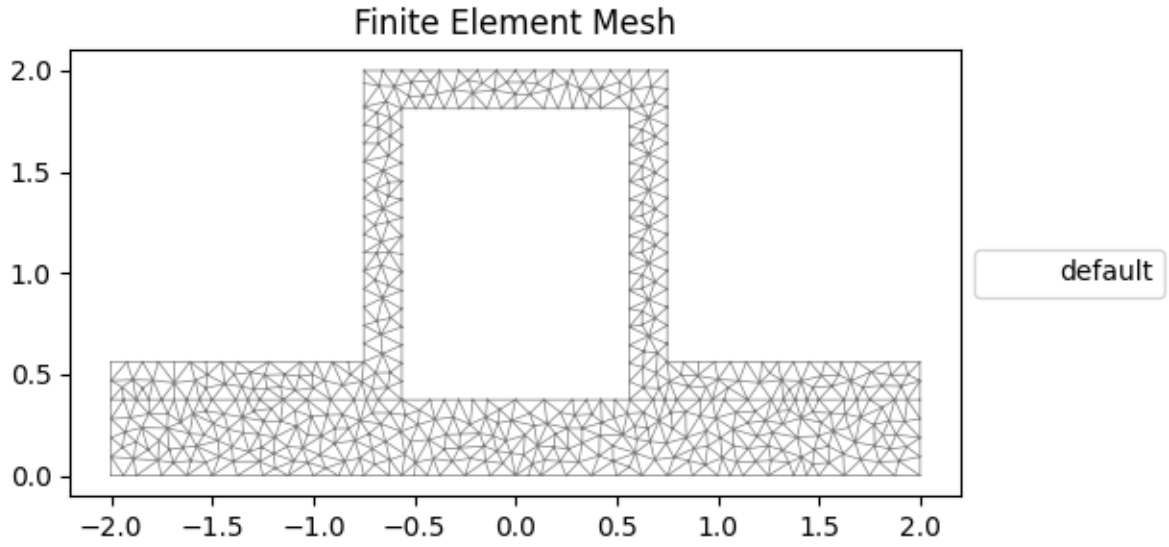



```
MULTIPOLYGON (((-2 0.375, -0.5625 0.375, -0.5625 1.8125, 0.5625 1.8125, 0.5625 0.375, 2␣
↪0.375, 2 0.5625, 0.75 0.5625, 0.75 2, -0.75 2, -0.75 0.5625, -2 0.5625, -2 0.375)), ((-
↪2 0, 2 0, 2 0.375, -2 0.375, -2 0)))
```

Create a mesh with a maximum elemental area of 0.005

```
geometry.create_mesh(mesh_sizes=[0.005])

section = Section(geometry, time_info=True)  # create a Section object
section.display_mesh_info()  # display the mesh information
section.plot_mesh()  # plot the generated mesh`
```




```
Mesh Statistics:
- 2038 nodes
- 926 elements
- 2 regions


<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Perform a geometric, warping and plastic analysis, displaying the time info

```
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties()

section.display_results()
```

```
──────────────────────────────── Geometric Analysis ────────────────────────────────
┌──────────────────────────────────────────────────────────────────────────────────┐
│  Geometric analysis         100% [ 0.9310 s ] │                                  │
│ complete                                      │                                  │
└──────────────────────────────────────────────────────────────────────────────────┘
──────────────────────────────── Warping Analysis ─────────────────────────────────
```

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```
╭──────────────────────────────────────────────────────────────────────────────╮
│                                                                              │
│   Warping analysis           100% [ 5.4325 s ] │
│  completed                                                                   │
│   2038x2038 stiffness        100% [ 1.1458 s ] │
│  matrix assembled                                                            │
│   Warping function solved    100% [ 0.0595 s ] │
│  (direct)                                                                    │
│   Shear function vectors     100% [ 1.2455 s ] │
│  assembled                                                                   │
│   Shear functions solved     100% [ 0.1092 s ] │
│  (direct)                                                                    │
│   Shear and warping          100% [ 0.8185 s ] │
│  integrals assembled                                                         │
│   Shear deformation          100% [ 1.1750 s ] │
│  coefficients assembled                                                      │
│   Monosymmetry integrals     100% [ 0.8734 s ] │
│  assembled                                                                   │
│                                                                              │
╰──────────────────────────────────────────────────────────────────────────────╯
──────────────────────────────── Plastic Analysis ────────────────────────────────
╭──────────────────────────────────────────────────────────────────────────────╮
│                                                                              │
│   Plastic analysis complete  100% [ 0.0714 s ] │
│                                                                              │
╰──────────────────────────────────────────────────────────────────────────────╯
    Section Properties

Property          Value

┌──────────┬──────────────┐
│ A        │  2.789062e+00 │
│ Perim.   │  1.200000e+01 │
│ Qx       │  1.626709e+00 │
│ Qy       │ -2.268151e-16 │
│ cx       │ -8.132306e-17 │
│ cy       │  5.832458e-01 │
│ Ixx_g    │  1.935211e+00 │
│ Iyy_g    │  3.233734e+00 │
│ Ixy_g    │ -3.109492e-16 │
│ Ixx_c    │  9.864400e-01 │
│ Iyy_c    │  3.233734e+00 │
│ Ixy_c    │ -1.786602e-16 │
│ Zxx+     │  6.962676e-01 │
│ Zxx-     │  1.691294e+00 │
│ Zyy+     │  1.616867e+00 │
│ Zyy-     │  1.616867e+00 │
│ rx       │  5.947113e-01 │
│ ry       │  1.076770e+00 │
│ phi      │ -9.000000e+01 │
│ I11_c    │  3.233734e+00 │
│ I22_c    │  9.864400e-01 │
│ Z11+     │  1.616867e+00 │
│ Z11-     │  1.616867e+00 │
│ Z22+     │  1.691294e+00 │
```

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```
| Z22-      |  6.962676e-01 |
| r11       |  1.076770e+00 |
| r22       |  5.947113e-01 |
| J         |  9.878443e-01 |
| Iw        |  1.160810e-01 |
| x_se      |  4.822719e-05 |
| y_se      |  4.674792e-01 |
| x_st      |  4.822719e-05 |
| y_st      |  4.674792e-01 |
| x1_se     |  1.157666e-01 |
| y2_se     |  4.822719e-05 |
| A_sx      |  1.648312e+00 |
| A_sy      |  6.979733e-01 |
| A_s11     |  6.979733e-01 |
| A_s22     |  1.648312e+00 |
| betax+    | -2.746928e-01 |
| betax-    |  2.746928e-01 |
| betay+    |  9.645438e-05 |
| betay-    | -9.645438e-05 |
| beta11+   |  9.645438e-05 |
| beta11-   | -9.645438e-05 |
| beta22+   |  2.746928e-01 |
| beta22-   | -2.746928e-01 |
| x_pc      | -8.132306e-17 |
| y_pc      |  3.486328e-01 |
| Sxx       |  1.140530e+00 |
| Syy       |  2.603760e+00 |
| SF_xx+    |  1.638062e+00 |
| SF_xx-    |  6.743533e-01 |
| SF_yy+    |  1.610373e+00 |
| SF_yy-    |  1.610373e+00 |
| x11_pc    | -6.695716e-17 |
| y22_pc    |  3.486328e-01 |
| S11       |  2.603760e+00 |
| S22       |  1.140530e+00 |
| SF_11+    |  1.610373e+00 |
| SF_11-    |  1.610373e+00 |
| SF_22+    |  6.743533e-01 |
| SF_22-    |  1.638062e+00 |
```

Get the second moments of area and the torsion constant

```
(ixx_c, iyy_c, ixy_c) = section.get_ic()
j = section.get_j()
print("Ixx + Iyy = {0:.3f}".format(ixx_c + iyy_c))
print("J = {0:.3f}".format(j))
```

```
Ixx + Iyy = 4.220
J = 0.988
```

Total running time of the script: (0 minutes 7.246 seconds)

Creating Custom Geometry

Calculate section properties of a user-defined section from points and facets.

The following example demonstrates how geometry objects can be created from a list of points, facets, holes and control points. An straight angle section with a plate at its base is created from a list of points and facets. The bottom plate is assigned a separate control point meaning two discrete regions are created. Creating separate regions allows the user to control the mesh size in each region and assign material properties to different regions. The geometry is cleaned to remove the overlapping facet at the junction of the angle and the plate. A geometric, warping and plastic analysis is then carried out.

The geometry and mesh are plotted before the analysis is carried out. Once the analysis is complete, a plot of the various calculated centroids is generated.

```
# sphinx_gallery_thumbnail_number = 2

from sectionproperties.pre.geometry import CompoundGeometry
from sectionproperties.analysis.section import Section
```

Define parameters for the angle section

```
a = 1
b = 2
t = 0.1
```

Build the lists of points, facets, holes and control points

```
points = [
    [-t / 2, -2 * a],
    [t / 2, -2 * a],
    [t / 2, -t / 2],
    [a, -t / 2],
    [a, t / 2],
    [-t / 2, t / 2],
    [-b / 2, -2 * a],
    [b / 2, -2 * a],
    [b / 2, -2 * a - t],
    [-b / 2, -2 * a - t],
]
facets = [
    [0, 1],
    [1, 2],
    [2, 3],
    [3, 4],
    [4, 5],
    [5, 0],
    [6, 7],
    [7, 8],
    [8, 9],
    [9, 6],
]
holes = []
control_points = [[0, 0], [0, -2 * a - t / 2]]
```

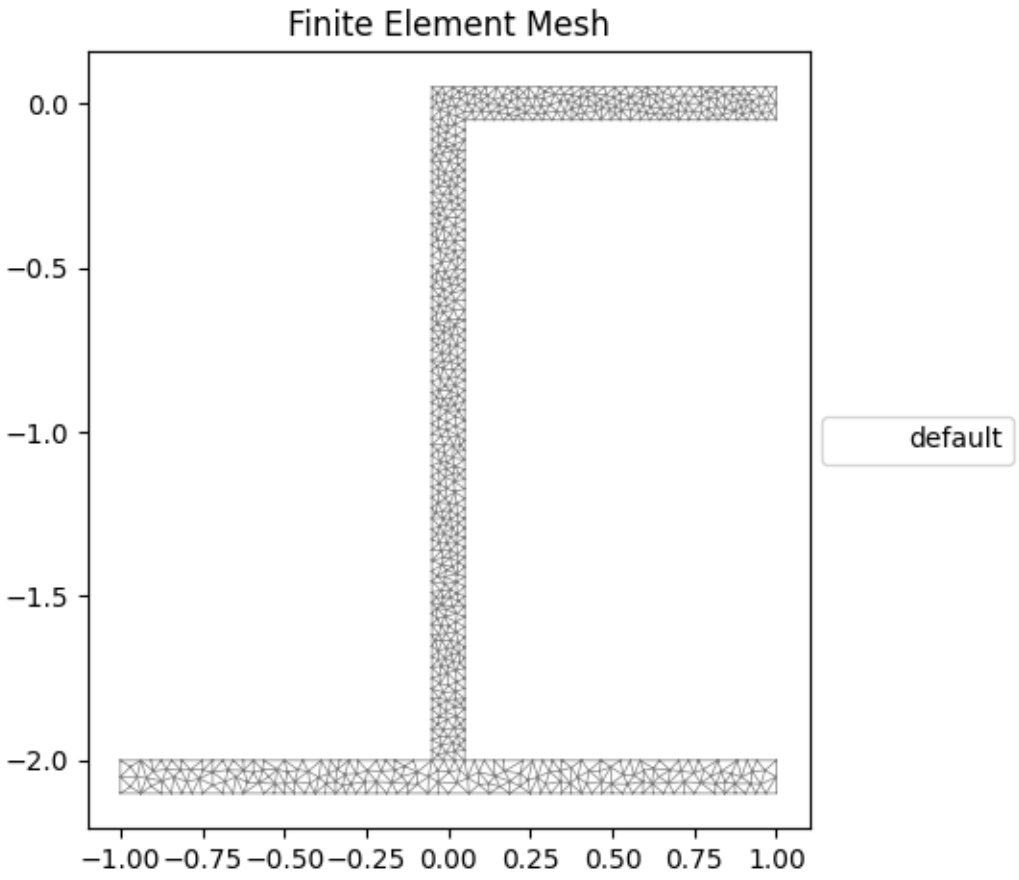
Because we have two separate geometry regions (as indicated by our control_points) we create a CompoundGeometry from points

```
geometry = CompoundGeometry.from_points(points, facets, control_points, holes)
```

Create the mesh and section. For the mesh, use a smaller refinement for the angle region.

```
geometry.create_mesh(mesh_sizes=[0.0005, 0.001])

section = Section(geometry)
section.plot_mesh()  # plot the generated mesh
```

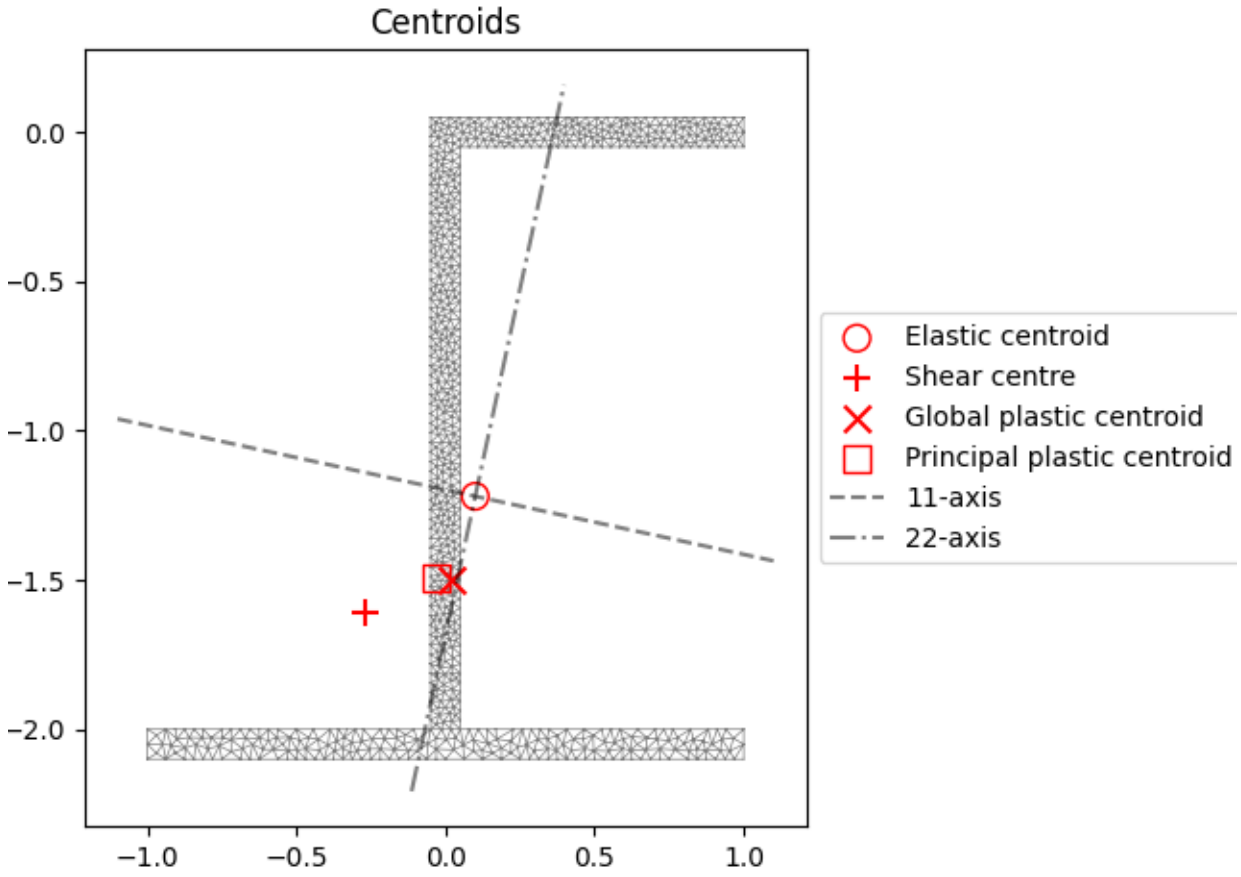



```
<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Perform a geometric, warping and plastic analysis

```
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties()

section.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Total running time of the script: (0 minutes 9.850 seconds)

Creating a Built-Up Section

Merge two sections together into a single larger section.

The following example demonstrates how to combine multiple geometry objects into a single geometry object. A 150x100x6 RHS is modelled with a solid 50x50 triangular section on its top and a 100x100x6 angle section on its right side. The three geometry objects are combined together as a *CompoundGeometry* object using the + operator.

To manipulate individual geometries into the final shape, there are a variety of methods available to move and align. This example uses *.align_center()*, *.align_to()*, and *.shift_section()*.

The geometry and mesh are plotted, and the mesh information printed to the terminal before the analysis is carried out. Detailed time information is printed to the terminal during the cross-section analysis stage. Once the analysis is complete, the centroids are plotted.

```
# sphinx_gallery_thumbnail_number = 1

import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.pre.geometry import Geometry
from sectionproperties.analysis.section import Section
```

Create a 150x100x6 RHS

```
rhs = steel_sections.rectangular_hollow_section(d=150, b=100, t=6, r_out=15, n_r=8)
```

Create a triangular section from points, facets, and control points

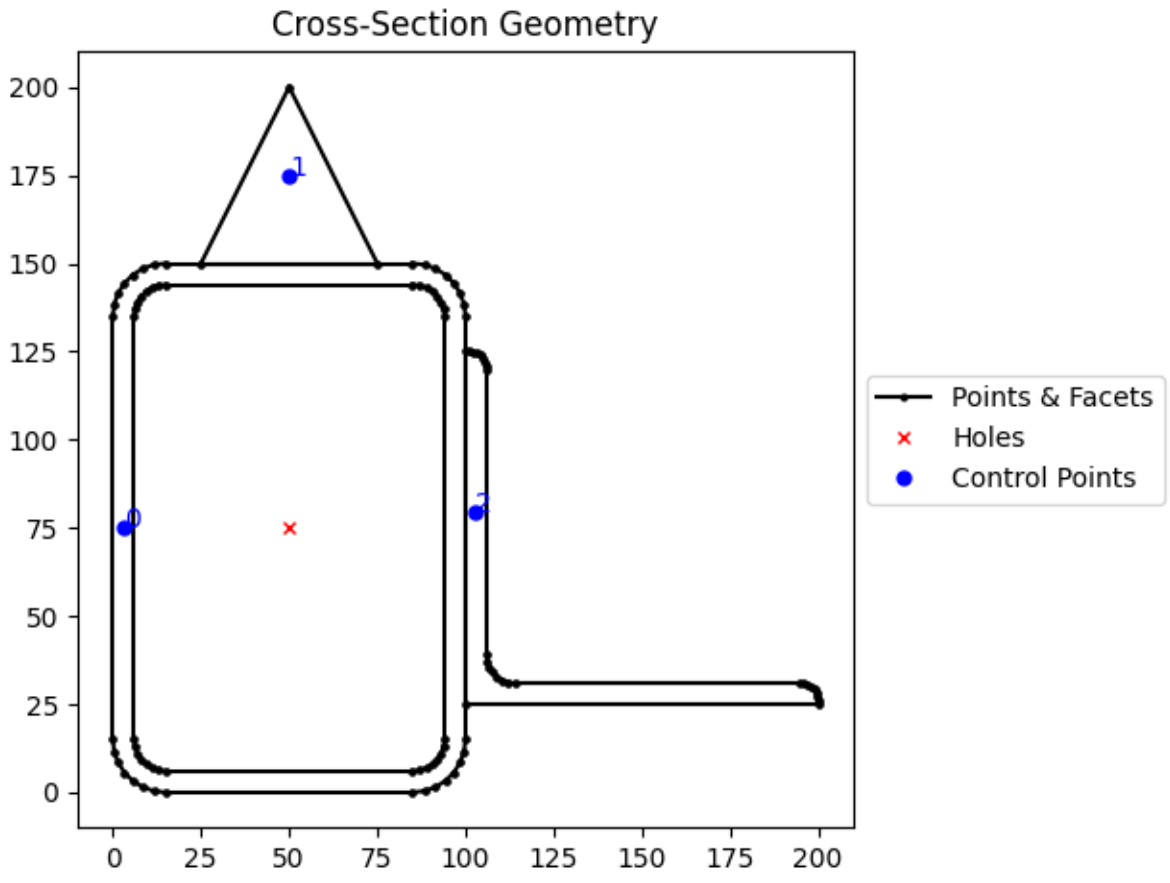
```
points = [[0, 0], [50, 0], [25, 50]]
facets = [[0, 1], [1, 2], [2, 0]]
control_points = [[25, 25]]
triangle = Geometry.from_points(points, facets, control_points)
triangle = triangle.align_center(rhs).align_to(rhs, on="top")
```

Create a 100x100x6 angle and position it on the right of the RHS

```
angle = steel_sections.angle_section(d=100, b=100, t=6, r_r=8, r_t=5, n_r=8)
angle = angle.shift_section(x_offset=100, y_offset=25)
```

Combine the sections into a CompoundGeometry with + operator

```
geometry = rhs + triangle + angle
geometry.plot_geometry()  # plot the geometry
```

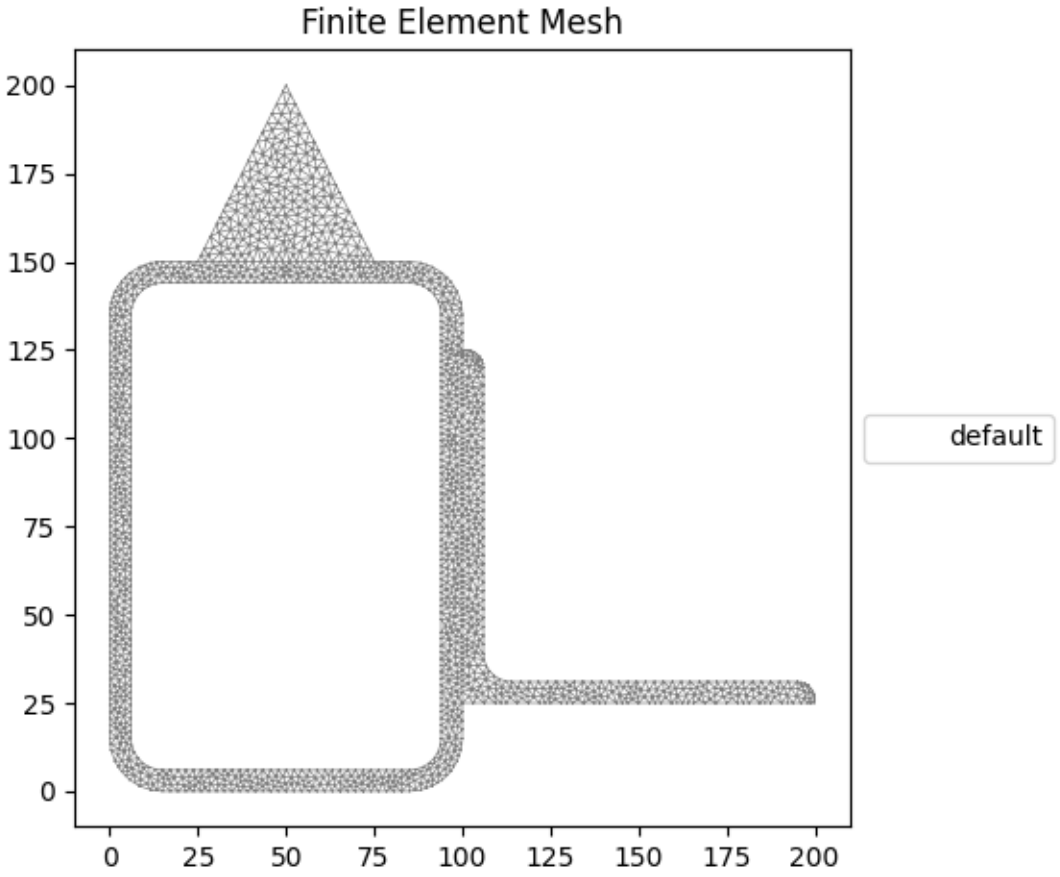



```
<AxesSubplot: title={'center': 'Cross-Section Geometry'}>
```

Create a mesh and section. For the mesh, use a mesh size of 2.5 for the RHS, 5 for the triangle and 3 for the angle.

```
geometry.create_mesh(mesh_sizes=[2.5, 5, 3])

section = Section(geometry, time_info=True)
section.display_mesh_info()  # display the mesh information
section.plot_mesh()  # plot the generated mesh
```



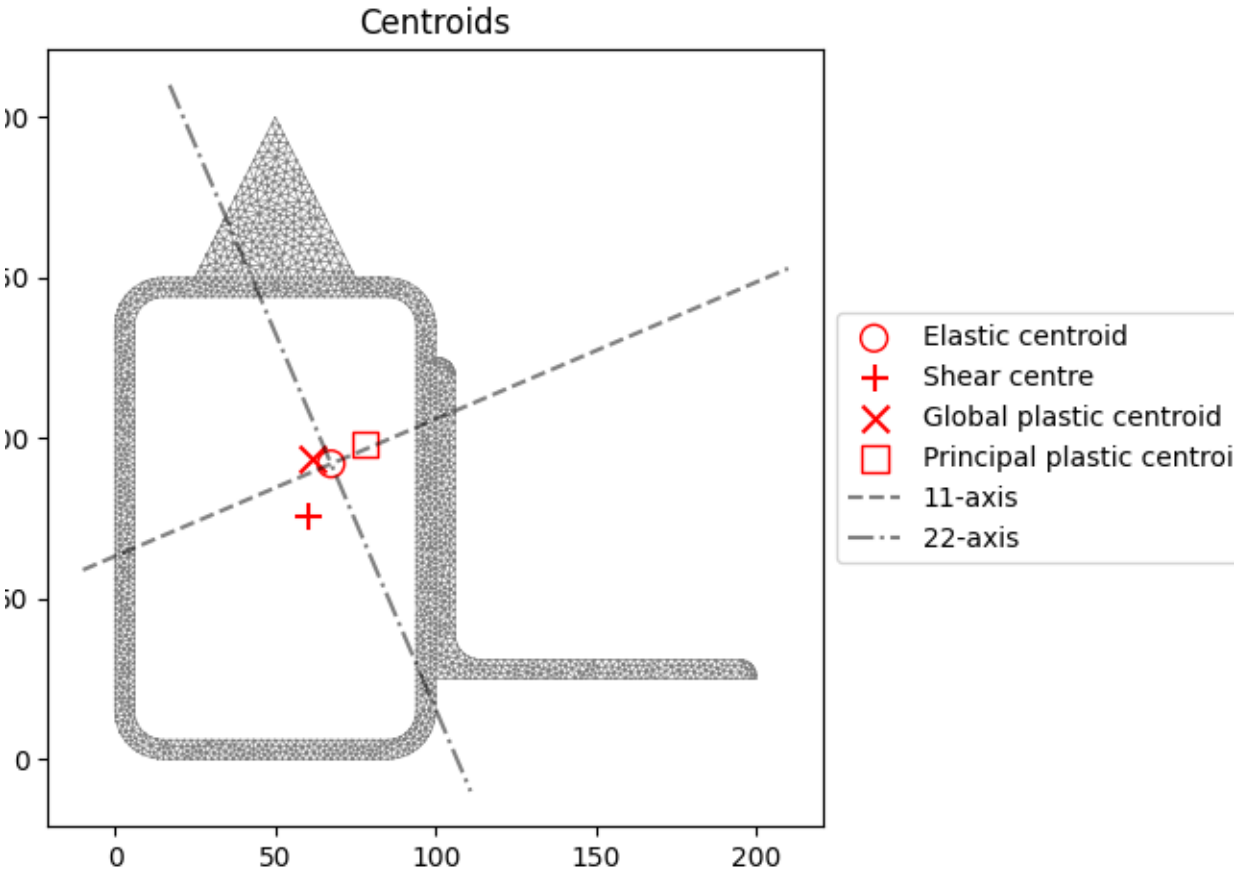

```
Mesh Statistics:
- 6020 nodes
- 2736 elements
- 3 regions


<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Perform a geometric, warping and plastic analysis, displaying the time info and the iteration info for the plastic analysis

```
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties(verbose=True)

# plot the centroids
section.plot_centroids()
```




```
──────────────────────────────── Geometric Analysis ────────────────────────────────
┌──────────────────────────────────────────────────────────────────────────────────┐
│                                                                                  │
│   Geometric analysis          100% [ 2.7682 s ] │                                │
│  complete                                                                        │
│                                                                                  │
└──────────────────────────────────────────────────────────────────────────────────┘
/home/docs/checkouts/readthedocs.org/user_builds/sectionproperties/envs/2.1.5/lib/
↪python3.9/site-packages/shapely/set_operations.py:133: RuntimeWarning: invalid value␣
↪encountered in intersection
  return lib.intersection(a, b, **kwargs)
──────────────────────────────── Warping Analysis ──────────────────────────────────
┌──────────────────────────────────────────────────────────────────────────────────┐
│                                                                                  │
│   Warping analysis            100% [ 17.0115 s ] │                               │
│  completed                                                                       │
│   6020x6020 stiffness         100% [ 3.7134 s ]  │                               │
│  matrix assembled                                                                │
│   Warping function solved     100% [ 0.3571 s ]  │                               │
│  (direct)                                                                        │
│   Shear function vectors      100% [ 3.7671 s ]  │                               │
│  assembled                                                                       │
│   Shear functions solved      100% [ 0.6567 s ]  │                               │
│  (direct)                                                                        │
│   Shear and warping           100% [ 2.4376 s ]  │
```

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```
    integrals assembled                                                    
     Shear deformation         100% [ 3.4618 s ]  
    coefficients assembled                                                 
     Monosymmetry integrals    100% [ 2.6060 s ]  
    assembled                                                              
                                                                           
──────────────────────────────────────────────────────────────────────────
d = -91.996268369166; f_norm = 1.0
d = 108.003731630834; f_norm = -1.0
d = 8.003731630833997; f_norm = -0.04433903442685115
d = 3.5610941851785247; f_norm = -0.013257745358062492
d = 1.6934563166381202; f_norm = -0.00019149964748535968
d = 1.666207547083575; f_norm = -8.635909458075013e-07
d = 1.6660841169311136; f_norm = -5.6994862781888e-11
d = 1.666082783889055; f_norm = 9.269146437289895e-09
---x-axis plastic centroid calculation converged at 1.66608e+00 in 7 iterations.
d = -67.409490017714; f_norm = 1.0
d = 132.590509982286; f_norm = -1.0
d = 32.590509982285994; f_norm = -0.546379783397773
d = -17.409490017714006; f_norm = 0.22681010830111303
d = -2.742322822682775; f_norm = -0.043022470763644426
d = -5.080875765477266; f_norm = -0.011205909614768165
d = -5.8657449981455345; f_norm = 0.0004252464243683582
d = -5.83704941245604; f_norm = -8.433780157994712e-06
d = -5.837607455591706; f_norm = -6.102942204925805e-09
d = -5.837610874395434; f_norm = 4.552923959027148e-08
---y-axis plastic centroid calculation converged at -5.83761e+00 in 9 iterations.
d = -106.16681282996605; f_norm = -1.0
d = 113.67282241177995; f_norm = 1.0
d = 3.7530047909069424; f_norm = 0.03943689434659865
d = -0.5886438731993531; f_norm = 0.006410315748600418
d = -1.4248809343865285; f_norm = 4.9126448281156184e-05
d = -1.4313306978318274; f_norm = 6.360698932422306e-08
d = -1.431339059474304; f_norm = 6.337318715773886e-13
d = -1.4313402751438338; f_norm = -9.24687090782149e-09
---11-axis plastic centroid calculation converged at -1.43134e+00 in 7 iterations.
d = -96.43010376150612; f_norm = -1.0
d = 93.41518522403122; f_norm = 1.0
d = -1.507459268737449; f_norm = 0.2499934430169193
d = -26.819412050693135; f_norm = -0.5136606142451586
d = -9.793701141307178; f_norm = 0.06969393886809541
d = -12.614043035926347; f_norm = -0.015439988274406884
d = -12.102542542006356; f_norm = 0.0009283409099111105
d = -12.13155264001933; f_norm = 1.1002850184137821e-05
d = -12.131900348758636; f_norm = -1.8901470638209108e-10
d = -12.131893782808461; f_norm = 2.075883947415O476e-07
---22-axis plastic centroid calculation converged at -1.21319e+01 in 9 iterations.
──────────────────────────────── Plastic Analysis ────────────────────────────────
                                                                           
    Plastic analysis complete  100% [ 0.1618 s ] 
                                                                           
──────────────────────────────────────────────────────────────────────────
```

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```
<AxesSubplot: title={'center': 'Centroids'}>
```

Total running time of the script: (0 minutes 21.523 seconds)

Mirroring and Rotating Geometry

Mirror and rotate a cross section.

The following example demonstrates how geometry objects can be mirrored and rotated. A 200PFC and 150PFC are placed back-to-back by using the *mirror_section()* method and are rotated counter-clockwise by 30 degrees by using the *rotate_section()* method. The geometry is cleaned to ensure there are no overlapping facets along the junction between the two PFCs. A geometric, warping and plastic analysis is then carried out.

The geometry and mesh are plotted, and the mesh information printed to the terminal before the analysis is carried out. Detailed time information is printed to the terminal during the cross-section analysis stage and iteration information printed for the plastic analysis. Once the analysis is complete, a plot of the various calculated centroids is generated.

```
# sphinx_gallery_thumbnail_number = 1

import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
```

Create a 200PFC and a 150PFC

```
pfc1 = steel_sections.channel_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=8)
pfc2 = steel_sections.channel_section(
    d=150, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=8
).shift_section(0, 26.5)
```

Mirror the 200 PFC about the y-axis

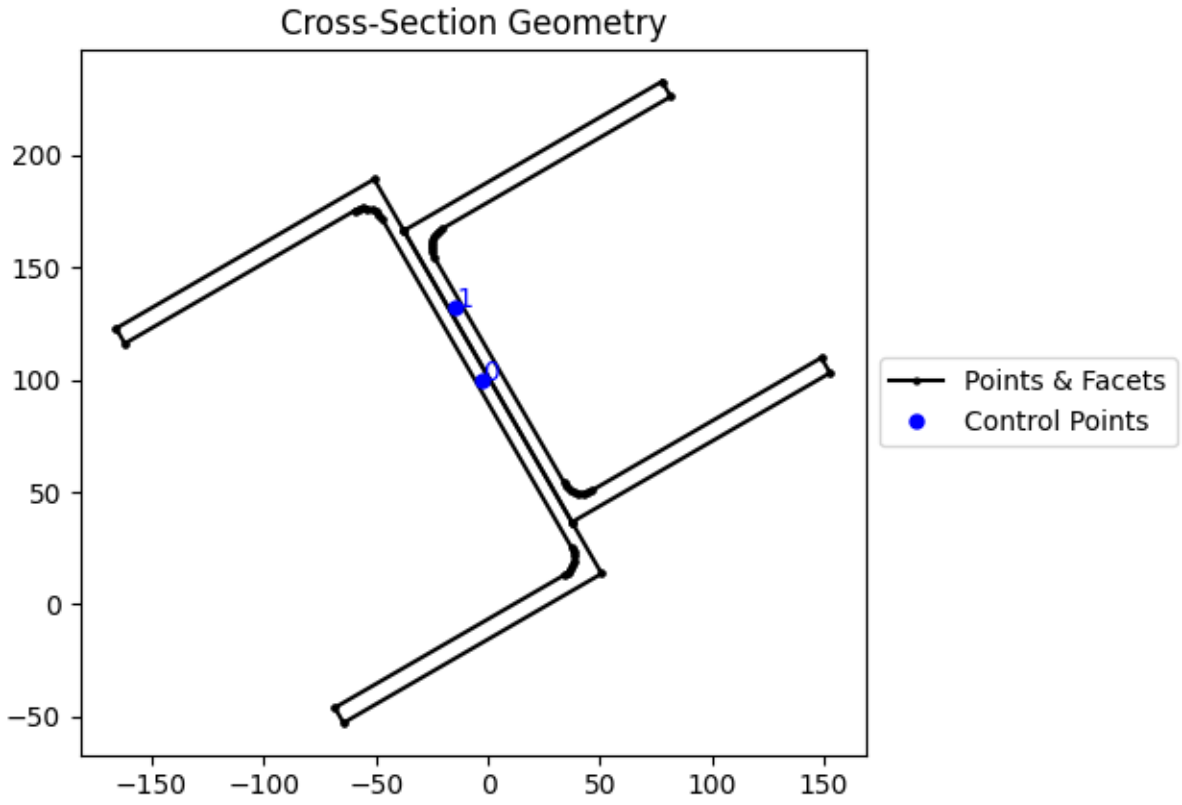
```
pfc1 = pfc1.mirror_section(axis="y", mirror_point=[0, 0])
```

Merge the pfc sections

```
geometry = ((pfc1 - pfc2) | pfc1) + pfc2
```

Rotate the geometry counter-clockwise by 30 degrees

```
geometry = geometry.rotate_section(angle=30)
geometry.plot_geometry()
```

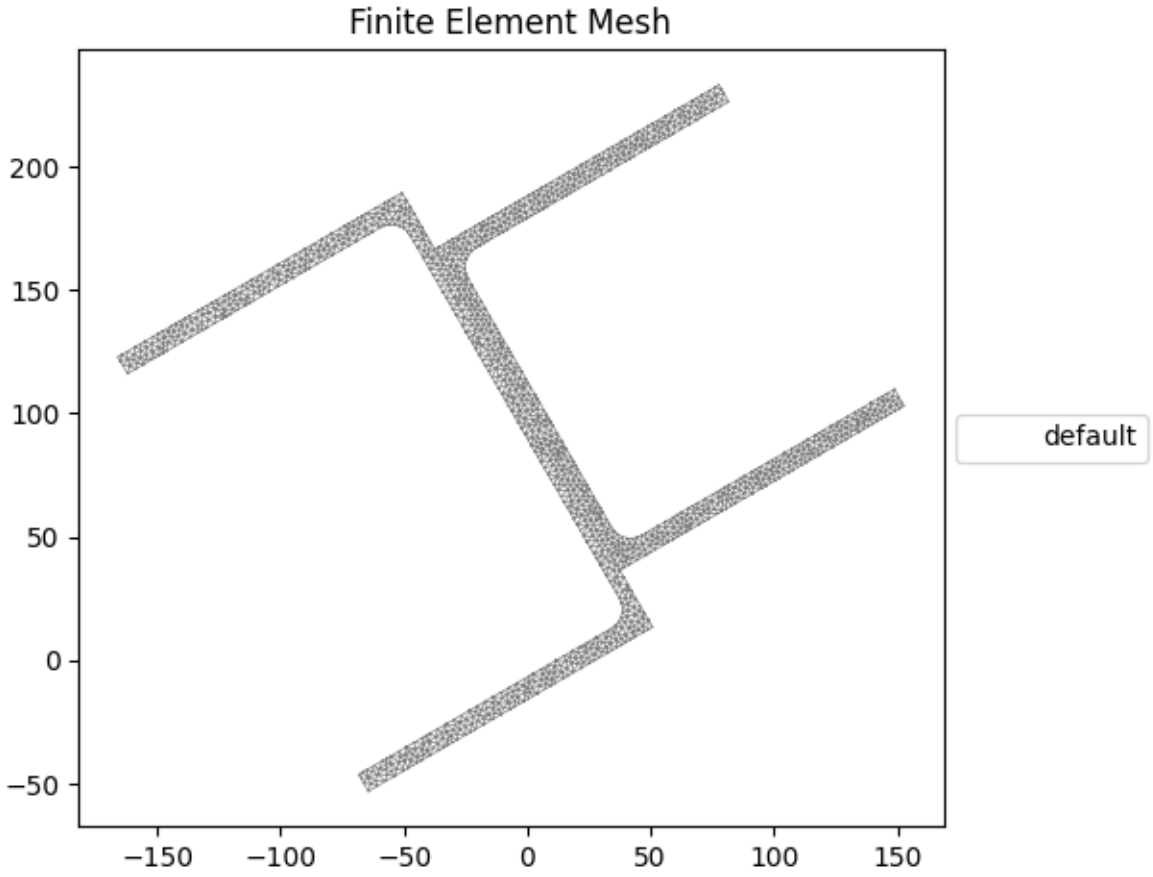



```
<AxesSubplot: title={'center': 'Cross-Section Geometry'}>
```

Create a mesh and section. For the mesh, use a mesh size of 5 for the 200PFC and 4 for the 150PFC

```
geometry.create_mesh(mesh_sizes=[5, 4])

section = Section(geometry, time_info=True)
section.display_mesh_info()  # display the mesh information
section.plot_mesh()  # plot the generated mesh
```



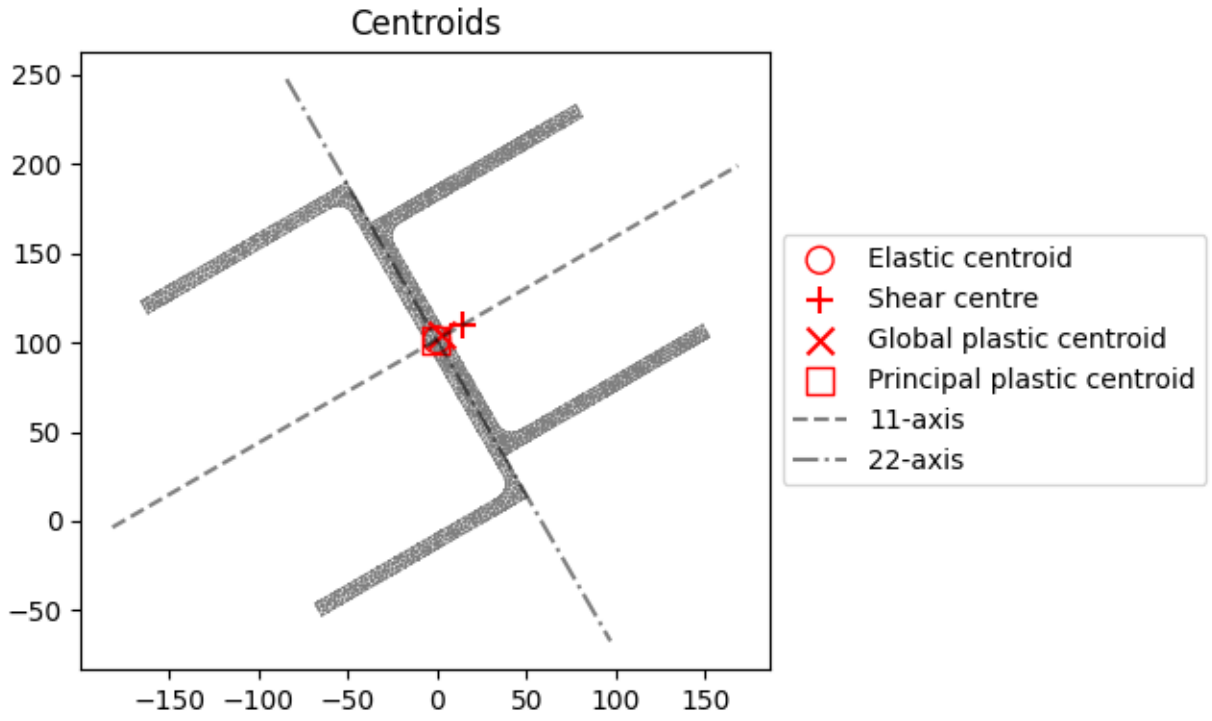

```
Mesh Statistics:
- 4850 nodes
- 2157 elements
- 2 regions


<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Perform a geometric, warping and plastic analysis, displaying the time info and the iteration info for the plastic analysis

```
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties(verbose=True)

section.plot_centroids()
```




```
──────────────────────────── Geometric Analysis ────────────────────────────
│                                                                          │
│   Geometric analysis          100% [ 2.1727 s ] │                        │
│  complete                                                                │
│                                                                          │
──────────────────────────────────────────────────────────────────────────
──────────────────────────── Warping Analysis ──────────────────────────────
│                                                                          │
│   Warping analysis            100% [ 13.6499 s ] │                       │
│  completed                                                               │
│   4850x4850 stiffness         100% [ 2.8322 s ]  │                       │
│  matrix assembled                                                        │
│   Warping function solved     100% [ 0.4012 s ]  │                       │
│  (direct)                                                                │
│   Shear function vectors      100% [ 2.9431 s ]  │                       │
│  assembled                                                               │
│   Shear functions solved      100% [ 0.7427 s ]  │                       │
│  (direct)                                                                │
│   Shear and warping           100% [ 1.9260 s ]  │                       │
│  integrals assembled                                                     │
│   Shear deformation           100% [ 2.7342 s ]  │                       │
│  coefficients assembled                                                  │
│   Monosymmetry integrals      100% [ 2.0610 s ]  │
```

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```
    assembled
```

```
d = -154.328341319365; f_norm = 1.0
d = 131.525142448589; f_norm = -1.0
d = -11.401599435387993; f_norm = 0.13458389687111885
d = 5.55231463203793; f_norm = -0.021090896125661042
d = 3.255390167671628; f_norm = -0.004055963678927537
d = 2.724499194243151; f_norm = 0.00045100079082289346
d = 2.7776241413031926; f_norm = -9.631031333251518e-06
d = 2.7765133886258857; f_norm = -2.1922919066993703e-08
d = 2.7765108545970905; f_norm = 5.977503270512459e-15
---x-axis plastic centroid calculation converged at 2.77651e+00 in 8 iterations.
d = -165.804528212969; f_norm = 1.0
d = 152.808229193691; f_norm = -1.0
d = -6.498149509638978; f_norm = 0.12894151075699403
d = 11.696951784368366; f_norm = -0.11748181846161726
d = 3.0224744372298193; f_norm = -5.230315361698405e-16
d = 3.022472425992601; f_norm = 2.7238961090721597e-08
---y-axis plastic centroid calculation converged at 3.02247e+00 in 5 iterations.
d = -101.49999999999997; f_norm = -1.0
d = 101.50000000000105; f_norm = 1.0
d = 5.400124791776761e-13; f_norm = 4.10953349847732e-15
d = -4.999994599877908e-07; f_norm = -1.905973318172039e-09
---11-axis plastic centroid calculation converged at 5.40012e-13 in 3 iterations.
d = -133.14647432951313; f_norm = -1.0
d = 132.8535256704871; f_norm = 1.0
d = -0.14647432951301198; f_norm = -0.05050838948742143
d = 6.248158849099496; f_norm = 0.3421731433506338
d = 0.6760309227208605; f_norm = 0.004360272672365583
d = 0.6106685276300443; f_norm = -3.810658334951715e-15
d = 0.6106693329643081; f_norm = 5.372319469831614e-08
---22-axis plastic centroid calculation converged at 6.10669e-01 in 6 iterations.
─────────────────────────────── Plastic Analysis ────────────────────────────────

    Plastic analysis complete  100% [ 0.1037 s ]

─────────────────────────────────────────────────────────────────────────────────

<AxesSubplot: title={'center': 'Centroids'}>
```

Total running time of the script: (0 minutes 17.261 seconds)

Performing a Stress Analysis

Calculate and plot stresses on a section.

The following example demonstrates how a stress analysis can be performed on a cross-section. A 150x100x6 RHS is modelled on its side with a maximum mesh area of 2 mm^2. The pre-requisite geometric and warping analyses are performed before two separate stress analyses are undertaken. The first combines bending and shear about the x-axis with a torsion moment and the second combines bending and shear about the y-axis with a torsion moment.

After the analysis is performed, various plots of the stresses are generated.

```
# sphinx_gallery_thumbnail_number = 1

import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
```

Create a 150x100x6 RHS on its side

```
geometry = steel_sections.rectangular_hollow_section(d=100, b=150, t=6, r_out=15, n_r=8)
```

Create a mesh and section object. For the mesh, use a maximum area of 2

```
geometry.create_mesh(mesh_sizes=[2])
section = Section(geometry)
```

Perform a geometry and warping analysis

```
section.calculate_geometric_properties()
section.calculate_warping_properties()
```

Perform a stress analysis with Mx = 5 kN.m; Vx = 10 kN and Mzz = 3 kN.m

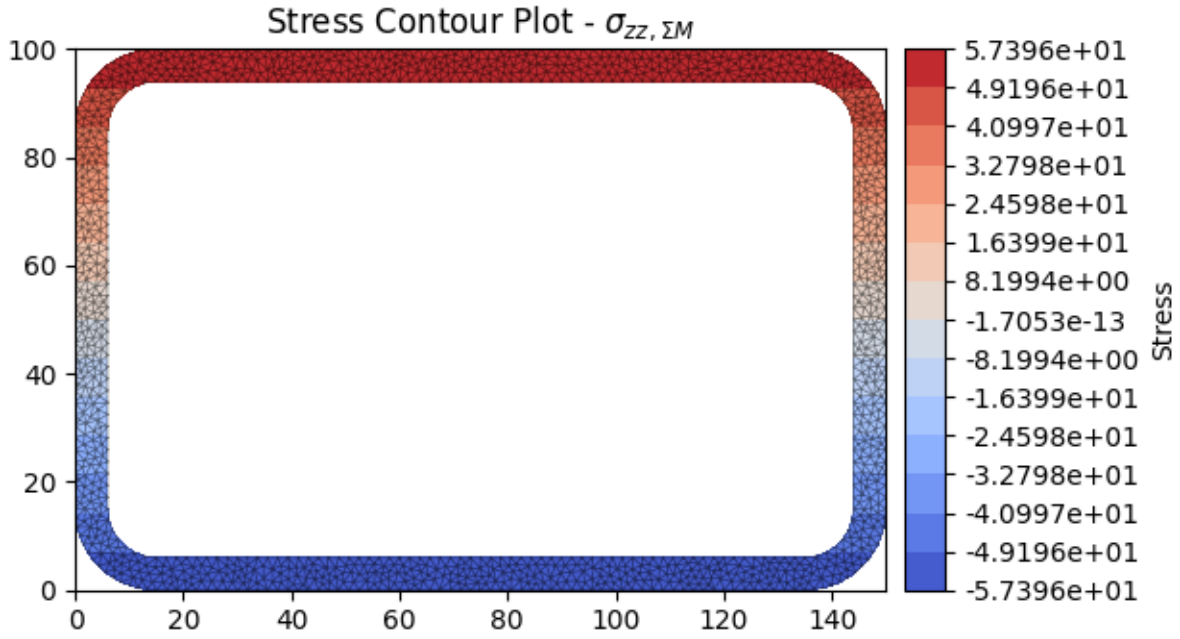
```
case1 = section.calculate_stress(Mxx=5e6, Vx=10e3, Mzz=3e6)
```

Perform a stress analysis with My = 15 kN.m; Vy = 30 kN and Mzz = 1.5 kN.m

```
case2 = section.calculate_stress(Myy=15e6, Vy=30e3, Mzz=1.5e6)
```

Plot the bending stress for case1

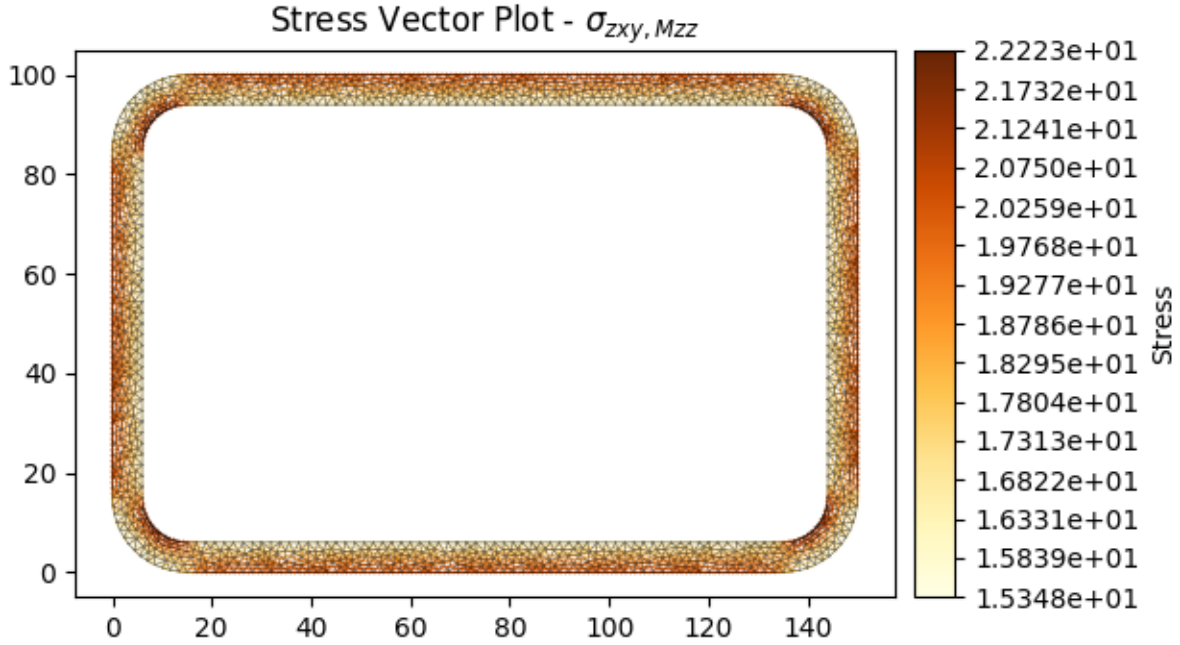
```
case1.plot_stress_m_zz(pause=False)
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{zz,\\Sigma M}$'}>
```

Plot the torsion vectors for case1

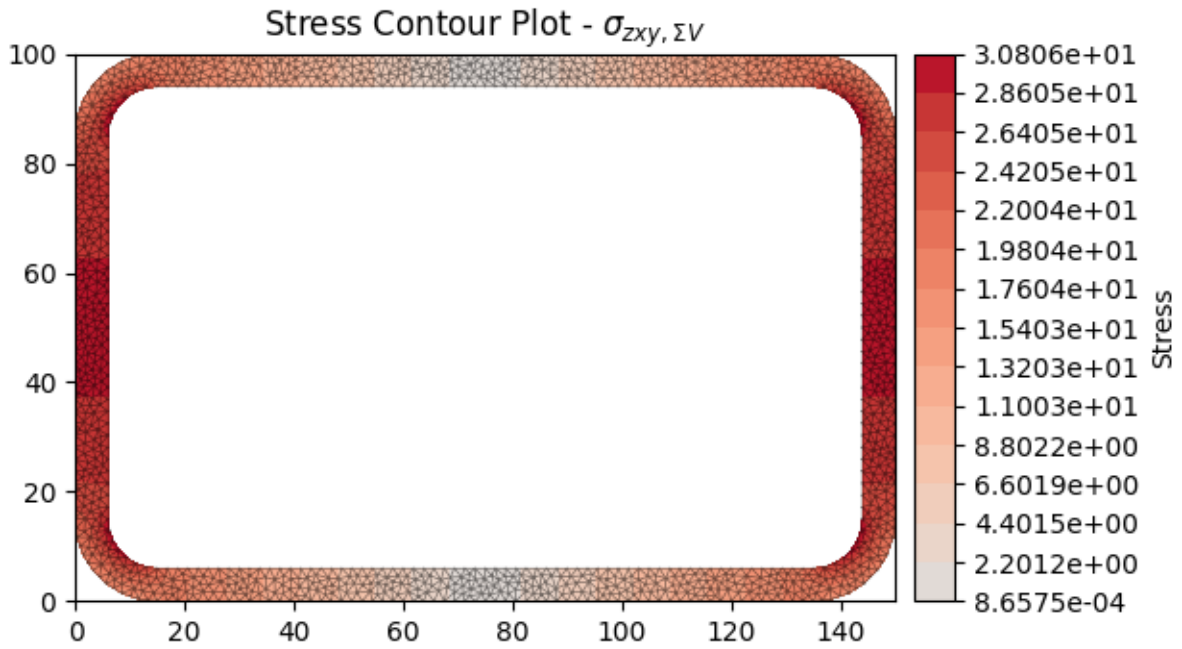
```
case1.plot_vector_mzz_zxy(pause=False)
```




```
<AxesSubplot: title={'center': 'Stress Vector Plot - $\\sigma_{zxy,Mzz}$'}>
```

Plot the shear stress for case2

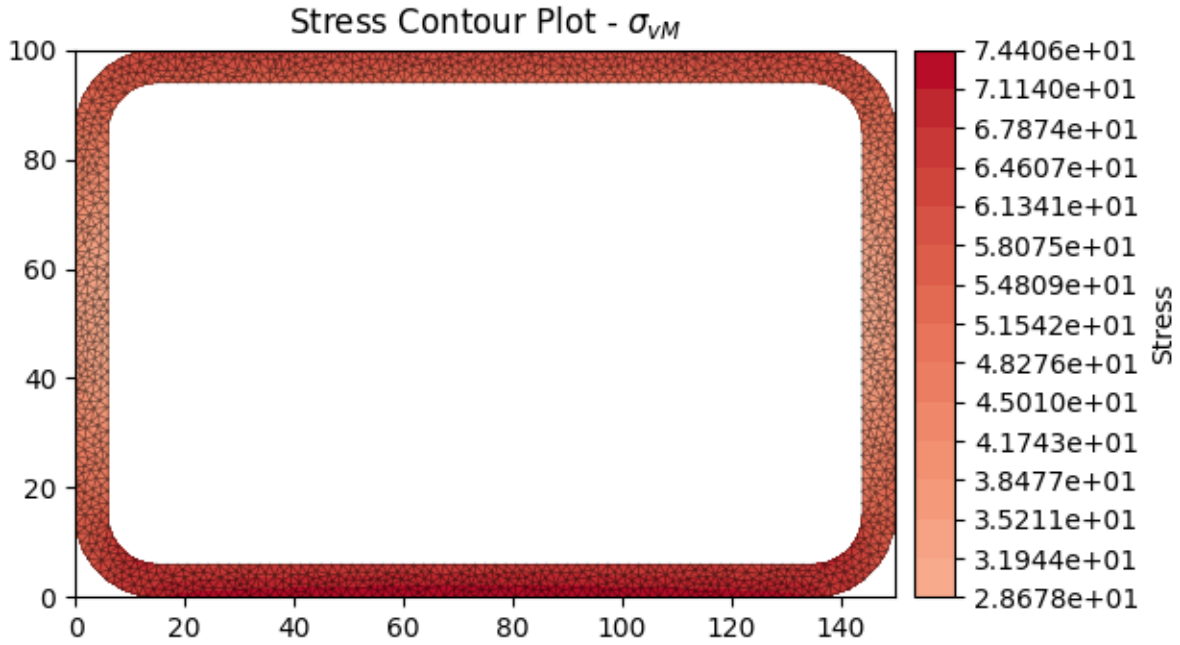
```
case2.plot_stress_v_zxy(pause=False)
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{zxy,\\Sigma V}$'}>
```

Plot the von mises stress for case1

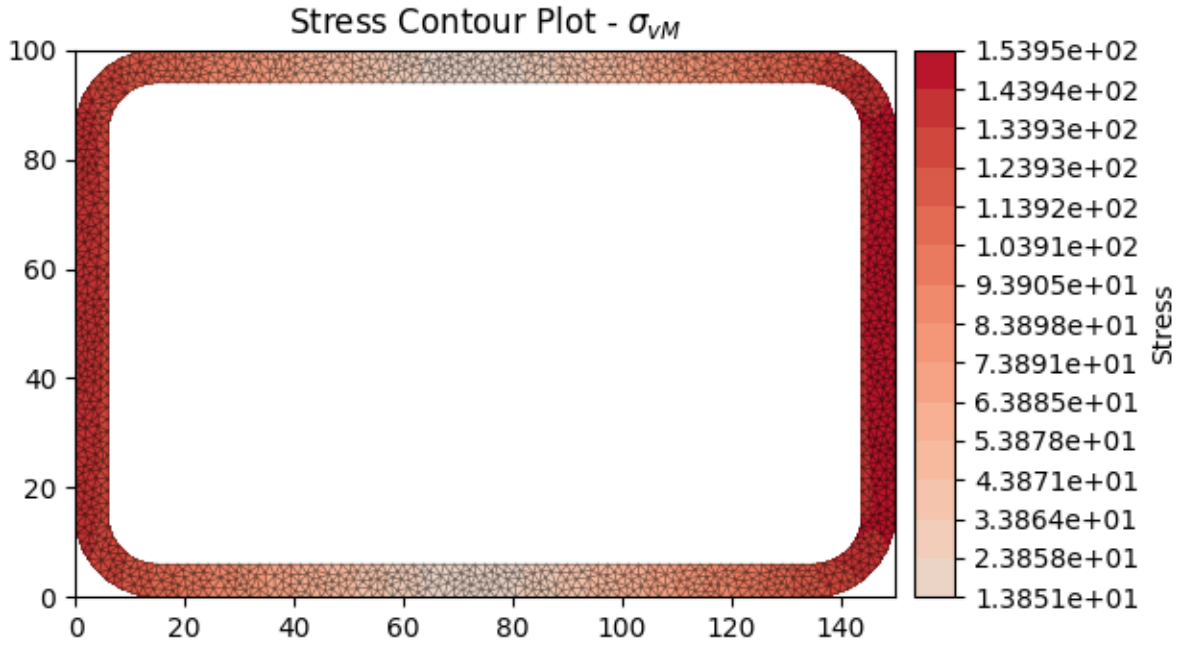
```
case1.plot_stress_vm(pause=False)
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{vM}$'}>
```

Plot the von mises stress for case2

```
case2.plot_stress_vm()
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{vM}$'}>
```

Total running time of the script: (0 minutes 26.155 seconds)

Creating a Composite Section

Create a section of mixed materials.

The following example demonstrates how to create a composite cross-section by assigning different material properties to various regions of the mesh. A steel 310UB40.4 is modelled with a 50Dx600W timber panel placed on its top flange.

The geometry and mesh are plotted, and the mesh information printed to the terminal before the analysis is carried out. All types of cross-section analyses are carried out, with an axial force, bending moment and shear force applied during the stress analysis. Once the analysis is complete, the cross-section properties are printed to the terminal and a plot of the centroids and cross-section stresses generated.

```
# sphinx_gallery_thumbnail_number = 2

import sectionproperties.pre.library.primitive_sections as sections
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.pre.geometry import CompoundGeometry
from sectionproperties.pre.pre import Material
from sectionproperties.analysis.section import Section
```

Create material properties

```
steel = Material(
    name="Steel",
    elastic_modulus=200e3,
    poissons_ratio=0.3,
    yield_strength=500,
    density=8.05e-6,
    color="grey",
)
timber = Material(
    name="Timber",
    elastic_modulus=8e3,
    poissons_ratio=0.35,
    yield_strength=20,
    density=0.78e-6,
    color="burlywood",
)
```

Create 310UB40.4

```
ub = steel_sections.i_section(
    d=304, b=165, t_f=10.2, t_w=6.1, r=11.4, n_r=8, material=steel
)
```

Create timber panel on top of the UB

```
panel = sections.rectangular_section(d=50, b=600, material=timber)
panel = panel.align_center(ub).align_to(ub, on="top")
# Create intermediate nodes in panel to match nodes in ub
panel = (panel - ub) | panel
```

Merge the two sections into one geometry object

```
section_geometry = CompoundGeometry([ub, panel])
```

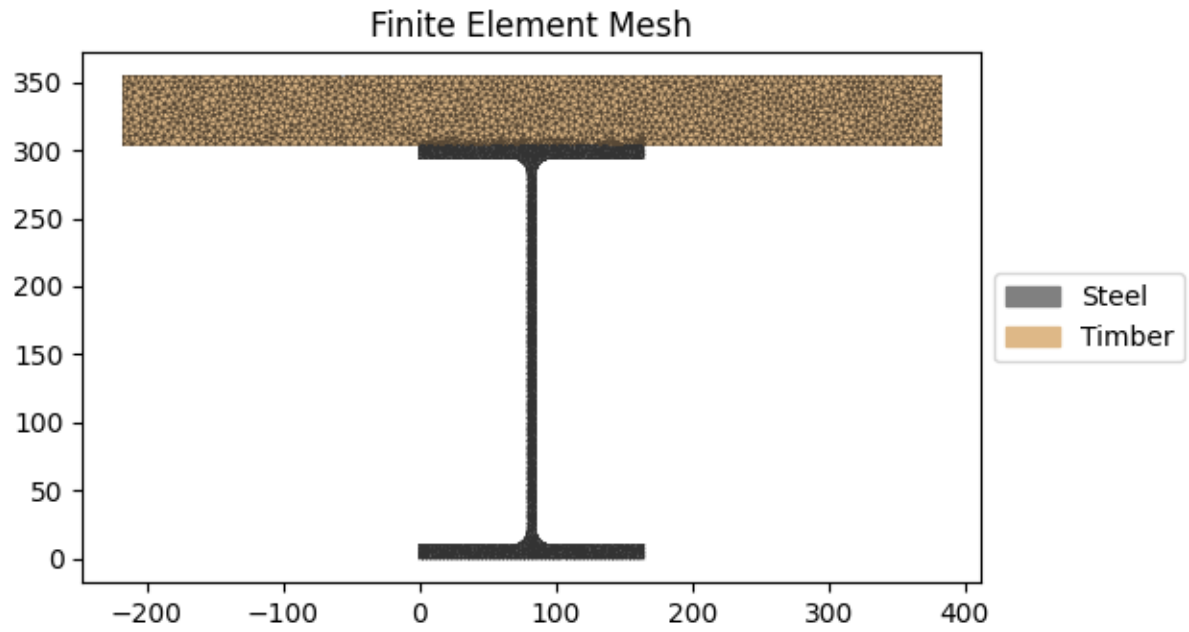
Create a mesh and a Section object. For the mesh use a mesh size of 5 for the UB, 20 for the panel

```
section_geometry.create_mesh(mesh_sizes=[5, 20])
comp_section = Section(section_geometry, time_info=True)
comp_section.display_mesh_info()  # display the mesh information
```

```
Mesh Statistics:
- 9083 nodes
- 4246 elements
- 2 regions
```

Plot the mesh with coloured materials and a line transparency of 0.6

```
comp_section.plot_mesh(materials=True, alpha=0.6)
```




```
<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Perform a geometric, warping and plastic analysis

```
comp_section.calculate_geometric_properties()
comp_section.calculate_warping_properties()
comp_section.calculate_plastic_properties(verbose=True)
```

```
──────────────────────────── Geometric Analysis ────────────────────────────
┌──────────────────────────────────────────────────────────────────────────┐
│  Geometric analysis          100% [ 4.2805 s ] │                         │
│ complete                                                                 │
└──────────────────────────────────────────────────────────────────────────┘
──────────────────────────── Warping Analysis ─────────────────────────────
┌──────────────────────────────────────────────────────────────────────────┐
│  Warping analysis            100% [ 25.1499 s ] │                        │
│ completed                                                                │
│  9083x9083 stiffness         100% [ 6.0900 s ]  │                        │
│ matrix assembled                                                         │
│  Warping function solved     100% [ 0.0652 s ]  │
```

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```
│   (direct)                                                                    │
│    Shear function vectors    100% [ 5.6407 s ]  │                           │
│   assembled                                                                  │
│    Shear functions solved    100% [ 0.1199 s ]  │                           │
│   (direct)                                                                   │
│    Shear and warping         100% [ 3.8104 s ]  │                           │
│   integrals assembled                                                        │
│    Shear deformation         100% [ 5.3781 s ]  │                           │
│   coefficients assembled                                                     │
│    Monosymmetry integrals    100% [ 4.0271 s ]  │                           │
│   assembled                                                                  │
│                                                                              │
──────────────────────────────────────────────────────────────────────────────
d = -185.130884950272; f_norm = 1.0
d = 168.869115049728; f_norm = -1.0
d = -8.13088495027199; f_norm = 0.13960518846748188
d = 13.5521668722403; f_norm = 0.0983423820518058
d = 60.60270845168366; f_norm = 0.008805290832494688
d = 64.90008929872147; f_norm = 0.000627383246551551
d = 65.22746216923495; f_norm = 4.393296043904811e-06
d = 65.22976962543778; f_norm = 2.21127426308038e-09
d = 65.2298027403226; f_norm = -6.080628894287011e-08
---x-axis plastic centroid calculation converged at 6.52298e+01 in 8 iterations.
d = -300.0; f_norm = 1.0
d = 300.0; f_norm = -1.0
d = 0.0; f_norm = 7.263545449900235e-17
d = 5e-07; f_norm = -4.773093592438759e-08
---y-axis plastic centroid calculation converged at 0.00000e+00 in 3 iterations.
d = -185.130884950272; f_norm = 1.0
d = 168.869115049728; f_norm = -1.0
d = -8.13088495027199; f_norm = 0.13960518846748188
d = 13.5521668722403; f_norm = 0.0983423820518058
d = 60.60270845168366; f_norm = 0.008805290832494688
d = 64.90008929872147; f_norm = 0.000627383246551551
d = 65.22746216923495; f_norm = 4.393296043904811e-06
d = 65.22976962543778; f_norm = 2.21127426308038e-09
d = 65.2298027403226; f_norm = -6.080628894287011e-08
---11-axis plastic centroid calculation converged at 6.52298e+01 in 8 iterations.
d = -300.0; f_norm = 1.0
d = 300.0; f_norm = -1.0
d = 0.0; f_norm = 7.263545449900235e-17
d = 5e-07; f_norm = -4.773093592438759e-08
---22-axis plastic centroid calculation converged at 0.00000e+00 in 3 iterations.
──────────────────────────── Plastic Analysis ────────────────────────────────
│                                                                              │
│    Plastic analysis complete  100% [ 0.0874 s ] │                           │
│                                                                              │
──────────────────────────────────────────────────────────────────────────────
```

Perform a stress analysis with N = 100 kN, Mxx = 120 kN.m and Vy = 75 kN

```
stress_post = comp_section.calculate_stress(N=-100e3, Mxx=-120e6, Vy=-75e3)
```

```
──────────────────────────────── Stress Analysis ────────────────────────────────
                                                                                
    Stress analysis complete   100% [ 6.0855 s ]                                
                                                                                
─────────────────────────────────────────────────────────────────────────────────
```

Print the results to the terminal

```
comp_section.display_results()
```

```
    Section Properties

 Property          Value

 A           3.521094e+04
 Perim.      2.206078e+03
 Mass        6.534804e-02
 E.A         1.282187e+09
 E.Qx        2.373725e+11
 E.Qy        1.057805e+11
 cx          8.250000e+01
 cy          1.851309e+02
 E.Ixx_g     6.740447e+13
 E.Iyy_g     1.745613e+13
 E.Ixy_g     1.958323e+13
 E.Ixx_c     2.345949e+13
 E.Iyy_c     8.729240e+12
 E.Ixy_c    -3.906250e-03
 E.Zxx+      1.389212e+11
 E.Zxx-      1.267184e+11
 E.Zyy+      2.909747e+10
 E.Zyy-      2.909747e+10
 rx          1.352644e+02
 ry          8.251112e+01
 phi         0.000000e+00
 E.I11_c     2.345949e+13
 E.I22_c     8.729240e+12
 E.Z11+      1.389212e+11
 E.Z11-      1.267184e+11
 E.Z22+      2.909747e+10
 E.Z22-      2.909747e+10
 r11         1.352644e+02
 r22         8.251112e+01
 E_eff       3.641446e+04
 G_eff       1.390847e+04
 nu_eff      3.090753e-01
 J           3.768367e+11
 Iw          6.687734e+16
 x_se        8.250105e+01
 y_se        2.863411e+02
 x_st        8.250104e+01
 y_st        2.857085e+02
 x1_se       1.049231e-03
```

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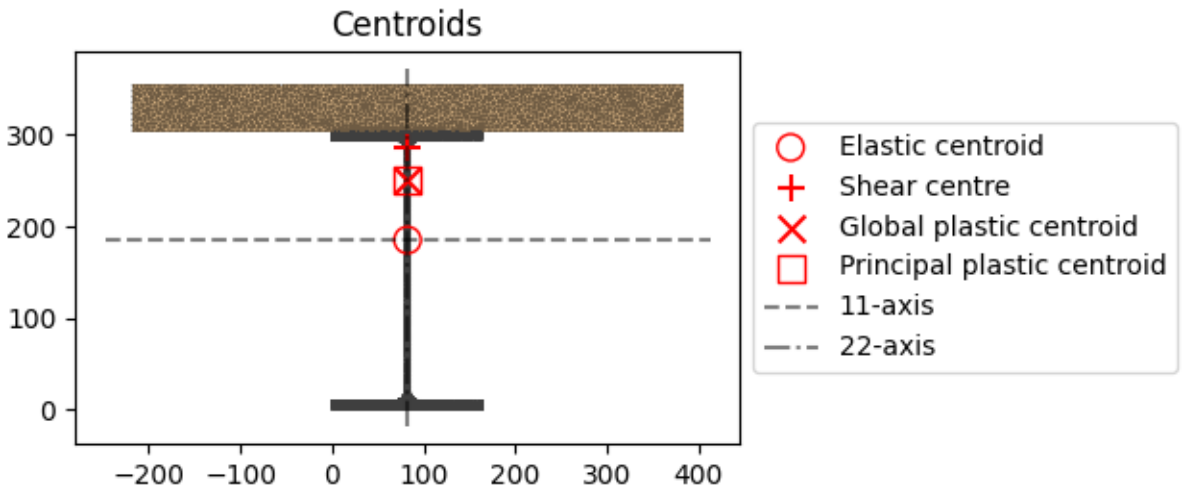
```
| y2_se     |  1.012102e+02 |
| E.A_sx    |  4.022647e+08 |
| E.A_sy    |  3.718571e+08 |
| E.A_s11   |  4.022647e+08 |
| E.A_s22   |  3.718571e+08 |
| betax+    |  2.039435e+02 |
| betax-    | -2.039435e+02 |
| betay+    |  2.098462e-03 |
| betay-    | -2.098462e-03 |
| beta11+   |  2.039435e+02 |
| beta11-   | -2.039435e+02 |
| beta22+   |  2.098462e-03 |
| beta22-   | -2.098462e-03 |
| x_pc      |  8.250000e+01 |
| y_pc      |  2.503607e+02 |
| M_p,xx    |  4.238101e+08 |
| M_p,yy    |  4.226021e+08 |
| x11_pc    |  8.250000e+01 |
| y22_pc    |  2.503607e+02 |
| M_p,11    |  4.238101e+08 |
| M_p,22    |  4.226021e+08 |
+-----------+---------------+
```

Plot the centroids

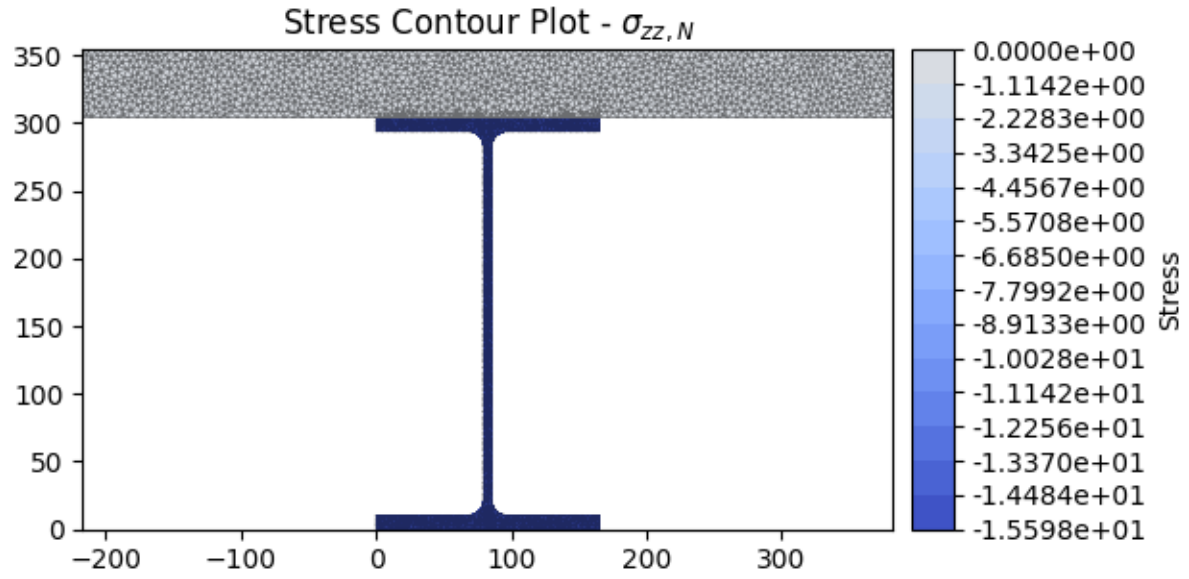
```
comp_section.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Plot the axial stress

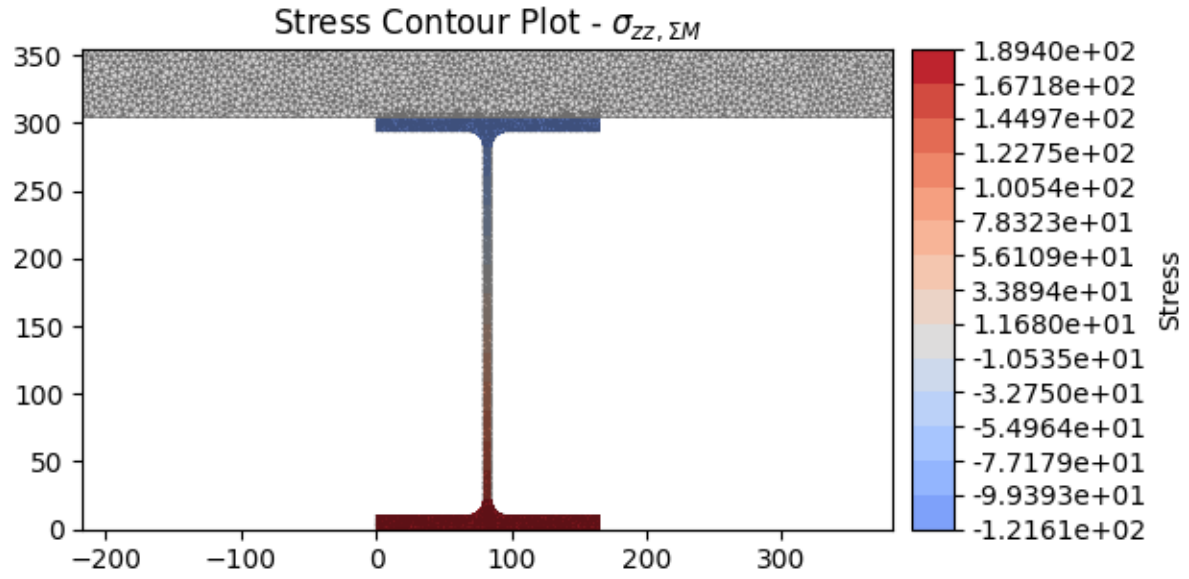
```
stress_post.plot_stress_n_zz(pause=False)
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{zz,N}$'}>
```

Plot the bending stress

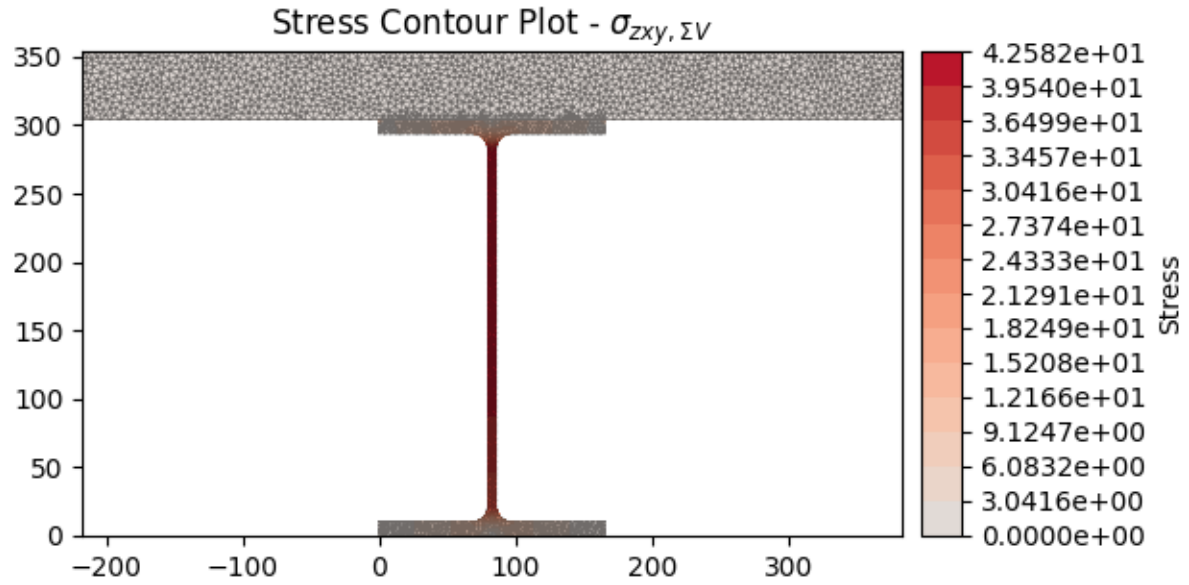
```
stress_post.plot_stress_m_zz(pause=False)
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{zz,\\Sigma M}$'}>
```

Plot the shear stress

```
stress_post.plot_stress_v_zxy()
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{zxy,\\Sigma V}$'}>
```

Total running time of the script: (0 minutes 38.558 seconds)

Frame Analysis Example

Analyse a cross-section to be used in frame analysis.

The following example demonstrates how *sectionproperties* can be used to calculate the cross-section properties required for a frame analysis. Using this method is preferred over executing a geometric and warping analysis as only variables required for a frame analysis are computed. In this example the torsion constant of a rectangular section is calculated for a number of different mesh sizes and the accuracy of the result compared with the time taken to obtain the solution.

```
# sphinx_gallery_thumbnail_number = 1

import time
import numpy as np
import matplotlib.pyplot as plt
import sectionproperties.pre.library.primitive_sections as sections
from sectionproperties.analysis.section import Section
```

Create a rectangular section

```
geometry = sections.rectangular_section(d=100, b=50)
```

Create a list of mesh sizes to analyse

```
mesh_sizes = [3, 4, 5, 10, 15, 20, 25, 30, 40, 50, 75, 100, 200]
j_calc = []  # list to store torsion constants
t_calc = []  # list to store computation times
```

Loop through mesh sizes

```
for mesh_size in mesh_sizes:
    geometry.create_mesh(mesh_sizes=[mesh_size])  # create mesh
    section = Section(geometry)  # create a Section object
    start_time = time.time()  # start timing
    # calculate the frame properties
    (_, _, _, _, j, _) = section.calculate_frame_properties()
    t = time.time() - start_time  # stop timing
    t_calc.append(t)  # save the time
    j_calc.append(j)  # save the torsion constant
    # print the result
    str = "Mesh Size: {0}; ".format(mesh_size)
    str += "Solution Time {0:.5f} s; ".format(t)
    str += "Torsion Constant: {0:.12e}".format(j)
    print(str)
```

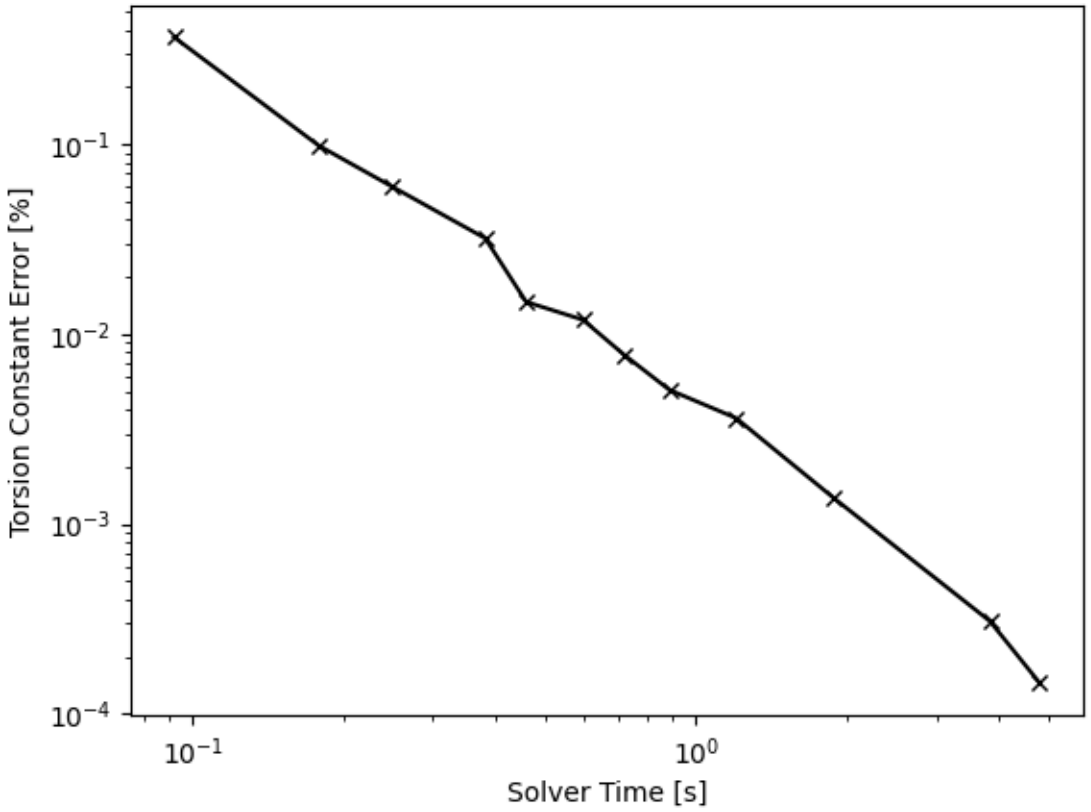
```
Mesh Size: 3; Solution Time 6.62067 s; Torsion Constant: 2.858525191518e+06
Mesh Size: 4; Solution Time 4.81756 s; Torsion Constant: 2.858529348617e+06
Mesh Size: 5; Solution Time 3.83594 s; Torsion Constant: 2.858533994778e+06
Mesh Size: 10; Solution Time 1.87892 s; Torsion Constant: 2.858564308063e+06
Mesh Size: 15; Solution Time 1.20050 s; Torsion Constant: 2.858628499542e+06
Mesh Size: 20; Solution Time 0.88515 s; Torsion Constant: 2.858670496343e+06
Mesh Size: 25; Solution Time 0.71865 s; Torsion Constant: 2.858748138885e+06
Mesh Size: 30; Solution Time 0.59589 s; Torsion Constant: 2.858865014806e+06
Mesh Size: 40; Solution Time 0.45969 s; Torsion Constant: 2.858947255775e+06
Mesh Size: 50; Solution Time 0.38113 s; Torsion Constant: 2.859438375764e+06
Mesh Size: 75; Solution Time 0.24807 s; Torsion Constant: 2.860241467603e+06
Mesh Size: 100; Solution Time 0.17823 s; Torsion Constant: 2.861326245766e+06
Mesh Size: 200; Solution Time 0.09177 s; Torsion Constant: 2.869013885610e+06
```

Compute the error, assuming that the finest mesh (index 0) gives the 'correct' value

```
correct_val = j_calc[0]
j_np = np.array(j_calc)
error_vals = (j_calc - correct_val) / j_calc * 100
```

Produce a plot of the accuracy of the torsion constant with computation time

```
plt.loglog(t_calc[1:], error_vals[1:], "kx-")
plt.xlabel("Solver Time [s]")
plt.ylabel("Torsion Constant Error [%]")
plt.show()
```



Total running time of the script: (0 minutes 23.172 seconds)

Importing Geometry from CAD

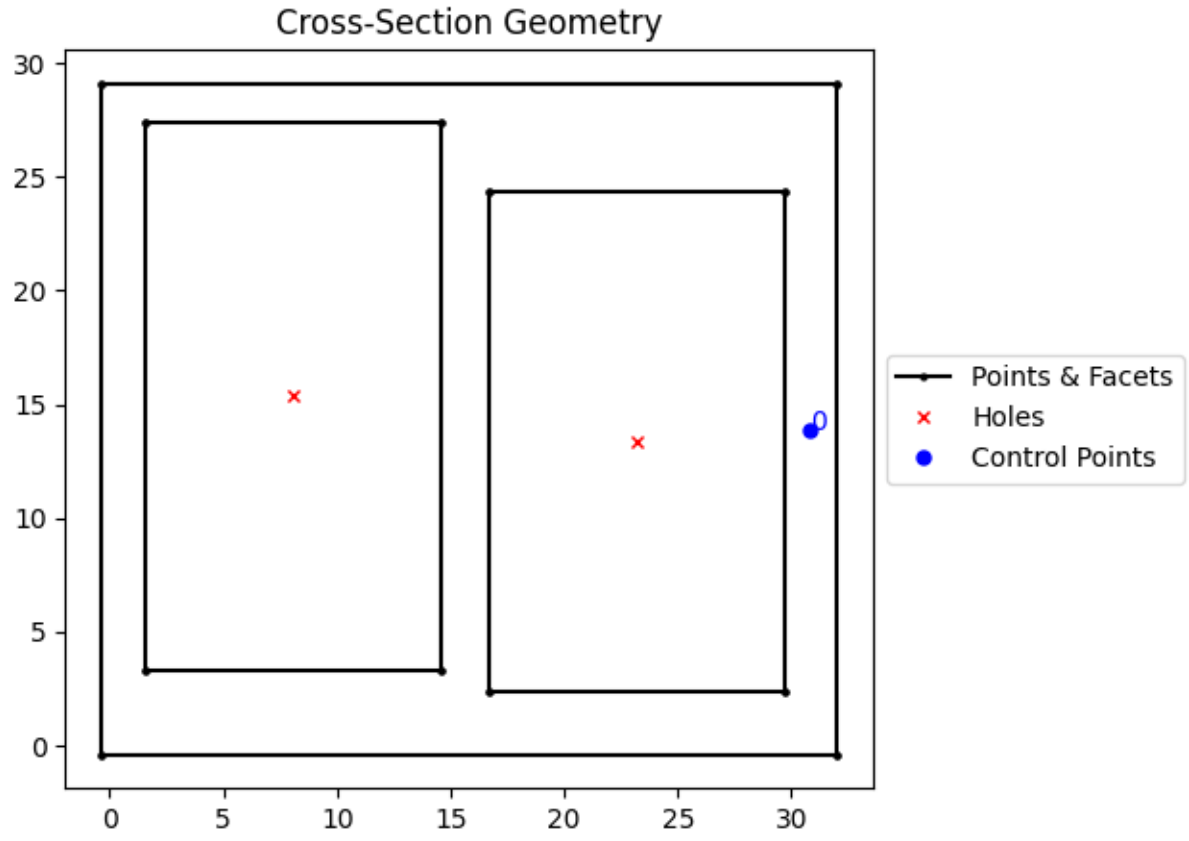
Demonstrates loading *Geometry* and *CompoundGeometry* objects from *.dxf* and *.3dm* (Rhino) files.

```
# sphinx_gallery_thumbnail_number = 8

from sectionproperties.pre.geometry import Geometry, CompoundGeometry
from sectionproperties.analysis.section import Section
```

Load a geometry with a single region from a dxf file

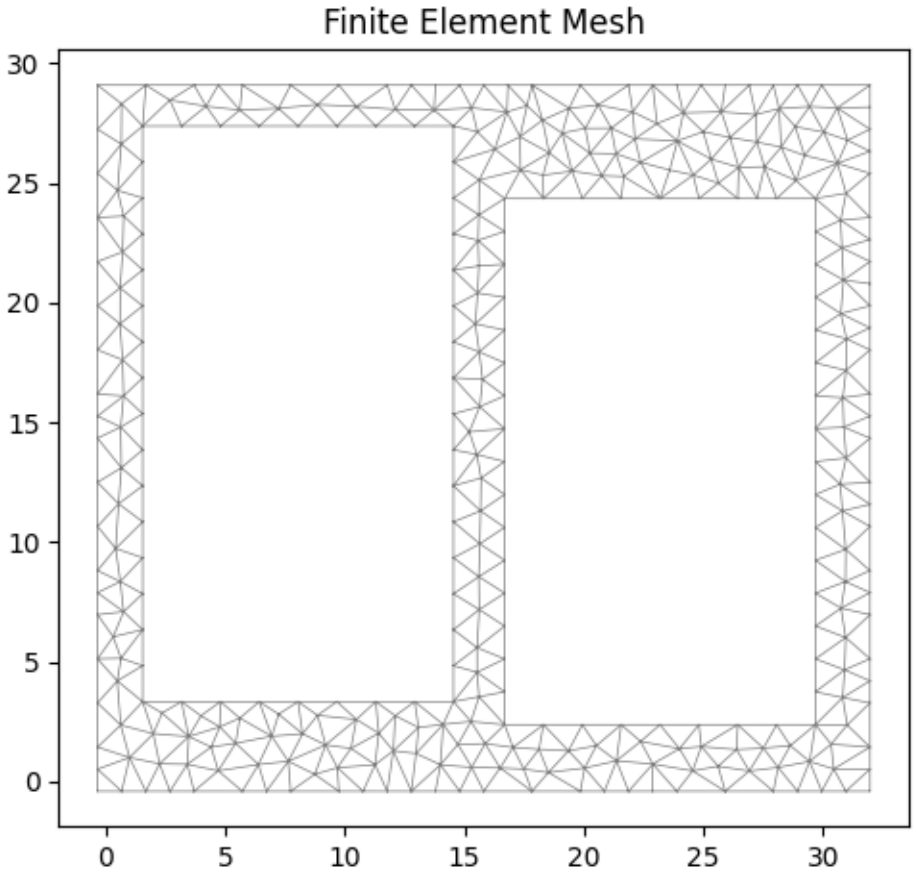
```
geom = Geometry.from_dxf(dxf_filepath="files/section_holes.dxf")
geom.plot_geometry()
```




```
<AxesSubplot: title={'center': 'Cross-Section Geometry'}>
```

Generate a mesh

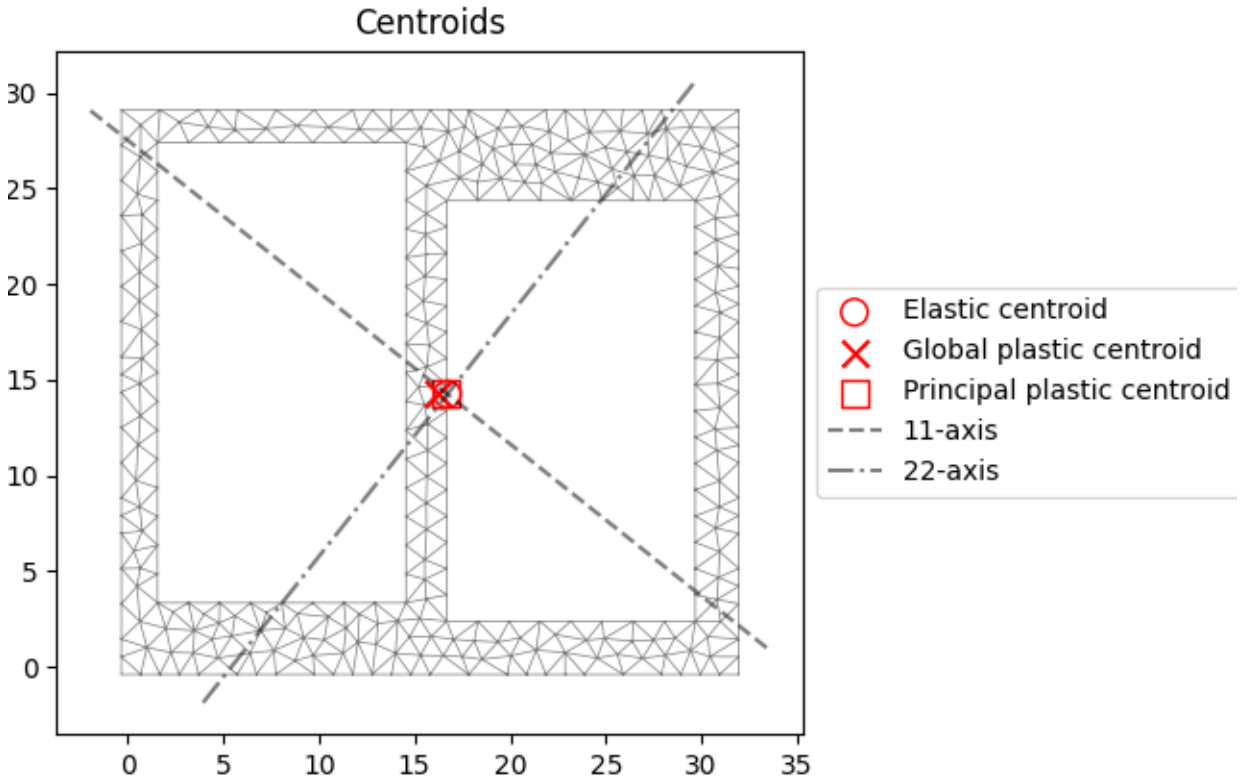
```
geom.create_mesh([1])
sec = Section(geom)
sec.plot_mesh(materials=False)
```




```
<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Conduct a geometric & plastic analysis

```
sec.calculate_geometric_properties()
sec.calculate_plastic_properties()
sec.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Display the geometric & plastic properties

```
sec.display_results()
```

```
    Section Properties

Property          Value

| A          |  3.543777e+02 |
| Perim.     |  1.235768e+02 |
| Qx         |  5.047390e+03 |
| Qy         |  5.923689e+03 |
| cx         |  1.671575e+01 |
| cy         |  1.424297e+01 |
| Ixx_g      |  1.137665e+05 |
| Iyy_g      |  1.387654e+05 |
| Ixy_g      |  8.892907e+04 |
| Ixx_c      |  4.187664e+04 |
| Iyy_c      |  3.974650e+04 |
| Ixy_c      |  4.558164e+03 |
| Zxx+       |  2.820099e+03 |
```

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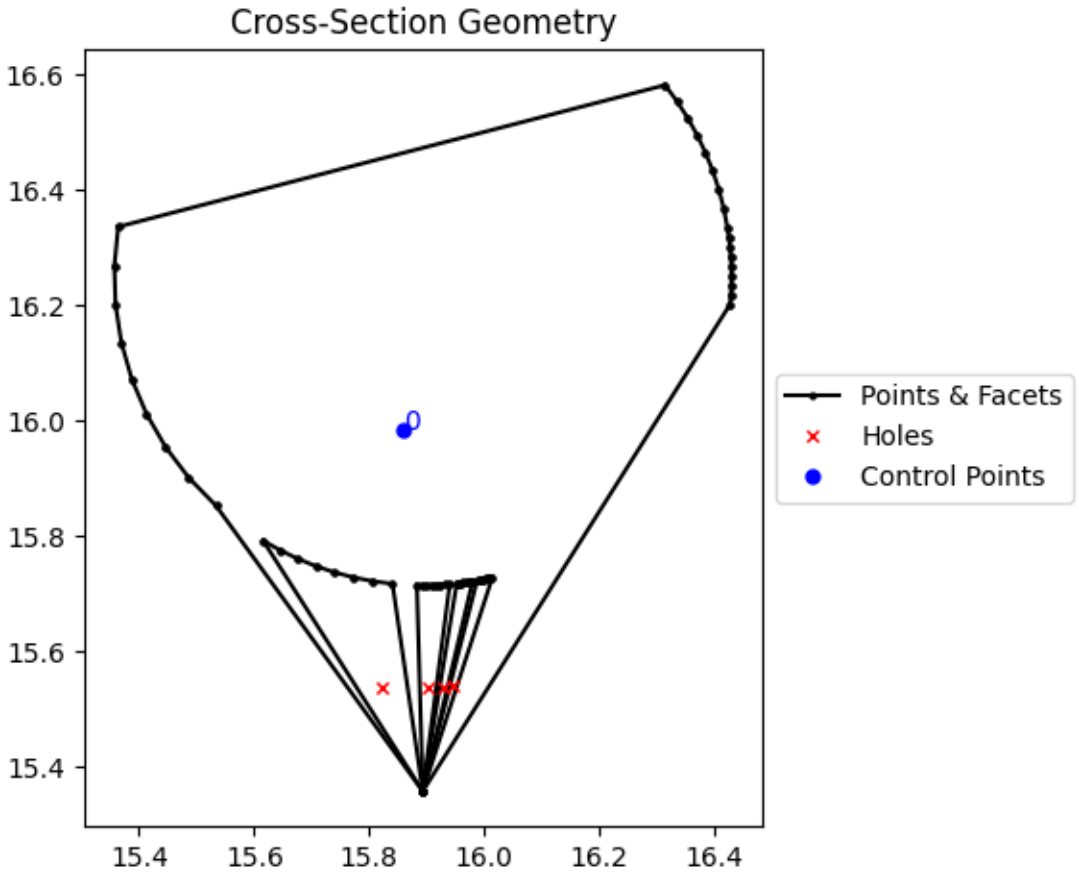
```
| Zxx-     |  2.860789e+03 |
| Zyy+     |  2.606925e+03 |
| Zyy-     |  2.330570e+03 |
| rx       |  1.087058e+01 |
| ry       |  1.059049e+01 |
| phi      | -3.842404e+01 |
| I11_c    |  4.549251e+04 |
| I22_c    |  3.613063e+04 |
| Z11+     |  2.155143e+03 |
| Z11-     |  2.061571e+03 |
| Z22+     |  1.717082e+03 |
| Z22-     |  1.599443e+03 |
| r11      |  1.133018e+01 |
| r22      |  1.009728e+01 |
| x_pc     |  1.626707e+01 |
| y_pc     |  1.427513e+01 |
| Sxx      |  3.548750e+03 |
| Syy      |  3.153834e+03 |
| SF_xx+   |  1.258378e+00 |
| SF_xx-   |  1.240479e+00 |
| SF_yy+   |  1.209791e+00 |
| SF_yy-   |  1.353246e+00 |
| x11_pc   |  1.662479e+01 |
| y22_pc   |  1.427103e+01 |
| S11      |  3.492177e+03 |
| S22      |  2.898242e+03 |
| SF_11+   |  1.620392e+00 |
| SF_11-   |  1.693940e+00 |
| SF_22+   |  1.687888e+00 |
| SF_22-   |  1.812032e+00 |
+----------+---------------+
```

Load a geometry with multiple holes from a dxf file

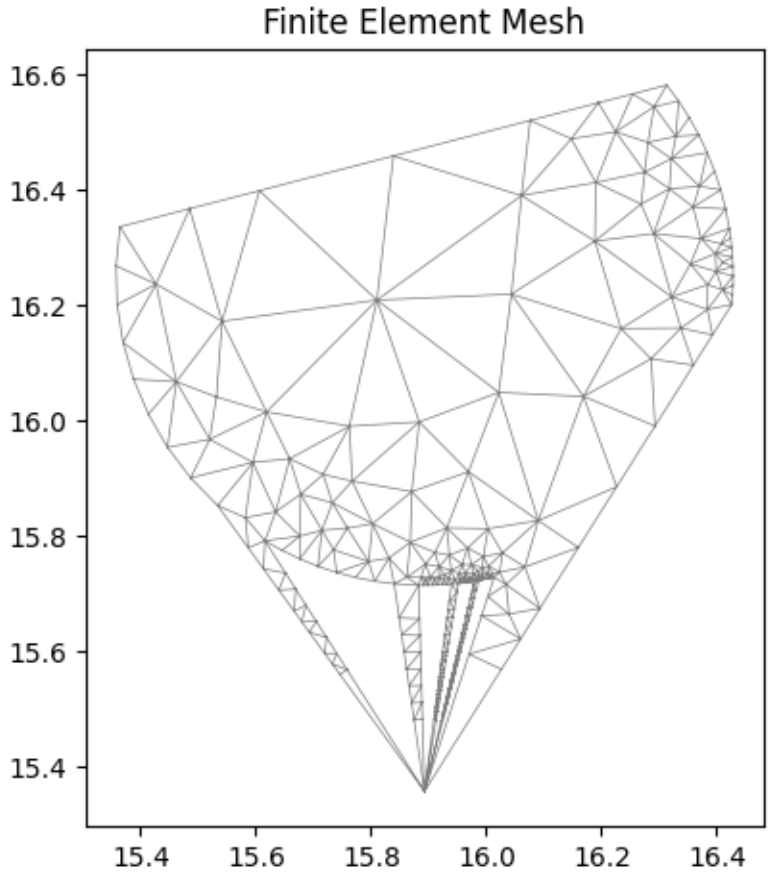
```
geom = Geometry.from_dxf(dxf_filepath="files/section_holes_complex.dxf")
geom.plot_geometry()
```




```
<AxesSubplot: title={'center': 'Cross-Section Geometry'}>
```

Generate a mesh

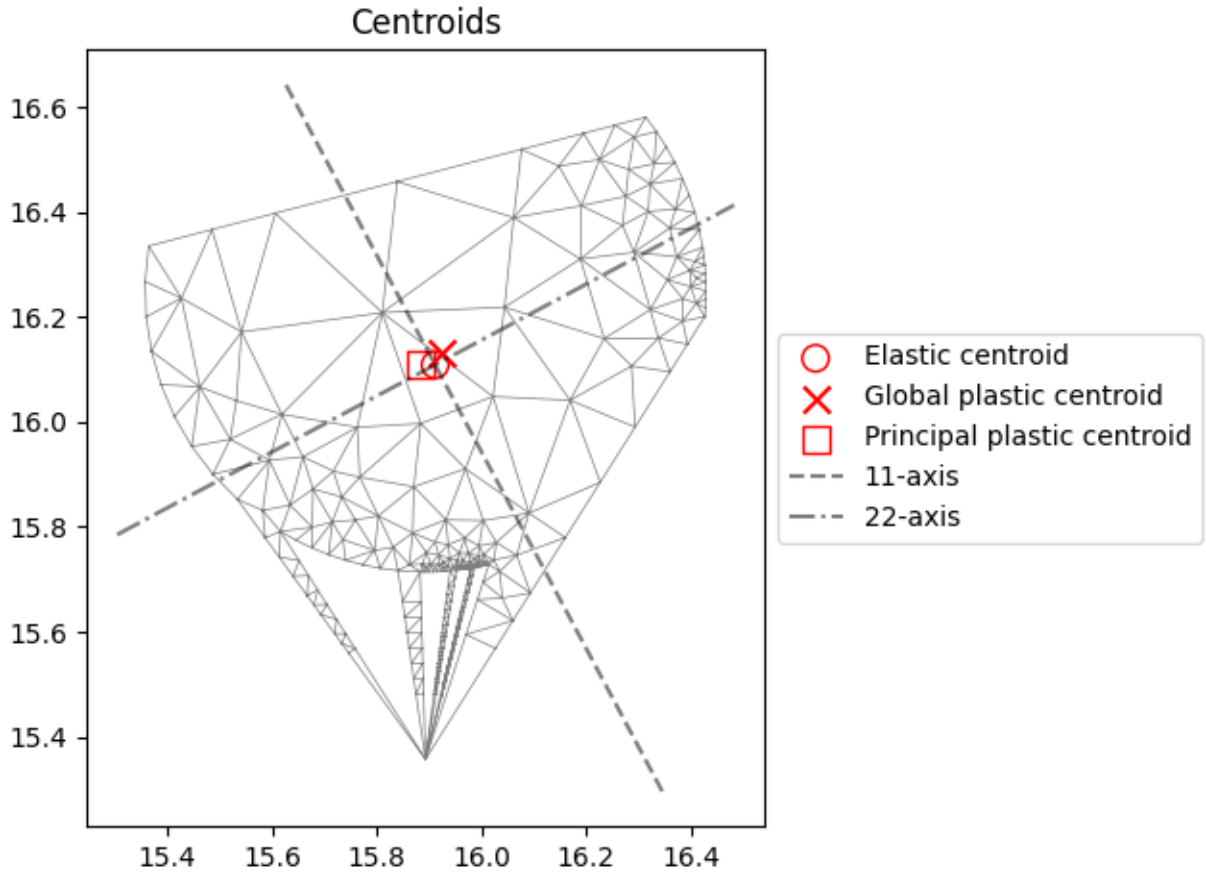
```
geom.create_mesh([1])
sec = Section(geom)
sec.plot_mesh(materials=False)
```




```
<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Conduct a geometric & plastic analysis

```
sec.calculate_geometric_properties()
sec.calculate_plastic_properties()
sec.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Display the geometric & plastic properties

```
sec.display_results()
```

```
    Section Properties

Property          Value

| A          |  6.997735e-01 |
| Perim.     |  3.531378e+00 |
| Qx         |  1.127282e+01 |
| Qy         |  1.113424e+01 |
| cx         |  1.591121e+01 |
| cy         |  1.610924e+01 |
| Ixx_g      |  1.816356e+02 |
| Iyy_g      |  1.772080e+02 |
| Ixy_g      |  1.793714e+02 |
| Ixx_c      |  3.907173e-02 |
| Iyy_c      |  4.872212e-02 |
| Ixy_c      |  7.193759e-03 |
| Zxx+       |  8.271797e-02 |
```

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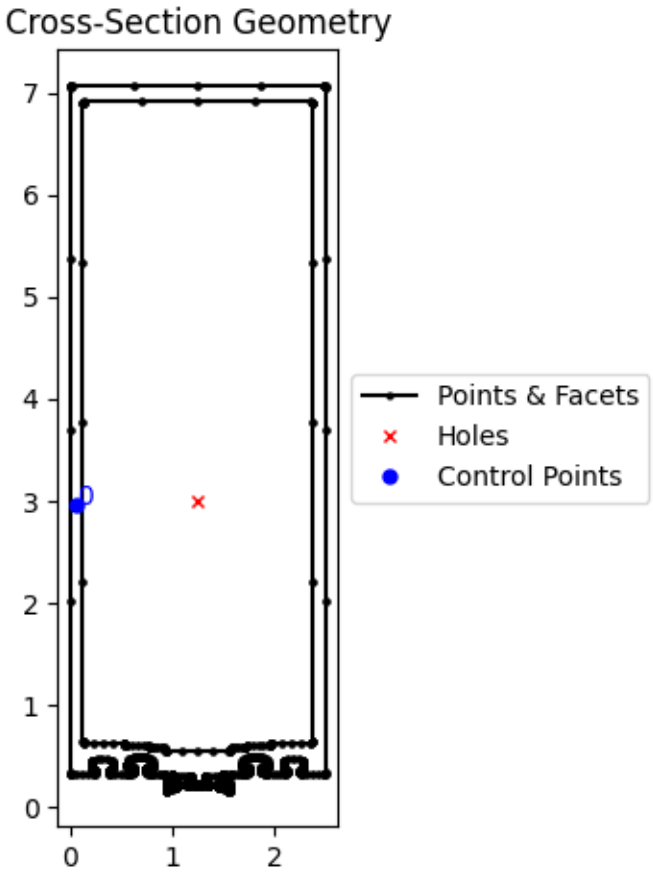
```
| Zxx-      |  5.195063e-02 |
| Zyy+      |  9.416195e-02 |
| Zyy-      |  8.798053e-02 |
| rx        |  2.362939e-01 |
| ry        |  2.638665e-01 |
| phi       | -6.192581e+01 |
| I11_c     |  5.255906e-02 |
| I22_c     |  3.523478e-02 |
| Z11+      |  8.973609e-02 |
| Z11-      |  1.082595e-01 |
| Z22+      |  5.379674e-02 |
| Z22-      |  7.702330e-02 |
| r11       |  2.740596e-01 |
| r22       |  2.243918e-01 |
| x_pc      |  1.592489e+01 |
| y_pc      |  1.613076e+01 |
| Sxx       |  1.364187e-01 |
| Syy       |  1.554974e-01 |
| SF_xx+    |  1.649203e+00 |
| SF_xx-    |  2.625929e+00 |
| SF_yy+    |  1.651383e+00 |
| SF_yy-    |  1.767407e+00 |
| x11_pc    |  1.588269e+01 |
| y22_pc    |  1.611059e+01 |
| S11       |  1.620643e-01 |
| S22       |  1.302126e-01 |
| SF_11+    |  1.806010e+00 |
| SF_11-    |  1.496999e+00 |
| SF_22+    |  2.420454e+00 |
| SF_22-    |  1.690561e+00 |
+-----------+---------------+
```

Load a geometry from a 3dm (Rhino) file

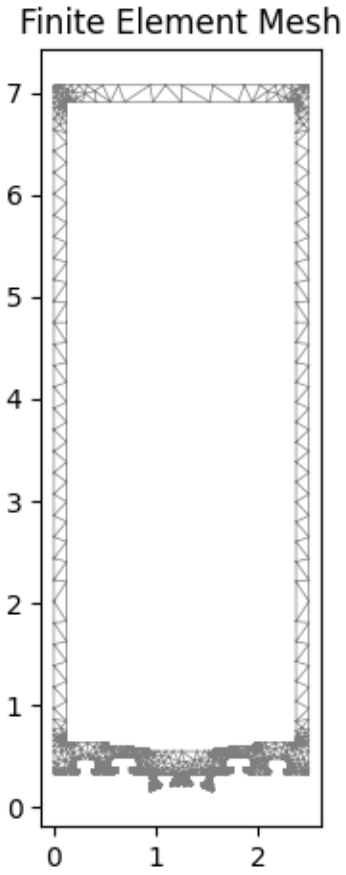
```
geom = Geometry.from_3dm(filepath="files/complex_shape.3dm")
geom.plot_geometry()
```




```
<AxesSubplot: title={'center': 'Cross-Section Geometry'}>
```

Generate a mesh

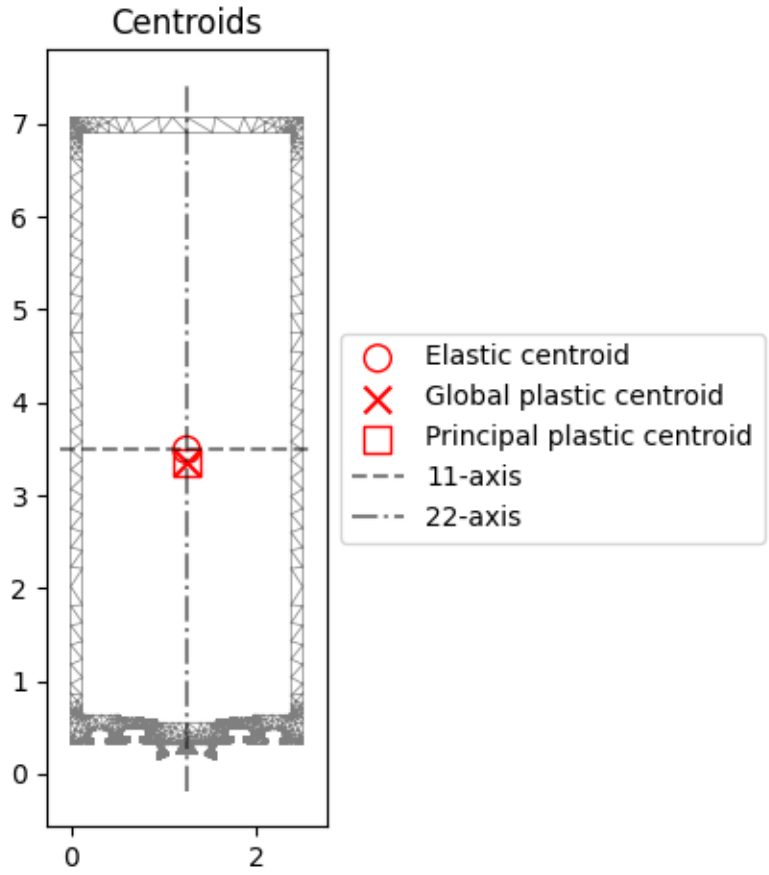
```
geom.create_mesh([1])
sec = Section(geom)
sec.plot_mesh(materials=False)
```




```
<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Conduct a geometric & plastic analysis

```
sec.calculate_geometric_properties()
sec.calculate_plastic_properties()
sec.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Display the geometric & plastic properties

```
sec.display_results()
```

```
    Section Properties

 Property         Value

| A          | 2.602049e+00 |
| Perim.     | 2.130651e+01 |
| Qx         | 9.082300e+00 |
| Qy         | 3.252560e+00 |
| cx         | 1.249999e+00 |
| cy         | 3.490441e+00 |
| Ixx_g      | 4.773758e+01 |
| Iyy_g      | 6.824622e+00 |
| Ixy_g      | 1.135287e+01 |
| Ixx_c      | 1.603635e+01 |
| Iyy_c      | 2.758925e+00 |
| Ixy_c      | 5.755182e-06 |
| Zxx+       | 4.476583e+00 |
```

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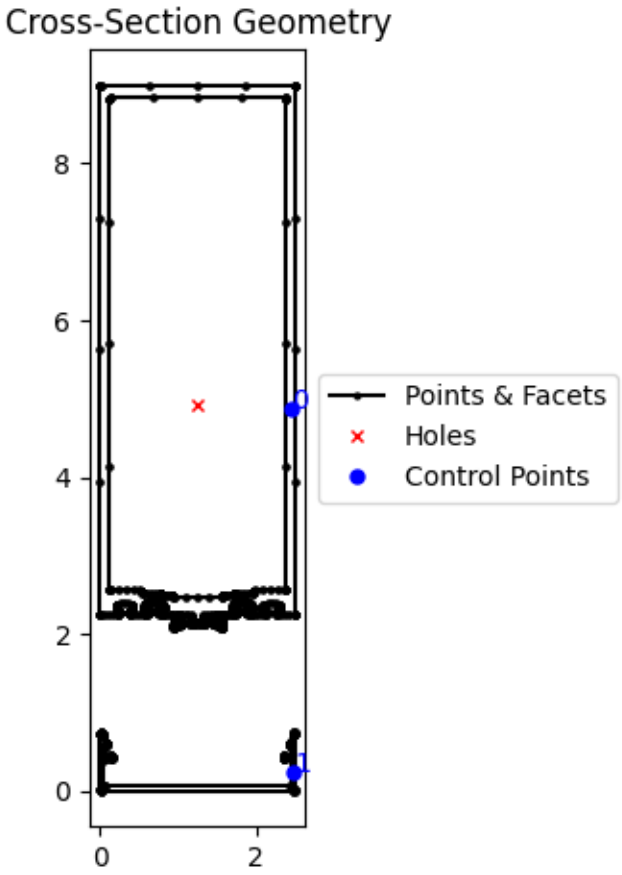
Zxx-	4.813398e+00
Zyy+	2.207139e+00
Zyy-	2.207142e+00
rx	2.482533e+00
ry	1.029704e+00
phi	0.000000e+00
I11_c	1.603635e+01
I22_c	2.758925e+00
Z11+	4.476583e+00
Z11-	4.813398e+00
Z22+	2.207139e+00
Z22-	2.207142e+00
r11	2.482533e+00
r22	1.029704e+00
x_pc	1.249996e+00
y_pc	3.353406e+00
Sxx	5.801661e+00
Syy	2.504936e+00
SF_xx+	1.296002e+00
SF_xx-	1.205315e+00
SF_yy+	1.134925e+00
SF_yy-	1.134923e+00
x11_pc	1.249996e+00
y22_pc	3.353406e+00
S11	5.801661e+00
S22	2.504936e+00
SF_11+	1.296002e+00
SF_11-	1.205315e+00
SF_22+	1.134925e+00
SF_22-	1.134923e+00

Load a compound geometry with multiple regions from a 3dm (Rhino) file

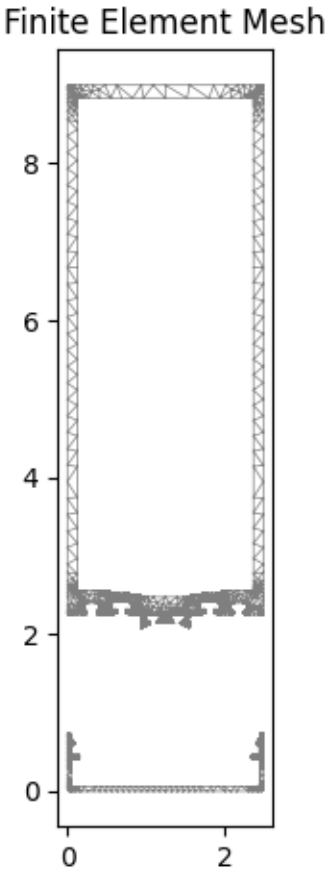
```
geom = CompoundGeometry.from_3dm(filepath="files/compound_shape.3dm")
geom.plot_geometry()
```




```
<AxesSubplot: title={'center': 'Cross-Section Geometry'}>
```

Generate a mesh

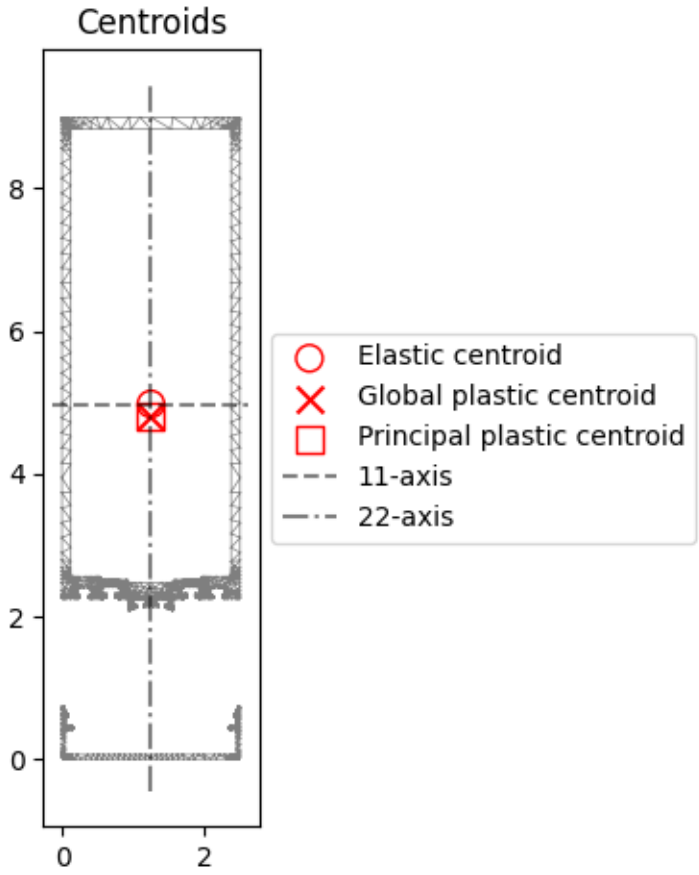
```
geom.create_mesh([1])
sec = Section(geom)
sec.plot_mesh(materials=False)
```




```
<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Conduct a geometric & plastic analysis N.B a warping analysis would be invalid due to the lack of connectivity between the two regions

```
sec.calculate_geometric_properties()
sec.calculate_plastic_properties()
sec.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Display the geometric & plastic properties

```
sec.display_results()
```

```
    Section Properties

Property          Value

| A          |  2.838725e+00 |
| Perim.     | -1.000000e+00 |
| Qx         |  1.413033e+01 |
| Qy         |  3.548404e+00 |
| cx         |  1.249999e+00 |
| cy         |  4.977702e+00 |
| Ixx_g      |  9.237156e+01 |
| Iyy_g      |  7.393179e+00 |
| Ixy_g      |  1.766291e+01 |
| Ixx_c      |  2.203499e+01 |
| Iyy_c      |  2.957676e+00 |
| Ixy_c      |  4.909920e-06 |
| Zxx+       |  5.480936e+00 |
```

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```
| Zxx-      |  4.426740e+00 |
| Zyy+      |  2.366139e+00 |
| Zyy-      |  2.366142e+00 |
| rx        |  2.786088e+00 |
| ry        |  1.020736e+00 |
| phi       |  0.000000e+00 |
| I11_c     |  2.203499e+01 |
| I22_c     |  2.957676e+00 |
| Z11+      |  5.480936e+00 |
| Z11-      |  4.426740e+00 |
| Z22+      |  2.366139e+00 |
| Z22-      |  2.366142e+00 |
| r11       |  2.786088e+00 |
| r22       |  1.020736e+00 |
| x_pc      |  1.249996e+00 |
| y_pc      |  4.805339e+00 |
| Sxx       |  6.956641e+00 |
| Syy       |  2.699474e+00 |
| SF_xx+    |  1.269243e+00 |
| SF_xx-    |  1.571504e+00 |
| SF_yy+    |  1.140877e+00 |
| SF_yy-    |  1.140876e+00 |
| x11_pc    |  1.249996e+00 |
| y22_pc    |  4.805339e+00 |
| S11       |  6.956641e+00 |
| S22       |  2.699474e+00 |
| SF_11+    |  1.269243e+00 |
| SF_11-    |  1.571504e+00 |
| SF_22+    |  1.140877e+00 |
| SF_22-    |  1.140876e+00 |
```

Total running time of the script: (0 minutes 16.655 seconds)

8.2.2 Advanced Examples

The following examples demonstrates how *sectionproperties* can be used for more academic purposes.

Advanced Plotting

Harness some of the plotting features provided by *plotting_context()*.

```
# sphinx_gallery_thumbnail_number = 1

import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
import matplotlib.pyplot as plt
```

The below example creates a 100x6 SHS and plots the geometry, mesh, centroids and torsion vectors in a 2x2 subplot arrangement.

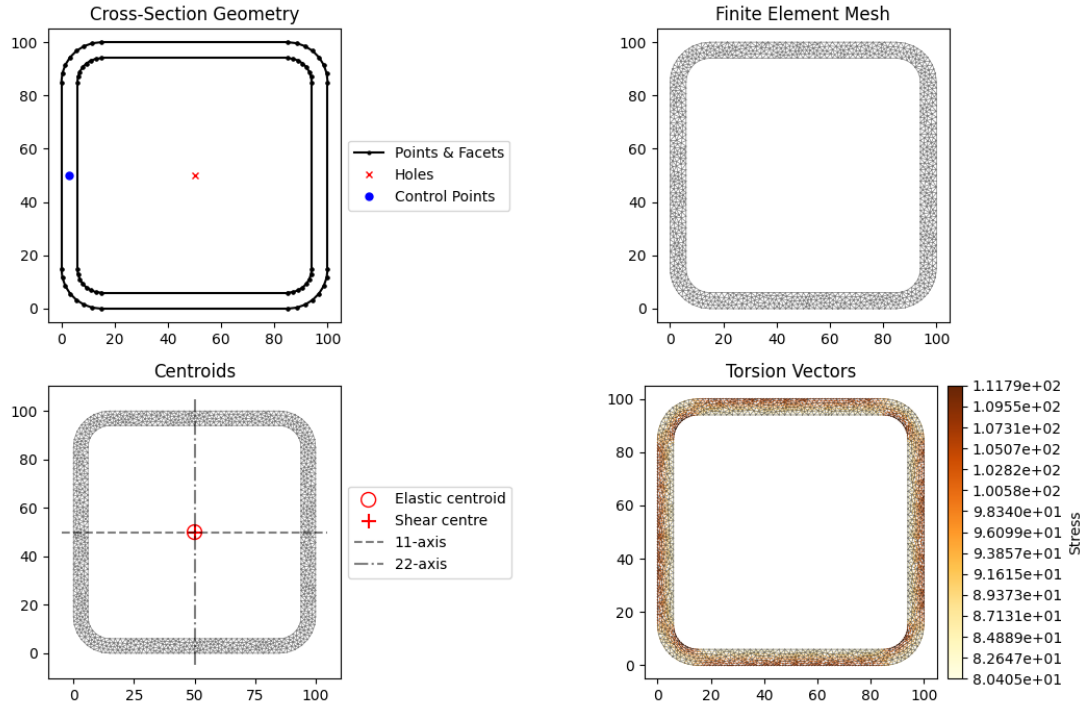
```
geometry = steel_sections.rectangular_hollow_section(d=100, b=100, t=6, r_out=15, n_r=8)

# Plot the geometry
ax = geometry.plot_geometry(nrows=2, ncols=2, figsize=(12, 7), render=False, labels=[])
fig = ax.get_figure()  # get the figure

# Create a mesh and section object, for the mesh, use a maximum area of 2
geometry.create_mesh(mesh_sizes=[2])
section = Section(geometry)
section.plot_mesh(ax=fig.axes[1], materials=False)  # plot the mesh

# Perform a geometry and warping analysis
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.plot_centroids(ax=fig.axes[2])  # plot the cetnroids

# Perform a stress analysis with Mzz = 10 kN.m
stress = section.calculate_stress(Mzz=10e6)
stress.plot_vector_mzz_zxy(
    ax=fig.axes[3], title="Torsion Vectors"
)  # plot the torsion vectors
plt.show()  # show the plot
```



Total running time of the script: (0 minutes 16.214 seconds)

Mesh Refinement

Perform a mesh refinement study.

In this example the convergence of the torsion constant is investigated through an analysis of an I Section. The mesh is refined both by modifying the mesh size and by specifying the number of points making up the root radius. The figure below the example code shows that mesh refinement adjacent to the root radius is a far more efficient method in obtaining fast convergence when compared to reducing the mesh area size for the entire section.

```
# sphinx_gallery_thumbnail_number = 1

import numpy as np
import matplotlib.pyplot as plt
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
```

Define mesh sizes

```
mesh_size_list = [200, 100, 50, 20, 10, 5]
nr_list = [4, 8, 12, 16, 20, 24, 32]
```

Initialise result lists

```
mesh_results = []
mesh_elements = []
nr_results = []
nr_elements = []
```

Calculate reference solution

```
geometry = steel_sections.i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=32)
geometry.create_mesh(mesh_sizes=[5])  # create mesh
section = Section(geometry)  # create a Section object
section.calculate_geometric_properties()
section.calculate_warping_properties()
j_reference = section.get_j()  # get the torsion constant
```

Run through mesh_sizes with n_r = 8

```
for mesh_size in mesh_size_list:
    geometry = steel_sections.i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=8)
    geometry.create_mesh(mesh_sizes=[mesh_size])  # create mesh
    section = Section(geometry)  # create a Section object
    section.calculate_geometric_properties()
    section.calculate_warping_properties()

    mesh_elements.append(len(section.elements))
    mesh_results.append(section.get_j())
```

Run through n_r with mesh_size = 10

```
for n_r in nr_list:
    geometry = steel_sections.i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=n_r)
    geometry.create_mesh(mesh_sizes=[10])  # create mesh
    section = Section(geometry)  # create a Section object
```

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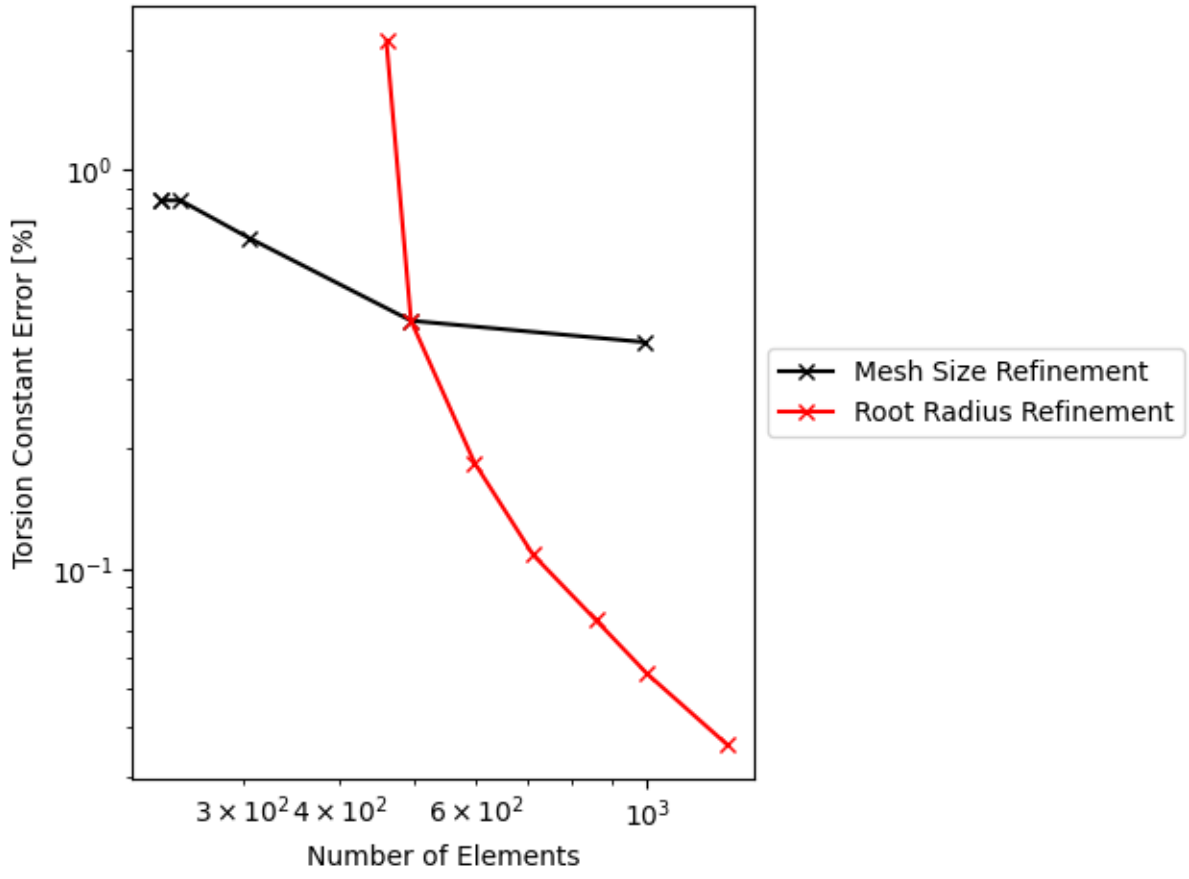
```
    section.calculate_geometric_properties()
    section.calculate_warping_properties()

    nr_elements.append(len(section.elements))
    nr_results.append(section.get_j())
```

Convert results to a numpy array and compute the error

```
mesh_results = np.array(mesh_results)
nr_results = np.array(nr_results)
mesh_error_vals = (mesh_results - j_reference) / mesh_results * 100
nr_error_vals = (nr_results - j_reference) / nr_results * 100
```

Plot the results

```
(fig, ax) = plt.subplots()
ax.loglog(mesh_elements, mesh_error_vals, "kx-", label="Mesh Size Refinement")
ax.loglog(nr_elements, nr_error_vals, "rx-", label="Root Radius Refinement")
plt.xlabel("Number of Elements")
plt.ylabel("Torsion Constant Error [%]")
plt.legend(loc="center left", bbox_to_anchor=(1, 0.5))
plt.tight_layout()
plt.show()
```



Total running time of the script: (1 minutes 8.805 seconds)

Torsion Constant of a Rectangle

Plot the variation of the torsion constant with aspect ratio of a rectangle section.

In this example, the aspect ratio of a rectangular section is varied whilst keeping a constant cross-sectional area and the torsion constant calculated. The variation of the torsion constant with the aspect ratio is then plotted.

```
# sphinx_gallery_thumbnail_number = 1

import numpy as np
import matplotlib.pyplot as plt
import sectionproperties.pre.library.primitive_sections as sections
from sectionproperties.analysis.section import Section
```

Rectangle dimensions

```
d_list = []
b_list = np.linspace(0.2, 1, 20)
j_list = []  # list holding torsion constant results
```

Number of elements for each analysis

```
n = 100
```

Loop through all the widths

```
for b in b_list:
    # calculate d assuming area = 1
    d = 1 / b
    d_list.append(d)

    # compute mesh size
    ms = d * b / n

    # perform a warping analysis on rectangle
    geometry = sections.rectangular_section(d=d, b=b)
    geometry.create_mesh(mesh_sizes=[ms])
    section = Section(geometry)
    section.calculate_geometric_properties()
    section.calculate_warping_properties()

    # get the torsion constant
    j = section.get_j()
    print("d/b = {0:.3f}; J = {1:.5e}".format(d / b, j))
    j_list.append(j)
```

```
d/b = 25.000; J = 1.30315e-02
d/b = 17.060; J = 1.88338e-02
d/b = 12.380; J = 2.55717e-02
d/b = 9.391; J = 3.31309e-02
d/b = 7.367; J = 4.14010e-02
```

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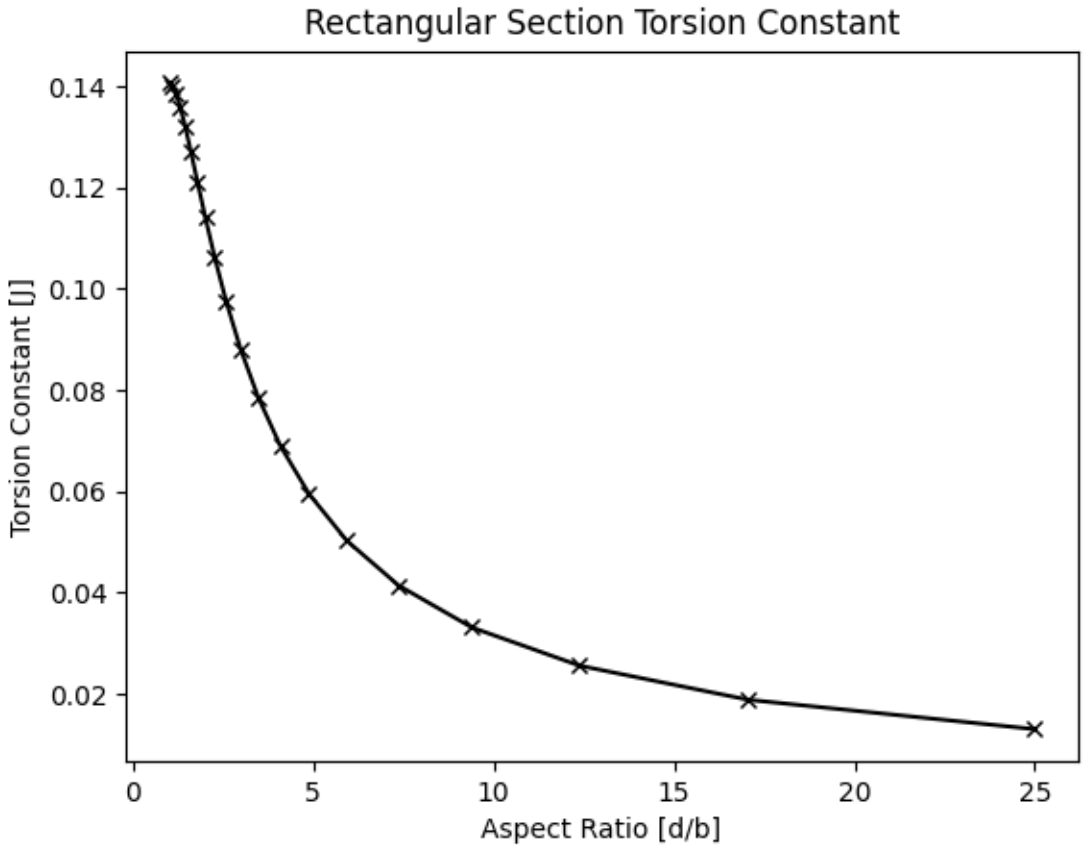
```
d/b = 5.934; J = 5.02289e-02
d/b = 4.881; J = 5.95117e-02
d/b = 4.086; J = 6.90261e-02
d/b = 3.470; J = 7.86512e-02
d/b = 2.983; J = 8.81645e-02
d/b = 2.593; J = 9.73762e-02
d/b = 2.274; J = 1.06053e-01
d/b = 2.010; J = 1.14042e-01
d/b = 1.790; J = 1.21141e-01
d/b = 1.604; J = 1.27224e-01
d/b = 1.446; J = 1.32194e-01
d/b = 1.310; J = 1.35982e-01
d/b = 1.192; J = 1.38616e-01
d/b = 1.090; J = 1.40125e-01
d/b = 1.000; J = 1.40617e-01
```

Plot the torsion constant as a function of the aspect ratio

```
(fig, ax) = plt.subplots()
ax.plot(np.array(d_list) / b_list, j_list, "kx-")
ax.set_xlabel("Aspect Ratio [d/b]")
ax.set_ylabel("Torsion Constant [J]")
ax.set_title("Rectangular Section Torsion Constant")
plt.show()
```



Total running time of the script: (0 minutes 21.878 seconds)

Symmetric and Unsymmetric Beams in Complex Bending

Calculate section properties of two different beams given in examples from 'Aircraft Structures,' by Peery. These cases have known results, and the output from *sectionproperties* can be compared for accuracy. These examples represent a more rigourous 'proof' against a 'real' problem. Only results that have values in the reference material are tested here.

BibTeX Entry for reference:

```
@Book{Peery,
    title = {Aircraft Structures},
    author = {David J. Peery},
    organization = {Pensylvania State University},
    publisher = {McGraw-Hill Book Company},
    year = {1950},
    edition = {First},
    ISBN = {978-0486485805}
}
```

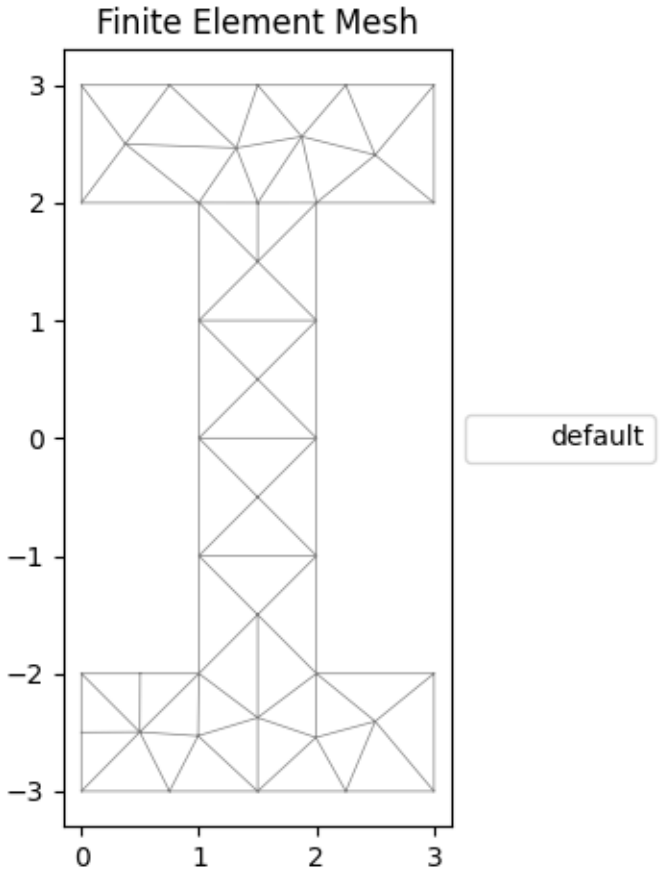
```
from sectionproperties.pre.library import nastran_sections
from sectionproperties.analysis.section import Section
```

Example 1 in Sec. 6.2 (Symmetric Bending)

This is a symmetric I-section with no lateral supports, undergoing pure unidirectional cantilever bending. Note that units here are **inches**, to match the text.

We'll use a very coarse mesh here, to show a conservative comparison for accuracy. Theoretically, with more discretization, we would capture the real results more accurately.

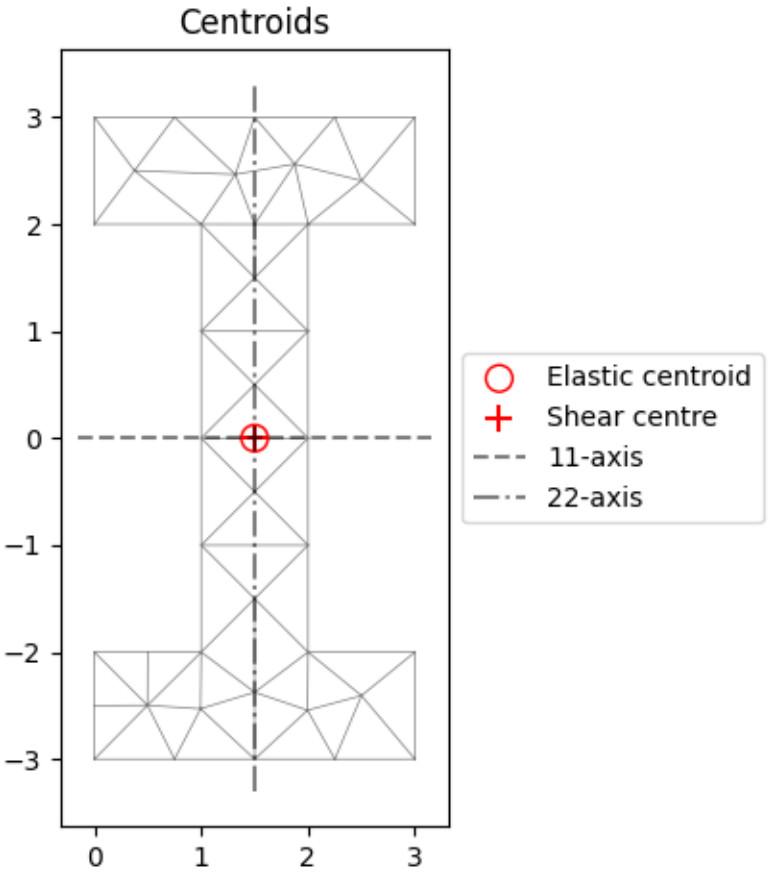
```
geometry = nastran_sections.nastran_i(6, 3, 3, 1, 1, 1)
geometry = geometry.shift_section(x_offset=0, y_offset=-3)
geometry = geometry.create_mesh(mesh_sizes=[0.25])
section = Section(geometry)
section.plot_mesh()
```




```
<AxesSubplot: title={'center': 'Finite Element Mesh'}>
```

Perform a geometric analysis on the section, and plot properties We don't need warping analysis for these simple checks, but sectionproperties needs them before evaluating stress.

```
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Directly from the example, we know that the 2nd moment of inertia resisting the bending is 43.3 in^4.

```
section.section_props.ixx_g
```

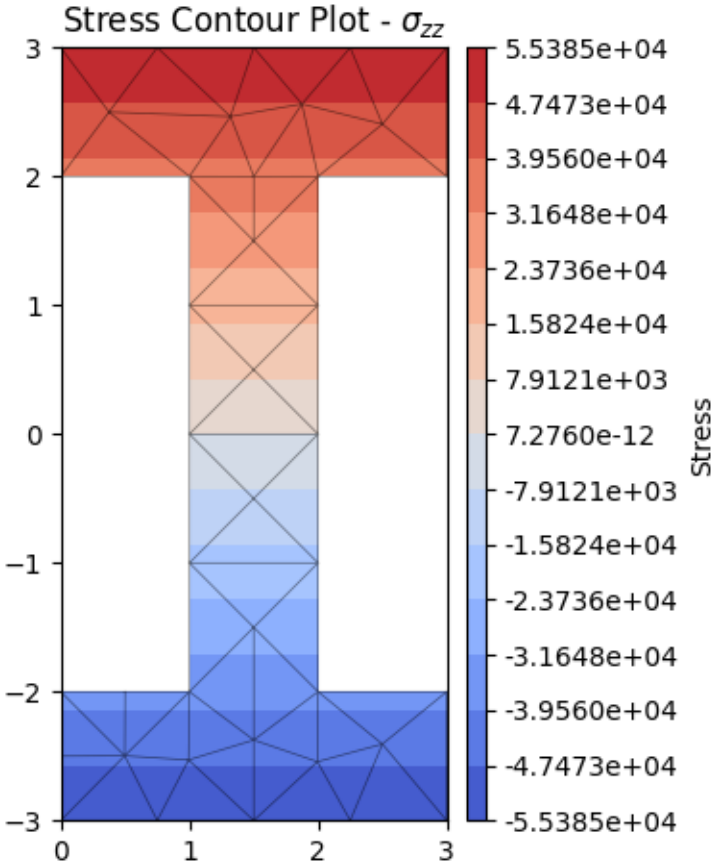
```
43.3333333333333
```

From statics, we know the max bending moment on the beam will be 80,000 in-lbs. We can apply this moment to the section, and evaluate stress.

```
moment = 8e5
stress = section.calculate_stress(Mxx=moment)
```

Next we can extract the max stress from the section, and let's go ahead and look at the calculated fringe plot. Refer to the stress example for details.

```
numerical_result = max(stress.get_stress()[0]["sig_zz"])
stress.plot_stress_zz()
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{zz}$'}>
```

From the book, and simple statics, we know the max stress is 55,427.3 psi.

```
numerical_result
```

```
55384.61538461558
```

This example is admittedly more simple, but it's still a nice check for the basics on validity of the package.

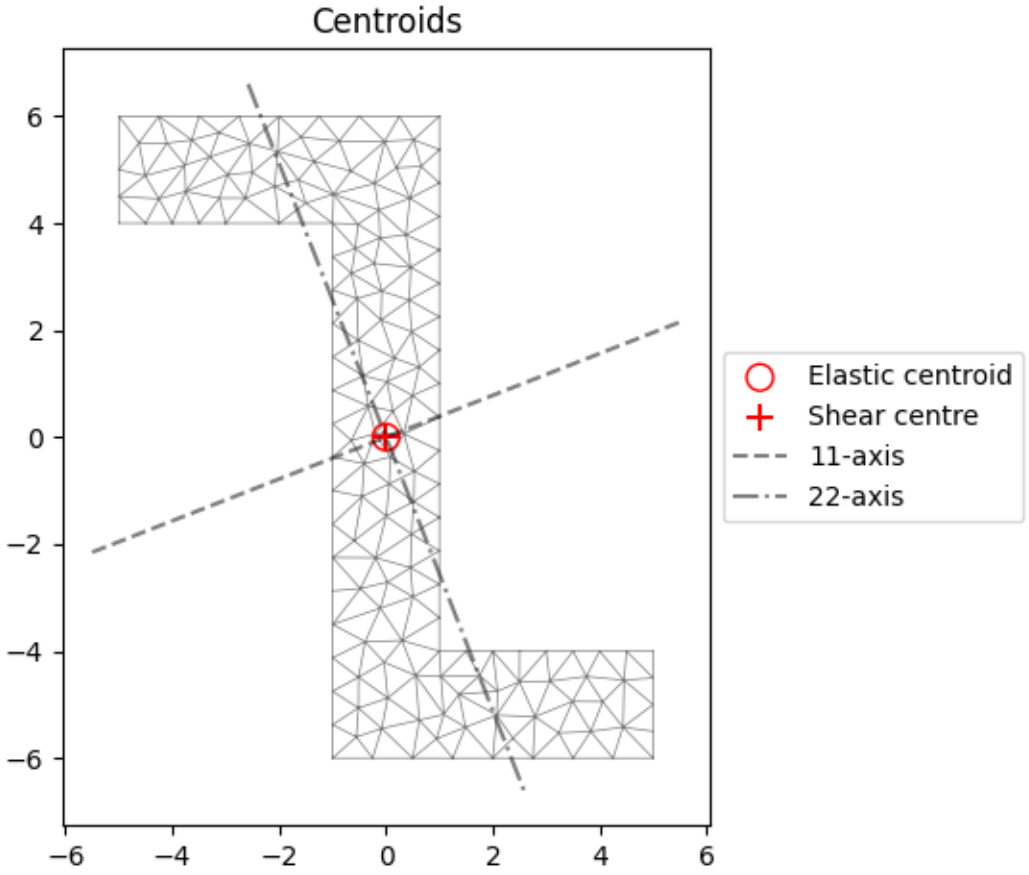
```
print(f"Theoretical Result = 55427 [psi]")
print(f"  Numerical Result = {numerical_result:.0f} [psi]")
acc = (55427 - numerical_result) / 55427
print(f"          Accuracy = {acc:%}")
```

```
Theoretical Result = 55427 [psi]
  Numerical Result = 55385 [psi]
          Accuracy = 0.076469%
```

Example 1 in Sec. 7.2. (Unsymmetric Bending)

Moving on to something a bit more advanced... This is an unsymmetric Z-section with no lateral supports. Note that units here are **inches**, to match the text.

```
base_geom = nastran_sections.nastran_zed(4, 2, 8, 12)
base_geom = base_geom.shift_section(-5, -6)
base_geom = base_geom.create_mesh([0.25])
section = Section(base_geom)
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.plot_centroids()
```




```
<AxesSubplot: title={'center': 'Centroids'}>
```

Checking each property against the reference text:

```
props = section.section_props
print("    Property | Theoretical | Numerical")
print(f"    ixx_g    | {693.3:<12.1f}| {props.ixx_g:<.1f}")
print(f"    iyy_g    | {173.3:<12.1f}| {props.iyy_g:<.1f}")
print(f"    ixy_g    | {-240:<12.1f}| {props.ixy_g:<.1f}")
print(f"    i11_c    | {787:<12.1f}| {props.i11_c:<.1f}")
print(f"    i22_c    | {79.5:<12.1f}| {props.i22_c:<.1f}")
```

```
Property | Theoretical | Numerical
ixx_g    | 693.3       | 693.3
iyy_g    | 173.3       | 173.3
ixy_g    | -240.0      | -240.0
i11_c    | 787.0       | 787.2
i22_c    | 79.5        | 79.5
```

Section properties all look good, so we can move on to some stress analysis. Before we do, we will need a quick function to pull stress at a certain point. This is a bit of a hack! Future versions of *sectionproperties* will have a much more robust system for getting stress at an arbitrary location. This particular function will work for the locations we need, since we *know* a node will be there.

```
from typing import Tuple


def get_node(nodes, coord) -> Tuple[int, tuple]:
    """
    This function will loop over the node list provided,
    finding the index of the coordinates you want.
    Returns the index in the nodes list, and the coords.
    """
    for index, var in enumerate(nodes):
        if all(var == coord):
            return index, var
        else:
            continue

    raise ValueError(f"No node found with coordinates: {coord}")
```

The load applied in the reference is -100,000 in-lbs about the x-axis, and 10,000 inb-lbs about the y-axis.

```
stress = section.calculate_stress(Mxx=-1e5, Myy=1e4)
```

Check stress at location A (see *docs* page for details)

```
A = (-5, 4)
text_result = 1210
n, _ = get_node(section.mesh_nodes, A)
numerical_result = stress.get_stress()[0]["sig_zz"][n]
print(text_result, numerical_result)
```

```
1210 1210.2272727272666
```

Check stress at location B (see *docs* page for details)

```
B = (-5, 6)
text_result = 580
n, _ = get_node(section.mesh_nodes, B)
numerical_result = stress.get_stress()[0]["sig_zz"][n]
print(text_result, numerical_result)
```

```
580 579.545454545451
```

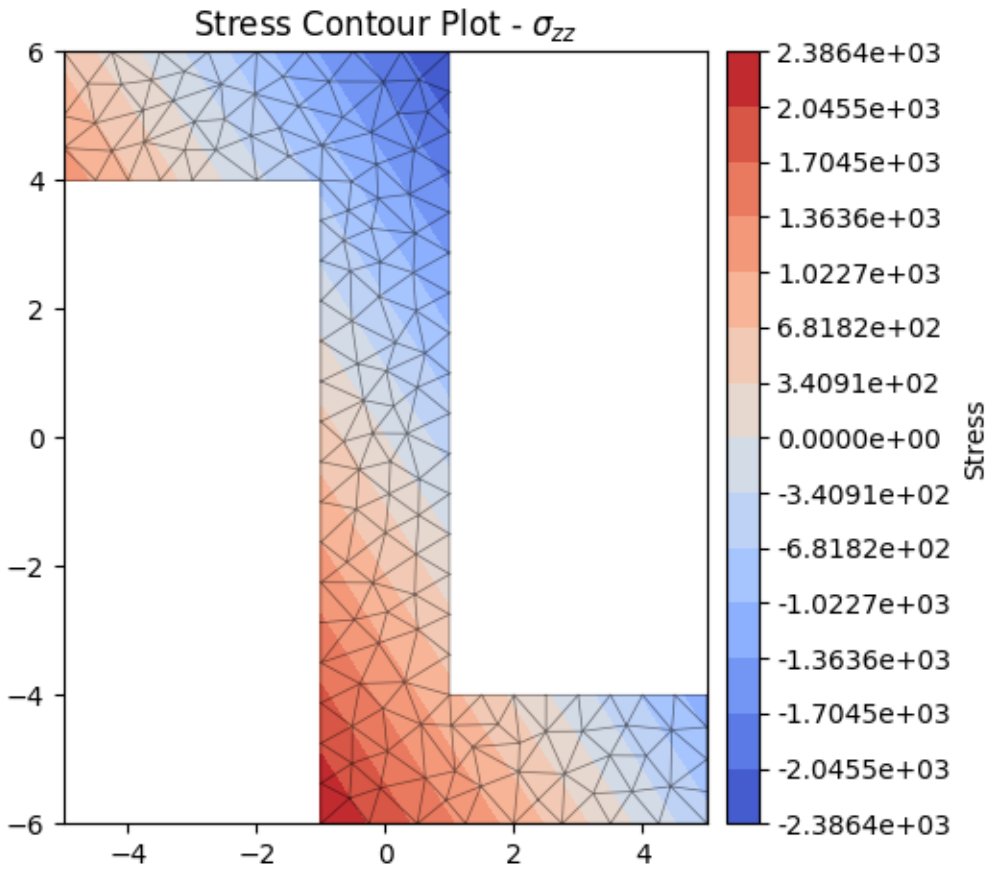
Check stress at location C (see *docs* page for details)

```
C = (1, 6)
text_result = -2384
n, _ = get_node(section.mesh_nodes, C)
numerical_result = stress.get_stress()[0]["sig_zz"][n]
print(text_result, numerical_result)
```

```
-2384 -2386.363636363626
```

Looking at total axial stress over the section.

```
stress.plot_stress_zz()
```




```
<AxesSubplot: title={'center': 'Stress Contour Plot - $\\sigma_{zz}$'}>
```

Total running time of the script: (0 minutes 4.063 seconds)

Approximation of Torsion Constant for Trapezoidal Sections

Trapezoidal elements or components of a cross-section are quite common in bridge structures, either concrete or steel composite construction. However, it's common to determine the torsion constant of the trapezoidal section by using a rectangular approximation. For example, this is done in the Autodesk Structural Bridge Design software when there is a haunch in a Steel Composite Beam

The question then arises, when is it appropriate to make the rectangular approximation to a trapezoidal section, and what might the expected error be?

Define the Imports

Here we bring in the primative section shapes and also the more generic Shapely *Polygon* object.

```
import numpy as np
import matplotlib.pyplot as plt
from shapely import Polygon
import sectionproperties.pre.geometry as geometry
import sectionproperties.pre.pre as pre
import sectionproperties.pre.library.primitive_sections as sections
from sectionproperties.analysis.section import Section
```

Define the Calculation Engine

It's better to collect the relevant section property calculation in a single function. We are only interested in the torsion constant, so this is straightforward enough. *geom* is the Section Property Geometry object; *ms* is the mesh size.

```
def get_section_j(geom, ms, plot_geom=False):
    geom.create_mesh(mesh_sizes=[ms])
    section = Section(geom)
    if plot_geom:
        section.plot_mesh()
    section.calculate_geometric_properties()
    section.calculate_warping_properties()
    return section.get_j()
```

Define the Mesh Density

The number of elements per unit area is an important input to the calculations even though we are only examining ratios of the results. A nominal value of 100 is reasonable.

```
n = 100  # mesh density
```

Create and Analyse the Section

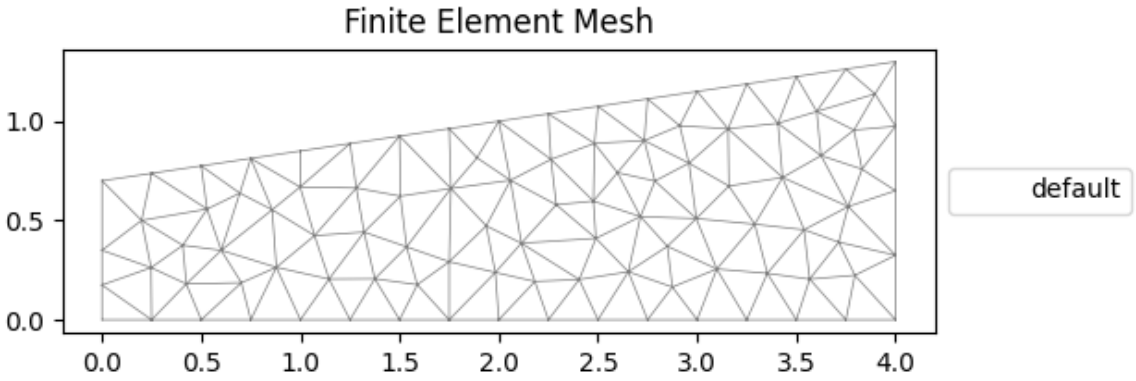
This function accepts the width b and a slope S to create the trapezoid. Since we are only interested in relative results, the nominal dimensions are immaterial. There are a few ways to parametrize the problem, but it has been found that setting the middle height of trapezoid (i.e. the average height) to a unit value works fine.

```
def do_section(b, S, d_mid=1, plot_geom=False):
    delta = S * d_mid
    d1 = d_mid - delta
    d2 = d_mid + delta

    # compute mesh size
    ms = d_mid * b / n

    points = []
    points.append([0, 0])
    points.append([0, d1])
    points.append([b, d2])
    points.append([b, 0])
    if S < 1.0:
        trap_geom = geometry.Geometry(Polygon(points))
    else:
        trap_geom = sections.triangular_section(h=d2, b=b)
    jt = get_section_j(trap_geom, ms, plot_geom)

    rect_geom = sections.rectangular_section(d=(d1 + d2) / 2, b=b)
    jr = get_section_j(rect_geom, ms, plot_geom)
    return jt, jr, d1, d2
```

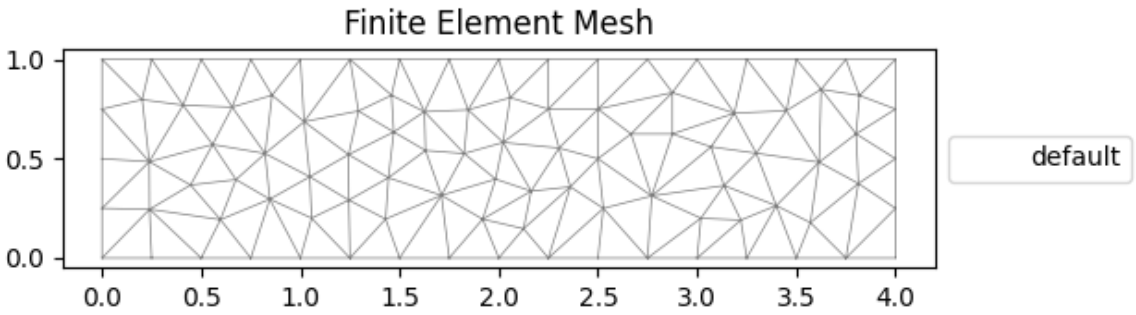
Example Section

The analysis for a particular section looks as follows:

```
b, S = 4.0, 0.3
jt, jr, d1, d2 = do_section(b, S, plot_geom=True)
print(f"{b=}; {S=}; {jr=}; {jt=}; {jr/jt}")
```


Finite Element Mesh
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
default

-



-

```
b=4.0; S=0.3; jr=1.1236937204755924; jt=1.1530949943412052; 0.9745022968533389
```

Create Loop Variables

The slope S is 0 for a rectangle, and 1 for a triangle and is defined per the plot below. A range of S, between 0.0 and 1.0 and a range of b are considered, between 1 and 10 here (but can be extended)

```
b_list = np.logspace(0, np.log10(10), 10)
S_list = np.linspace(0.0, 1.0, 10)
j_rect = np.zeros((len(b_list), len(S_list)))
j_trap = np.zeros((len(b_list), len(S_list)))
```

The Main Loop

Execute the double loop to get the ratios for combinations of S and b.

An optional deugging line is left in for development but commented out.

```
for i, b in enumerate(b_list):
    for j, S in enumerate(S_list):
        jt, jr, d1, d2 = do_section(b, S)
        j_trap[i][j] = jt
```

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```
        j_rect[i][j] = jr

        # print(f"{b=:.3}; {S=:.3}; {d1=:.5}; {d2=:.5}; J rect = {j_rect[i][j]:.5e}; J␣
↪trap = {j_trap[i][j]:.5e}; J ratio = {j_rect[i][j]/j_trap[i][j]:.5}")
```

Calculate the Ratios

Courtesy of numpy, this is easy:

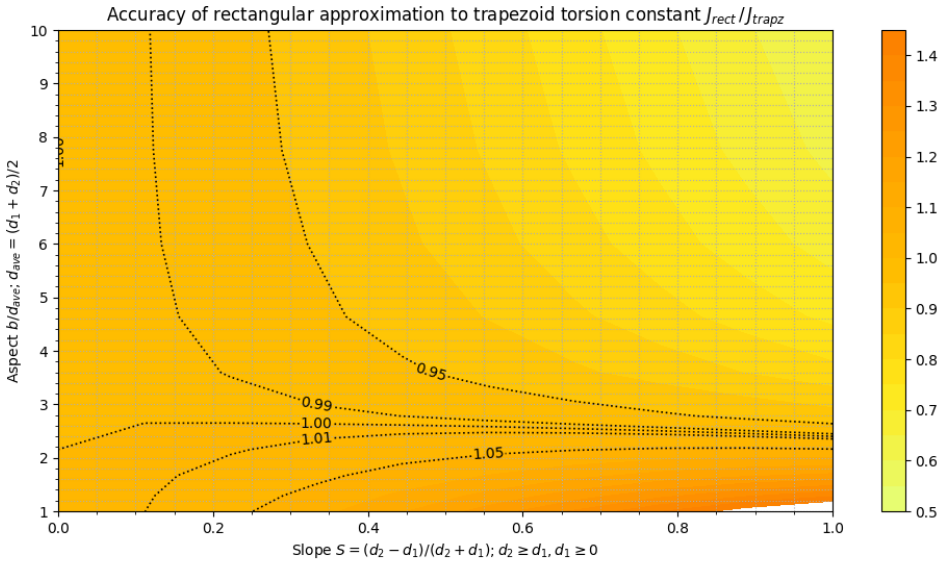
```
j_ratio = j_rect / j_trap
```

Plot the Results

Here we highlight a few of the contours to illustrate the accuracy and behaviour of the approximation.

As expected, when the section is rectangular, the error is small, but as it increases towards a triangle the accuracy generally reduces. However, there is an interesting line at an aspect ratio of about 2.7 where the rectangular approximation is always equal to the trapezoid's torsion constant.

```
levels = np.arange(start=0.5, stop=1.5, step=0.05)
plt.figure(figsize=(12, 6))
cs = plt.contour(
    S_list,
    b_list,
    j_ratio,
    levels=[0.95, 0.99, 1.00, 1.01, 1.05],
    colors=("k",),
    linestyles=(":",),
    linewidths=(1.2,),
)
plt.clabel(cs, colors="k", fontsize=10)
plt.contourf(S_list, b_list, j_ratio, 25, cmap="Wistia", levels=levels)
# plt.yscale('log')
minor_x_ticks = np.linspace(min(S_list), max(S_list), 10)
# plt.xticks(minor_x_ticks,minor=True)
plt.minorticks_on()
plt.grid(which="both", ls=":")
plt.xlabel(r"Slope $S = (d_2-d_1)/(d_2+d_1); d_2\geq d_1, d_1\geq 0$")
plt.ylabel("Aspect $b/d_{ave}; d_{ave} = (d_1 + d_2)/2$")
plt.colorbar()
plt.title(
    r"Accuracy of rectangular approximation to trapezoid torsion constant $J_{rect}\, /\,
↪ J_{trapz}$",
    multialignment="center",
)
plt.show()
```



Total running time of the script: (3 minutes 42.275 seconds)

CHAPTER

NINE

PYTHON API REFERENCE

9.1 Pre-Processor Package

9.1.1 *geometry* Module

Geometry Class

class sectionproperties.pre.geometry.**Geometry**(*geom: Polygon*, *material:* pre.Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *control_points: Optional[Union[Point, List[float, float]]] = None*, *tol=12*)

Class for defining the geometry of a contiguous section of a single material.

Provides an interface for the user to specify the geometry defining a section. A method is provided for generating a triangular mesh, transforming the section (e.g. translation, rotation, perimeter offset, mirroring), aligning the geometry to another geometry, and designating stress recovery points.

Variables

- **geom** (shapely.geometry.Polygon) – a Polygon object that defines the geometry
- **material** (Optional[Material]) – Optional, a Material to associate with this geometry
- **control_point** – Optional, an *(x, y)* coordinate within the geometry that represents a pre-assigned control point (aka, a region identification point) to be used instead of the automatically assigned control point generated with shapely.geometry.Polygon.representative_point().
- **tol** – Optional, default is 12. Number of decimal places to round the geometry vertices to. A lower value may reduce accuracy of geometry but increases precision when aligning geometries to each other.

align_center(*align_to: Optional[Union[*Geometry*, Tuple[float, float]]] = None*)

Returns a new Geometry object, translated in both x and y, so that the the new object's centroid will be aligned with the centroid of the object in 'align_to'. If 'align_to' is an x, y coordinate, then the centroid will be aligned to the coordinate. If 'align_to' is None then the new object will be aligned with its centroid at the origin.

Parameters

align_to (Optional[Union[*Geometry*, Tuple[float, float]]]) – Another Geometry to align to or None (default is None)

Returns

Geometry object translated to new alignment

Return type
Geometry

align_to(*other: Union[Geometry, Tuple[float, float]]*, *on: str*, *inner: bool = False*) → *Geometry*

Returns a new Geometry object, representing 'self' translated so that is aligned 'on' one of the outer bounding box edges of 'other'.

If 'other' is a tuple representing an *(x,y)* coordinate, then the new Geometry object will represent 'self' translated so that it is aligned 'on' that side of the point.

Parameters

- **other** (Union[*Geometry*, Tuple[float, float]]) – Either another Geometry or a tuple representing an *(x,y)* coordinate point that 'self' should align to.
- **on** – A str of either "left", "right", "bottom", or "top" indicating which side of 'other' that self should be aligned to.
- **inner** (*bool*) – Default False. If True, align 'self' to 'other' in such a way that 'self' is aligned to the "inside" of 'other'. In other words, align 'self' to 'other' on the specified edge so they overlap.

Returns
Geometry object translated to alignment location

Return type
Geometry

assign_control_point(*control_point: List[float, float]*)

Returns a new Geometry object with 'control_point' assigned as the control point for the new Geometry. The assignment of a control point is intended to replace the control point automatically generated by shapely.geometry.Polygon.representative_point().

An assigned control point is carried through and transformed with the Geometry whenever it is shifted, aligned, mirrored, unioned, and/or rotated. If a perimeter_offset operation is applied, a check is performed to see if the assigned control point is still valid (within the new region) and, if so, it is kept. If not, a new control point is auto-generated.

The same check is performed when the geometry undergoes a difference operation (with the '-' operator) or a shift_points operation. If the assigned control point is valid, it is kept. If not, a new one is auto-generated.

For all other operations (e.g. symmetric difference, intersection, split,), the assigned control point is discarded and a new one auto-generated.

Variables
control_points – An *(x, y)* coordinate that describes the distinct, contiguous, region of a single material within the geometry. Exactly one point is required for each geometry with a distinct material.

calculate_area()

Calculates the area of the geometry.

Returns
Geometry area.

Return type
float

calculate_centroid()

Calculates the centroid of the geometry as a tuple of *(x,y)* coordinates.

Returns
Geometry centroid.

Return type
Tuple[float, float]

`calculate_extents()`

Calculates the minimum and maximum x and y-values amongst the list of points; the points that describe the bounding box of the Geometry instance.

Returns
Minimum and maximum x and y-values *(x_min, x_max, y_min, y_max)*

Return type
tuple(float, float, float, float)

`calculate_perimeter()`

Calculates the exterior perimeter of the geometry.

Returns
Geometry perimeter.

Return type
float

`compile_geometry()`

Alters attributes .points, .facets, .holes, .control_points to represent the data in the shapely geometry.

`create_mesh(mesh_sizes: Union[float, List[float]], coarse: bool = False)`

Creates a quadratic triangular mesh from the Geometry object.

Parameters

- **mesh_sizes** (*Union[float, List[float]]*) – A float describing the maximum mesh element area to be used within the Geometry-object finite-element mesh.
- **coarse** (*bool*) – If set to True, will create a coarse mesh (no area or quality constraints)

Returns
Geometry-object with mesh data stored in .mesh attribute. Returned Geometry-object is self, not a new instance.

Return type
Geometry

The following example creates a circular cross-section with a diameter of 50 with 64 points, and generates a mesh with a maximum triangular area of 2.5:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections

geometry = primitive_sections.circular_section(d=50, n=64)
geometry = geometry.create_mesh(mesh_sizes=2.5)
```

`classmethod from_3dm(filepath: Union[str, Path], **kwargs) → Geometry`

Class method to create a *Geometry* from the objects in a Rhino *.3dm* file.

Parameters

- **filepath** (*Union[str, pathlib.Path]*) – File path to the rhino *.3dm* file.
- **kwargs** – See below.

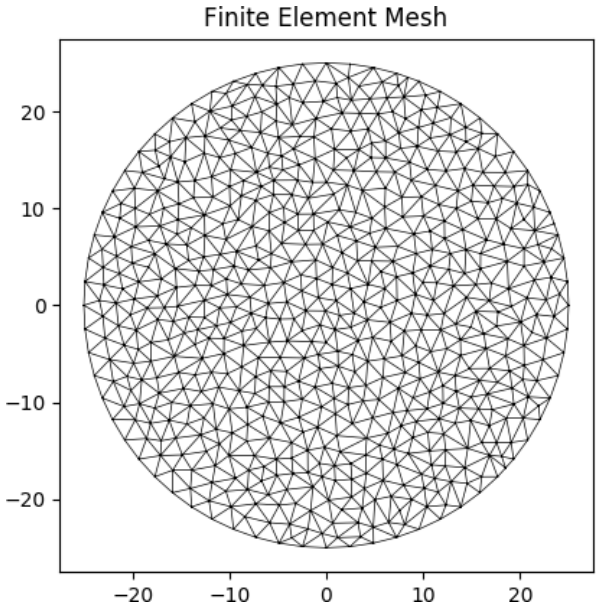


Fig. 1: Mesh generated from the above geometry.

Raises

RuntimeError – A RuntimeError is raised if two or more polygons are found. This is dependent on the keyword arguments. Try adjusting the keyword arguments if this error is raised.

Returns

A Geometry object.

Return type

Geometry

Keyword Arguments

- ***refine_num*** **(int, optional)** **–**
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.
- ***vec1*** **(numpy.ndarray, optional)** **–**
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** **(numpy.ndarray, optional)** **–**
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** **(float, optional)** **–**
  The distance to the Shapely plane. Default is 0.
- ***project*** **(boolean, optional)** **–**
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.

- ***parallel*** **(boolean, optional) –**
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

static from_dxf(*dxf_filepath: Union[str, Path]*) → Union[*Geometry*, *CompoundGeometry*]

An interface for the creation of Geometry objects from CAD .dxf files.

Variables
dxf_filepath (*Union[str, pathlib.Path]*) – A path-like object for the dxf file

static from_points(*points: List[List[float]]*, *facets: List[List[int]]*, *control_points: List[List[float]]*, *holes: Optional[List[List[float]]] = None*, *material: Optional[*Material*] = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*)

An interface for the creation of Geometry objects through the definition of points, facets, and holes.

Variables

- **points** (*list[list[float, float]]*) – List of points *(x, y)* defining the vertices of the section geometry. If facets are not provided, it is a assumed the that the list of points are ordered around the perimeter, either clockwise or anti-clockwise.
- **facets** (*list[list[int, int]]*) – A list of *(start, end)* indexes of vertices defining the edges of the section geoemtry. Can be used to define both external and internal perimeters of holes. Facets are assumed to be described in the order of exterior perimeter, interior perimeter 1, interior perimeter 2, etc.
- **control_points** – An *(x, y)* coordinate that describes the distinct, contiguous, region of a single material within the geometry. Must be entered as a list of coordinates, e.g. [[0.5, 3.2]] Exactly one point is required for each geometry with a distinct material. If there are multiple distinct regions, then use CompoundGeometry.from_points()
- **holes** (*list[list[float, float]]*) – Optional. A list of points *(x, y)* that define interior regions as being holes or voids. The point can be located anywhere within the hole region. Only one point is required per hole region.
- **material** – Optional. A *Material* object that is to be assigned. If not given, then the DEFAULT_MATERIAL will be used.

classmethod from_rhino_encoding(*r3dm_brep: str*, ***kwargs*) → *Geometry*

Load an encoded single surface planer brep.

Parameters

- **r3dm_brep** (*str*) – A Rhino3dm.Brep encoded as a string.
- **kwargs** – See below.

Returns
A Geometry object found in the encoded string.

Return type
Geometry

Keyword Arguments

- ***refine_num*** **(int, optional) –**
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.

- ***vec1*** **(numpy.ndarray, optional) –**
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** **(numpy.ndarray, optional) –**
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** **(float, optional) –**
  The distance to the Shapely plane. Default is 0.
- ***project*** **(boolean, optional) –**
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.
- ***parallel*** **(boolean, optional) –**
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

mirror_section(*axis: str = 'x'*, *mirror_point: Union[List[float], str] = 'center'*)

Mirrors the geometry about a point on either the x or y-axis.

Parameters

- **axis** (*string*) – Axis about which to mirror the geometry, *'x'* or *'y'*
- **mirror_point** (*Union[list[float, float], str]*) – Point about which to mirror the geometry *(x, y)*. If no point is provided, mirrors the geometry about the centroid of the shape's bounding box. Default = 'center'.

Returns
New Geometry-object mirrored on 'axis' about 'mirror_point'

Return type
Geometry

The following example mirrors a 200PFC section about the y-axis and the point (0, 0):

```
import sectionproperties.pre.library.steel_sections as steel_sections

geometry = steel_sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12, n_
↪r=8)
new_geometry = geometry.mirror_section(axis='y', mirror_point=[0, 0])
```

offset_perimeter(*amount: float = 0*, *where: str = 'exterior'*, *resolution: float = 12*)

Dilates or erodes the section perimeter by a discrete amount.

Parameters

- **amount** (*float*) – Distance to offset the section by. A -ve value "erodes" the section. A +ve value "dilates" the section.
- **where** (*str*) – One of either "exterior", "interior", or "all" to specify which edges of the geometry to offset. If geometry has no interiors, then this parameter has no effect. Default is "exterior".
- **resolution** (*float*) – Number of segments used to approximate a quarter circle around a point

Returns
Geometry object translated to new alignment

Return type
Geometry

The following example erodes a 200PFC section by 2 mm:

```
import sectionproperties.pre.library.steel_sections as steel_sections

geometry = sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12, n_r=8)
new_geometry = geometry.offset_perimeter(amount=-2)
```

plot_geometry(*labels=['control_points']*, *title='Cross-Section Geometry'*, *cp=True*, *legend=True*, ***kwargs*)

Plots the geometry defined by the input section.

Parameters

- **labels** (*list[str]*) – A list of str which indicate which labels to plot. Can be one or a combination of "points", "facets", "control_points", or an empty list to indicate no labels. Default is ["control_points"]
- **title** (*string*) – Plot title
- **cp** (*bool*) – If set to True, plots the control points
- **legend** (*bool*) – If set to True, plots the legend
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example creates a CHS discretised with 64 points, with a diameter of 48 and thickness of 3.2, and plots the geometry:

```
import sectionproperties.pre.library.steel_sections as steel_sections

geometry = steel_sections.circular_hollow_section(d=48, t=3.2, n=64)
geometry.plot_geometry()
```

rotate_section(*angle: float*, *rot_point: Union[List[float], str] = 'center'*, *use_radians: bool = False*)

Rotates the geometry and specified angle about a point. If the rotation point is not provided, rotates the section about the center of the geometry's bounding box.

Parameters

- **angle** (*float*) – Angle (degrees by default) by which to rotate the section. A positive angle leads to a counter-clockwise rotation.
- **rot_point** (*list[float, float]*) – Optional. Point *(x, y)* about which to rotate the section. If not provided, will rotate about the center of the geometry's bounding box. Default = 'center'.
- **use_radians** – Boolean to indicate whether 'angle' is in degrees or radians. If True, 'angle' is interpreted as radians.

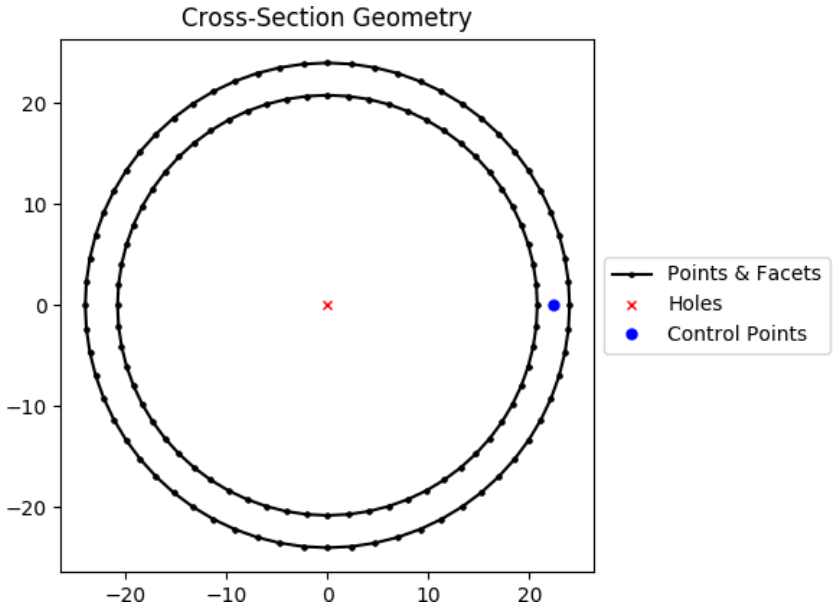


Fig. 2: Geometry generated by the above example.

Returns
New Geometry-object rotated by 'angle' about 'rot_point'

Return type
Geometry

The following example rotates a 200UB25 section clockwise by 30 degrees:

```
import sectionproperties.pre.library.steel_sections as steel_sections

geometry = steel_sections.i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_
↪r=8)
new_geometry = geometry.rotate_section(angle=-30)
```

shift_points(*point_idxs: Union[int, List[int]]*, *dx: float = 0*, *dy: float = 0*, *abs_x: Optional[float] = None*, *abs_y: Optional[float] = None*) → *Geometry*

Translates one (or many points) in the geometry by either a relative amount or to a new absolute location. Returns a new Geometry representing the original with the selected point(s) shifted to the new location.

Points are identified by their index, their relative location within the points list found in `self.points`. You can call `self.plot_geometry(labels="points")` to see a plot with the points labeled to find the appropriate point indexes.

Parameters

- **point_idxs** (*Union[int, List[int]]*) – An integer representing an index location or a list of integer index locations.
- **dx** (*float*) – The number of units in the x-direction to shift the point(s) by
- **dy** (*float*) – The number of units in the y-direction to shift the point(s) by
- **abs_x** (*Optional[float]*) – Absolute x-coordinate in coordinate system to shift the point(s) to. If abs_x is provided, dx is ignored. If providing a list to point_idxs, all points will be moved to this absolute location.

- **abs_y** (*Optional[float]*) – Absolute y-coordinate in coordinate system to shift the point(s) to. If abs_y is provided, dy is ignored. If providing a list to point_idxs, all points will be moved to this absolute location.

Returns
Geometry object with selected points translated to the new location.

Return type
Geometry

The following example expands the sides of a rectangle, one point at a time, to make it a square:

```
import sectionproperties.pre.library.primitive_sections as primitive_sections

geometry = primitive_sections.rectangular_section(d=200, b=150)

# Using relative shifting
one_pt_shifted_geom = geometry.shift_points(point_idxs=1, dx=50)

# Using absolute relocation
both_pts_shift_geom = one_pt_shift_geom.shift_points(point_idxs=2, abs_x=200)
```

shift_section(*x_offset=0.0*, *y_offset=0.0*)

Returns a new Geometry object translated by 'x_offset' and 'y_offset'.

Parameters

- **x_offset** (*float*) – Distance in x-direction by which to shift the geometry.
- **y_offset** (*float*) – Distance in y-direction by which to shift the geometry.

Returns
New Geometry-object shifted by 'x_offset' and 'y_offset'

Return type
Geometry

split_section(*point_i: Tuple[float, float]*, *point_j: Optional[Tuple[float, float]] = None*, *vector: Union[Tuple[float, float], None, ndarray] = None*) → Tuple[List[*Geometry*], List[*Geometry*]]

Splits, or bisects, the geometry about a line, as defined by two points on the line or by one point on the line and a vector. Either `point_j` or `vector` must be given. If `point_j` is given, `vector` is ignored.

Returns a tuple of two lists each containing new Geometry instances representing the "top" and "bottom" portions, respectively, of the bisected geometry.

If the line is a vertical line then the "right" and "left" portions, respectively, are returned.

Parameters

- **point_i** (*Tuple[float, float]*) – A tuple of *(x, y)* coordinates to define a first point on the line
- **point_j** (*Tuple[float, float]*) – Optional. A tuple of *(x, y)* coordinates to define a second point on the line
- **vector** (*Union[Tuple[float, float], numpy.ndarray]*) – Optional. A tuple or numpy ndarray of *(x, y)* components to define the line direction.

Returns
A tuple of lists containing Geometry objects that are bisected about the line defined by the two given points. The first item in the tuple represents the geometries on the "top" of the line

(or to the "right" of the line, if vertical) and the second item represents the geometries to the "bottom" of the line (or to the "left" of the line, if vertical).

Return type
Tuple[List[*Geometry*], List[*Geometry*]]

The following example splits a 200PFC section about the y-axis:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from shapely import LineString

geometry = steel_sections.channel_section(d=200, b=75, t_f=12, t_w=6, r=12, n_
↪r=8)
right_geom, left_geom = geometry.split_section((0, 0), (0, 1))
```

CompoundGeometry Class

class sectionproperties.pre.geometry.**CompoundGeometry**(*geoms: Union[MultiPolygon, List[*Geometry*]]*)

Bases: *Geometry*

Class for defining a geometry of multiple distinct regions, each potentially having different material properties.

CompoundGeometry instances are composed of multiple Geometry objects. As with Geometry objects, CompoundGeometry objects have methods for generating a triangular mesh over all geometries, transforming the collection of geometries as though they were one (e.g. translation, rotation, and mirroring), and aligning the CompoundGeometry to another Geometry (or to another CompoundGeometry).

CompoundGeometry objects can be created directly between two or more Geometry objects by using the + operator.

Variables
geoms (Union[shapely.geometry.MultiPolygon, List[*Geometry*]]) – either a list of Geometry objects or a shapely.geometry.MultiPolygon instance.

load_dxf

sectionproperties.pre.geometry.**load_dxf**(*dxf_filepath: Path*)

Import any-old-shape in dxf format for analysis. Code by aegis1980 and connorferster

create_facets

sectionproperties.pre.geometry.**create_facets**(*points_list: list*, *connect_back: bool = False*, *offset: int = 0*) → list

Returns a list of lists of integers representing the "facets" connecting the list of coordinates in 'loc'. It is assumed that 'loc' coordinates are already in their order of connectivity.

'loc': a list of coordinates 'connect_back': if True, then the last facet pair will be [len(loc), offset] 'offset': an integer representing the value that the facets should begin incrementing from.

create_exterior_points

sectionproperties.pre.geometry.**create_exterior_points**(*shape: Polygon*) → list

Return a list of lists representing x,y pairs of the exterior perimeter of *polygon*.

create_interior_points

sectionproperties.pre.geometry.**create_interior_points**(*lr: LinearRing*) → list

Return a list of lists representing x,y pairs of the exterior perimeter of *polygon*.

create_points_and_facets

sectionproperties.pre.geometry.**create_points_and_facets**(*shape: Polygon*, *tol=12*) → tuple

Return a list of lists representing x,y pairs of the exterior perimeter of *polygon*.

9.1.2 *pre* Module

Material Class

class sectionproperties.pre.pre.**Material**(*name: str*, *elastic_modulus: float*, *poissons_ratio: float*, *yield_strength: float*, *density: float*, *color: str*)

Bases: `object`

Class for structural materials.

Provides a way of storing material properties related to a specific material. The color can be a multitude of different formats, refer to https://matplotlib.org/api/colors_api.html and https://matplotlib.org/examples/color/named_colors.html for more information.

Parameters

- **name** (*string*) – Material name
- **elastic_modulus** (*float*) – Material modulus of elasticity
- **poissons_ratio** (*float*) – Material Poisson's ratio
- **yield_strength** (*float*) – Material yield strength
- **density** (*float*) – Material density (mass per unit volume)
- **color** (`matplotlib.colors`) – Material color for rendering

Variables

- **name** (*string*) – Material name
- **elastic_modulus** (*float*) – Material modulus of elasticity
- **poissons_ratio** (*float*) – Material Poisson's ratio
- **shear_modulus** (*float*) – Material shear modulus, derived from the elastic modulus and Poisson's ratio assuming an isotropic material
- **density** (*float*) – Material density (mass per unit volume)
- **yield_strength** (*float*) – Material yield strength

- **color** (matplotlib.colors) – Material color for rendering

The following example creates materials for concrete, steel and timber:

```
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, density=2.
↪4e-6,
        yield_strength=32, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, density=7.85e-
↪6,
        yield_strength=500, color='grey'
)
timber = Material(
    name='Timber', elastic_modulus=8e3, poissons_ratio=0.35, density=6.5e-
↪7,
        yield_strength=20, color='burlywood'
)
```

create_mesh

sectionproperties.pre.pre.**create_mesh**(*points: List[List[float]]*, *facets: List[List[float]]*, *holes: List[List[float]]*, *control_points: List[List[float]]*, *mesh_sizes: Union[List[float], float]*, *coarse: bool*)

Creates a quadratic triangular mesh using the triangle module, which utilises the code 'Triangle', by Jonathan Shewchuk.

Parameters

- **points** (*list[list[int, int]]*) – List of points *(x, y)* defining the vertices of the cross-section
- **facets** – List of point index pairs *(p1, p2)* defining the edges of the cross-section
- **holes** (*list[list[float, float]]*) – List of points *(x, y)* defining the locations of holes within the cross-section. If there are no holes, provide an empty list [].
- **control_points** (*list[list[float, float]]*) – A list of points *(x, y)* that define different regions of the cross-section. A control point is an arbitrary point within a region enclosed by facets.
- **mesh_sizes** (*list[float]*) – List of maximum element areas for each region defined by a control point
- **coarse** (*bool*) – If set to True, will create a coarse mesh (no area or quality constraints)

Returns

Dictionary containing mesh data

Return type

dict()

9.1.3 *rhino* Module

load_3dm

sectionproperties.pre.rhino.**load_3dm**(*r3dm_filepath: Union[Path, str]*, ***kwargs*) → List[Polygon]

Load a Rhino *.3dm* file and import the single surface planer breps.

Parameters

- **r3dm_filepath** (*pathlib.Path or string*) – File path to the rhino *.3dm* file.
- **kwargs** – See below.

Raises

RuntimeError – A RuntimeError is raised if no polygons are found in the file. This is dependent on the keyword arguments. Try adjusting the keyword arguments if this error is raised.

Returns

List of Polygons found in the file.

Return type

List[shapely.geometry.Polygon]

Keyword Arguments

- ***refine_num*** (**int, optional**) –
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.
- ***vec1*** (**numpy.ndarray, optional**) –
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** (**numpy.ndarray, optional**) –
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** (**float, optional**) –
  The distance to the Shapely plane. Default is 0.
- ***project*** (**boolean, optional**) –
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.
- ***parallel*** (**boolean, optional**) –
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

load_brep_encoding

sectionproperties.pre.rhino.**load_brep_encoding**(*brep: str*, ***kwargs*) → Polygon

Load an encoded single surface planer brep.

Parameters

- **brep** (*str*) – Rhino3dm.Brep encoded as a string.
- **kwargs** – See below.

Raises

RuntimeError – A RuntimeError is raised if no polygons are found in the encoding. This is dependent on the keyword arguments. Try adjusting the keyword arguments if this error is raised.

Returns

The Polygons found in the encoding string.

Return type

shapely.geometry.Polygon

Keyword Arguments

- ***refine_num*** **(int, optional) –**
  Bézier curve interpolation number. In Rhino a surface's edges are nurb based curves. Shapely does not support nurbs, so the individual Bézier curves are interpolated using straight lines. This parameter sets the number of straight lines used in the interpolation. Default is 1.
- ***vec1*** **(numpy.ndarray, optional) –**
  A 3d vector in the Shapely plane. Rhino is a 3D geometry environment. Shapely is a 2D geometric library. Thus a 2D plane needs to be defined in Rhino that represents the Shapely coordinate system. *vec1* represents the 1st vector of this plane. It will be used as Shapely's x direction. Default is [1,0,0].
- ***vec2*** **(numpy.ndarray, optional) –**
  Continuing from *vec1*, *vec2* is another vector to define the Shapely plane. It must not be [0,0,0] and it's only requirement is that it is any vector in the Shapely plane (but not equal to *vec1*). Default is [0,1,0].
- ***plane_distance*** **(float, optional) –**
  The distance to the Shapely plane. Default is 0.
- ***project*** **(boolean, optional) –**
  Controls if the breps are projected onto the plane in the direction of the Shapley plane's normal. Default is True.
- ***parallel*** **(boolean, optional) –**
  Controls if only the rhino surfaces that have the same normal as the Shapely plane are yielded. If true, all non parallel surfaces are filtered out. Default is False.

9.1.4 *bisect_section* Module

create_line_segment

sectionproperties.pre.bisect_section.**create_line_segment**(*point_on_line: Union[Tuple[float, float], ndarray]*, *vector: ndarray*, *bounds: tuple*)

Return a LineString of a line that contains 'point_on_line' in the direction of 'unit_vector' bounded by 'bounds'. 'bounds' is a tuple of float containing a max ordinate and min ordinate.

group_top_and_bottom_polys

sectionproperties.pre.bisect_section.**group_top_and_bottom_polys**(*polys: GeometryCollection*, *line: LineString*) → Tuple[list, list]

Returns tuple of two lists representing the list of Polygons in 'polys' on the "top" side of 'line' and the list of Polygons on the "bottom" side of the 'line' after the original geometry has been split by 'line'.

The 0-th tuple element is the "top" polygons and the 1-st element is the "bottom" polygons.

In the event that 'line' is a perfectly vertical line, the "top" polys are the polygons on the "right" of the 'line' and the "bottom" polys are the polygons on the "left" of the 'line'.

line_mx_plus_b

sectionproperties.pre.bisect_section.**line_mx_plus_b**(*line: LineString*) → Tuple[float, float]

Returns a tuple representing the values of "m" and "b" from the definition of 'line' as "y = mx + b".

perp_mx_plus_b

sectionproperties.pre.bisect_section.**perp_mx_plus_b**(*m_slope: float*, *point_on_line: Tuple[float, float]*) → Tuple[float, float]

Returns a tuple representing the values of "m" and "b" from for a line that is perpendicular to 'm_slope' and contains the 'point_on_line', which represents an (x, y) coordinate.

line_intersection

sectionproperties.pre.bisect_section.**line_intersection**(*m_1: float*, *b_1: float*, *m_2: float*, *b_2: float*) → Optional[float]

Returns a float representing the x-ordinate of the intersection point of the lines defined by y = m_1*x + b_1 and y = m_2*x + b_2.

Returns None if the lines are parallel.

sum_poly_areas

sectionproperties.pre.bisect_section.**sum_poly_areas**(*lop: List[Polygon]*) → float

Returns a float representing the total area of all polygons in 'lop', the list of polygons.

9.1.5 *primitive_sections* Module

rectangular_section

sectionproperties.pre.library.primitive_sections.**rectangular_section**(*b: float*, *d: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a rectangular section with the bottom left corner at the origin *(0, 0)*, with depth *d* and width *b*.

Parameters

- **d** (*float*) – Depth (y) of the rectangle
- **b** (*float*) – Width (x) of the rectangle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a rectangular cross-section with a depth of 100 and width of 50, and generates a mesh with a maximum triangular area of 5:

```
from sectionproperties.pre.library.primitive_sections import rectangular_section

geometry = rectangular_section(d=100, b=50)
geometry.create_mesh(mesh_sizes=[5])
```

circular_section

sectionproperties.pre.library.primitive_sections.**circular_section**(*d: float*, *n: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a solid circle centered at the origin *(0, 0)* with diameter *d* and using *n* points to construct the circle.

Parameters

- **d** (*float*) – Diameter of the circle
- **n** (*int*) – Number of points discretising the circle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

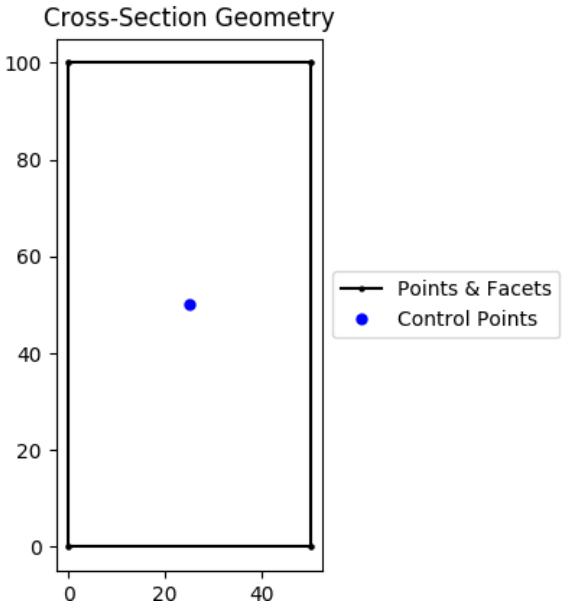


Fig. 3: Rectangular section geometry.

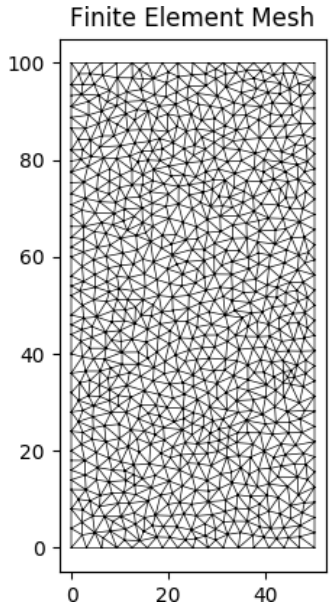


Fig. 4: Mesh generated from the above geometry.

The following example creates a circular geometry with a diameter of 50 with 64 points, and generates a mesh with a maximum triangular area of 2.5:

```
from sectionproperties.pre.library.primitive_sections import circular_section

geometry = circular_section(d=50, n=64)
geometry.create_mesh(mesh_sizes=[2.5])
```

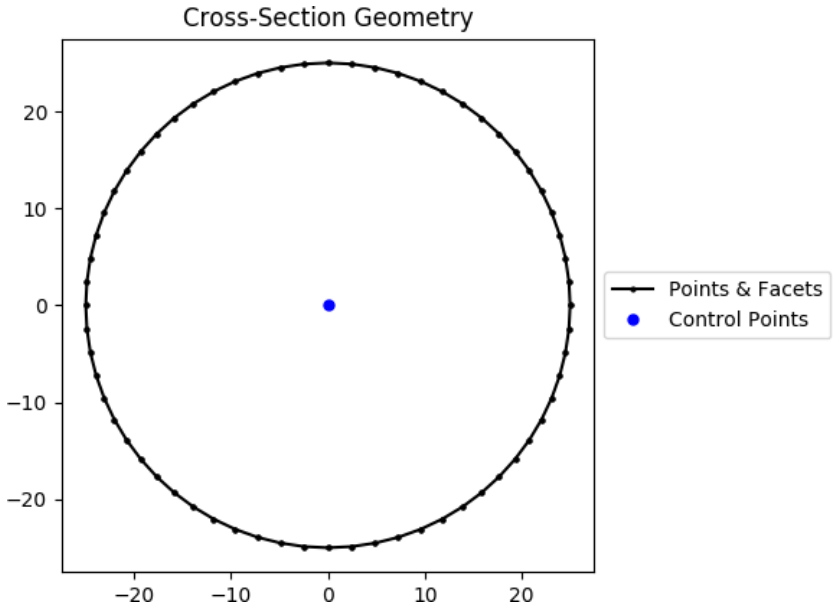


Fig. 5: Circular section geometry.

circular_section_by_area

sectionproperties.pre.library.primitive_sections.**circular_section_by_area**(*area: float*, *n: int*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a solid circle centered at the origin *(0, 0)* defined by its *area*, using *n* points to construct the circle.

Parameters

- **area** (*float*) – Area of the circle
- **n** (*int*) – Number of points discretising the circle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a circular geometry with an area of 200 with 32 points, and generates a mesh with a maximum triangular area of 5:

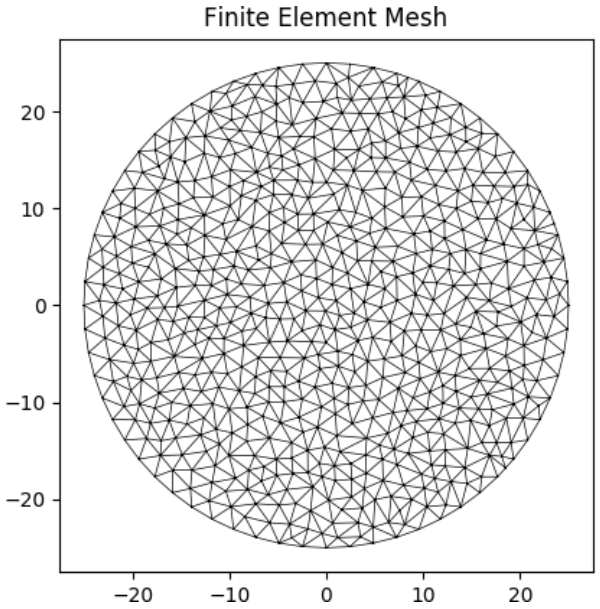


Fig. 6: Mesh generated from the above geometry.

```
from sectionproperties.pre.library.primitive_sections import circular_section_by_
↪area

geometry = circular_section_by_area(area=310, n=32)
geometry.create_mesh(mesh_sizes=[5])
```

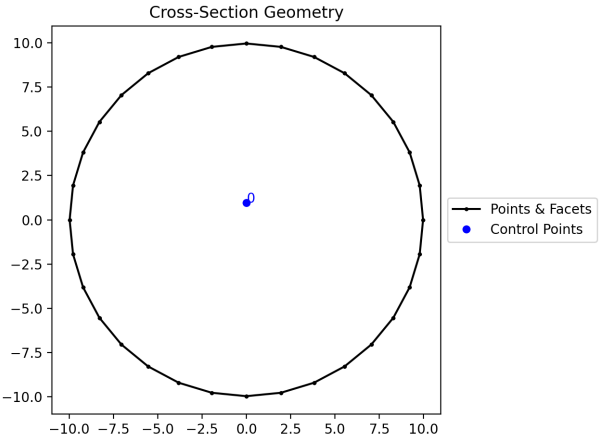


Fig. 7: Circular section by area geometry.

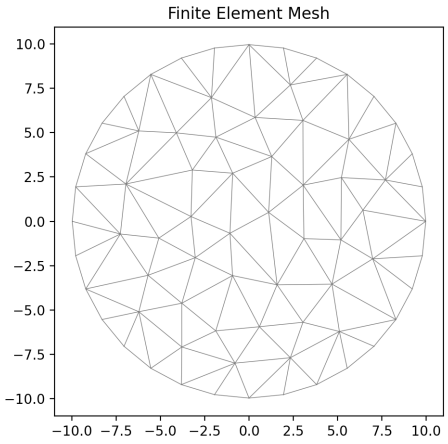


Fig. 8: Mesh generated from the above geometry.

elliptical_section

sectionproperties.pre.library.primitive_sections.**elliptical_section**(*d_y: float*, *d_x: float*, *n: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a solid ellipse centered at the origin *(0, 0)* with vertical diameter *d_y* and horizontal diameter *d_x*, using *n* points to construct the ellipse.

Parameters

- **d_y** (*float*) – Diameter of the ellipse in the y-dimension
- **d_x** (*float*) – Diameter of the ellipse in the x-dimension
- **n** (*int*) – Number of points discretising the ellipse
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an elliptical cross-section with a vertical diameter of 25 and horizontal diameter of 50, with 40 points, and generates a mesh with a maximum triangular area of 1.0:

```
from sectionproperties.pre.library.primitive_sections import elliptical_section

geometry = elliptical_section(d_y=25, d_x=50, n=40)
geometry.create_mesh(mesh_sizes=[1.0])
```

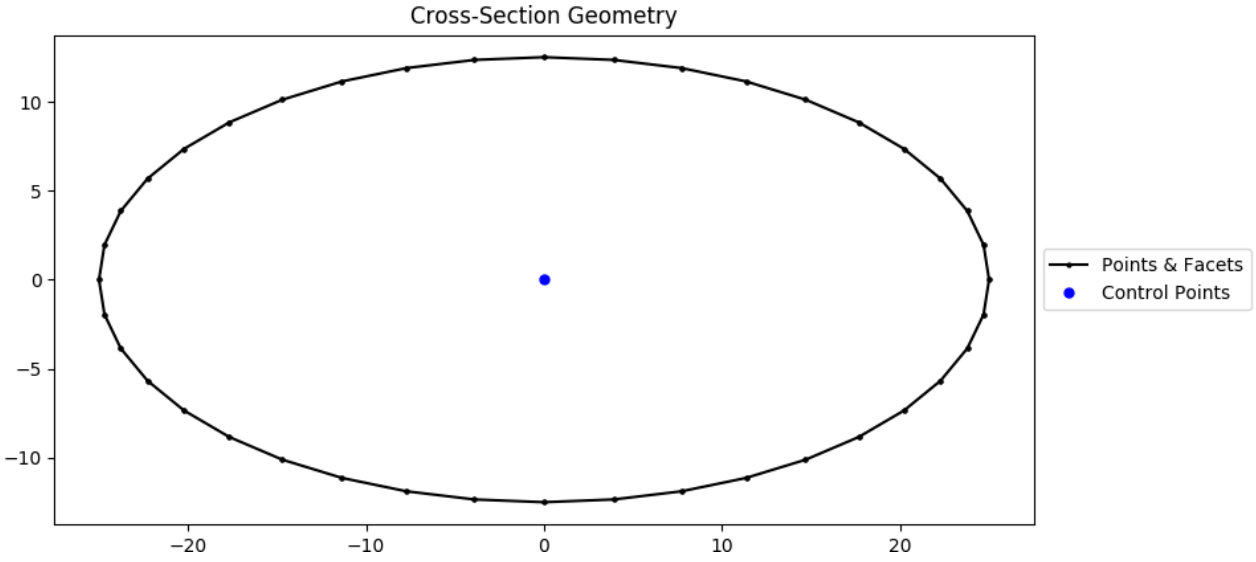


Fig. 9: Elliptical section geometry.

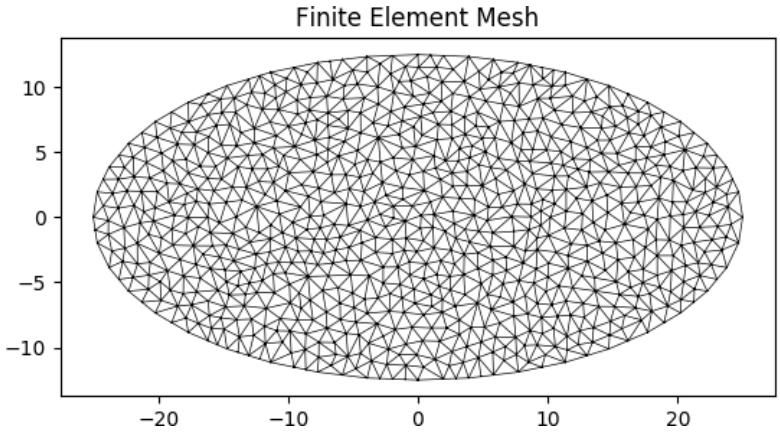


Fig. 10: Mesh generated from the above geometry.

triangular_section

sectionproperties.pre.library.primitive_sections.**triangular_section**(*b: float*, *h: float*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a right angled triangle with points *(0, 0)*, *(b, 0)*, *(0, h)*.

Parameters

- **b** (*float*) – Base length of triangle
- **h** (*float*) – Height of triangle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a triangular cross-section with a base width of 10 and height of 10, and generates a mesh with a maximum triangular area of 0.5:

```
from sectionproperties.pre.library.primitive_sections import triangular_section

geometry = triangular_section(b=10, h=10)
geometry.create_mesh(mesh_sizes=[0.5])
```

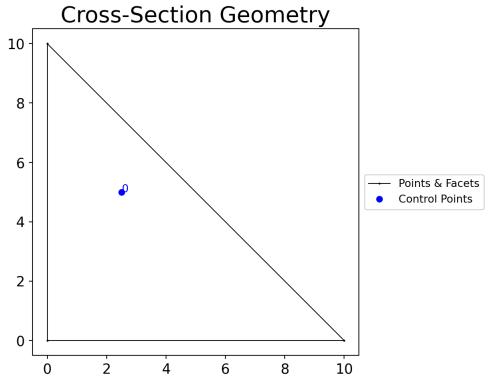


Fig. 11: Triangular section geometry.

triangular_radius_section

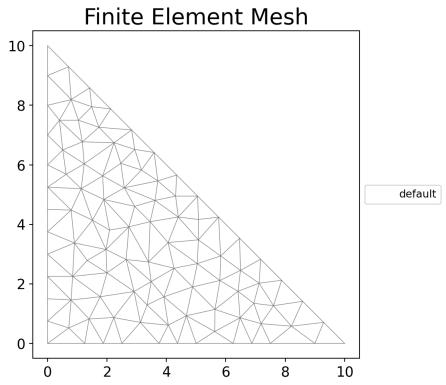


Fig. 12: Mesh generated from the above geometry.

sectionproperties.pre.library.primitive_sections.**triangular_radius_section**(*b: float*, *n_r: float*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a right angled isosceles triangle with points *(0, 0)*, *(b, 0)*, *(0, h)* and a concave radius on the hypotenuse.

Parameters

- **b** (*float*) – Base length of triangle
- **n_r** (*int*) – Number of points discretising the radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a triangular radius cross-section with a base width of 6, using *n_r* points to construct the radius, and generates a mesh with a maximum triangular area of 0.5:

```
from sectionproperties.pre.library.primitive_sections import triangular_radius_
↪section

geometry = triangular_radius_section(b=6, n_r=16)
geometry.create_mesh(mesh_sizes=[0.5])
```

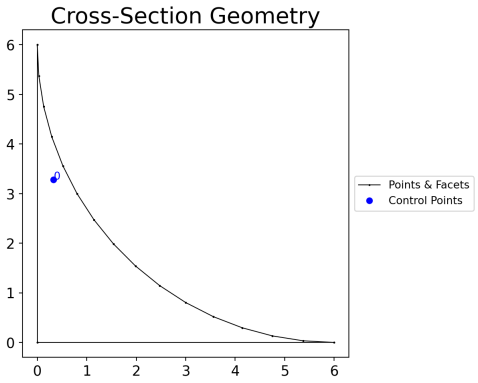


Fig. 13: Triangular radius section geometry.

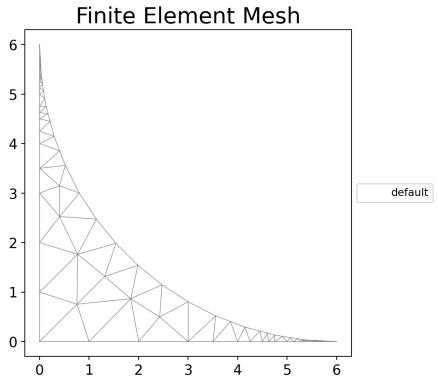


Fig. 14: Mesh generated from the above geometry.

cruciform_section

sectionproperties.pre.library.primitive_sections.**cruciform_section**(*d: float*, *b: float*, *t: float*, *r: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a cruciform section centered at the origin *(0, 0)*, with depth *d*, width *b*, thickness *t* and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the cruciform section
- **b** (*float*) – Width of the cruciform section
- **t** (*float*) – Thickness of the cruciform section
- **r** (*float*) – Root radius of the cruciform section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a cruciform section with a depth of 250, a width of 175, a thickness of 12 and a root radius of 16, using 16 points to discretise the radius. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.primitive_sections import cruciform_
↪section

geometry = cruciform_section(d=250, b=175, t=12, r=16, n_r=16)
geometry.create_mesh(mesh_sizes=[5.0])
```

9.1.6 *steel_sections* Module

circular_hollow_section

sectionproperties.pre.library.steel_sections.**circular_hollow_section**(*d: float*, *t: float*, *n: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

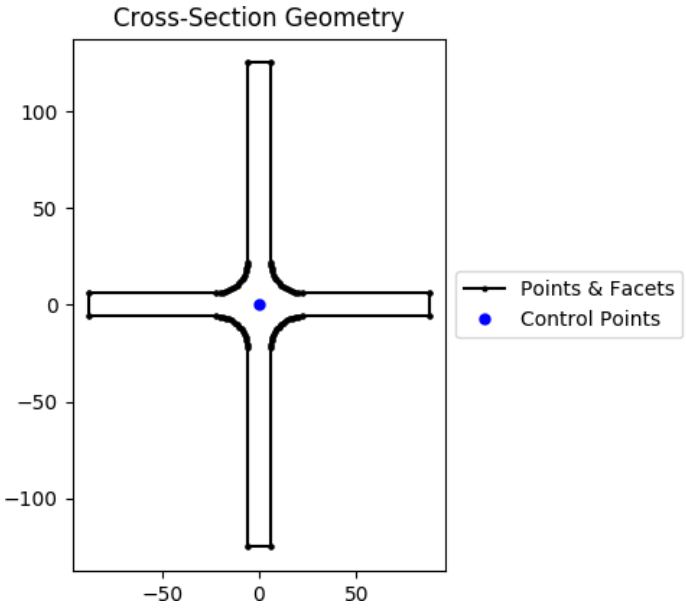


Fig. 15: Cruciform section geometry.

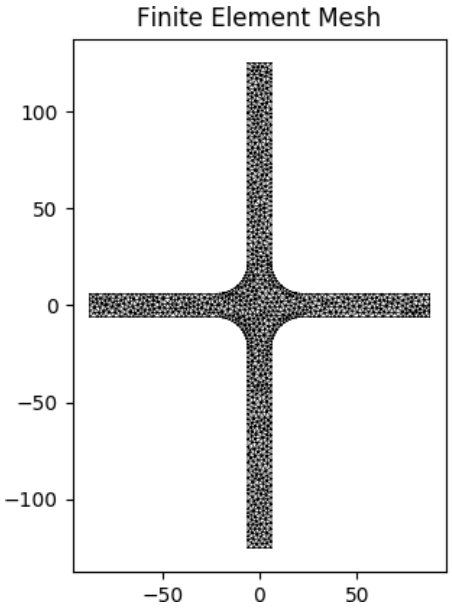


Fig. 16: Mesh generated from the above geometry.

Constructs a circular hollow section (CHS) centered at the origin *(0, 0)*, with diameter *d* and thickness *t*, using *n* points to construct the inner and outer circles.

Parameters

- **d** (*float*) – Outer diameter of the CHS
- **t** (*float*) – Thickness of the CHS
- **n** (*int*) – Number of points discretising the inner and outer circles
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a CHS discretised with 64 points, with a diameter of 48 and thickness of 3.2, and generates a mesh with a maximum triangular area of 1.0:

```
from sectionproperties.pre.library.steel_sections import circular_hollow_section

geometry = circular_hollow_section(d=48, t=3.2, n=64)
geometry.create_mesh(mesh_sizes=[1.0])
```

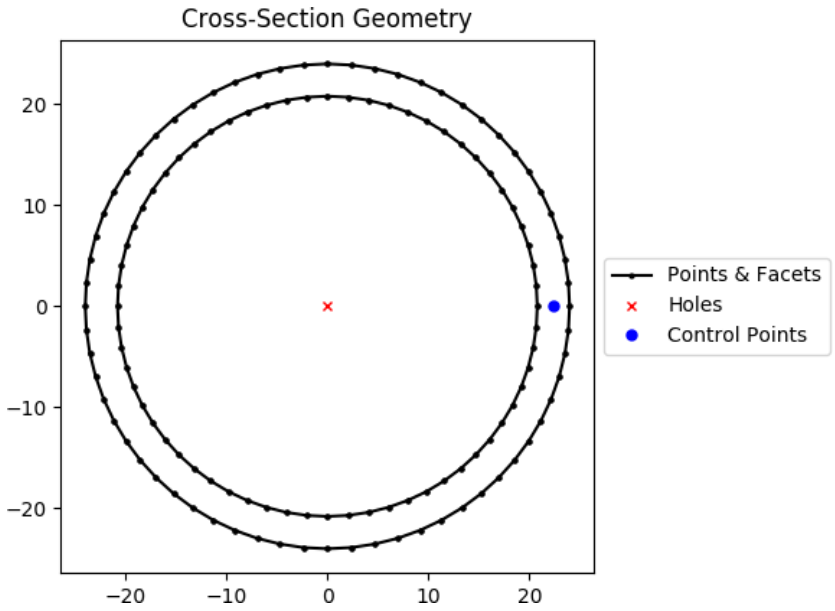


Fig. 17: CHS geometry.

elliptical_hollow_section

sectionproperties.pre.library.steel_sections.**elliptical_hollow_section**(*d_y: float*, *d_x: float*, *t: float*, *n: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

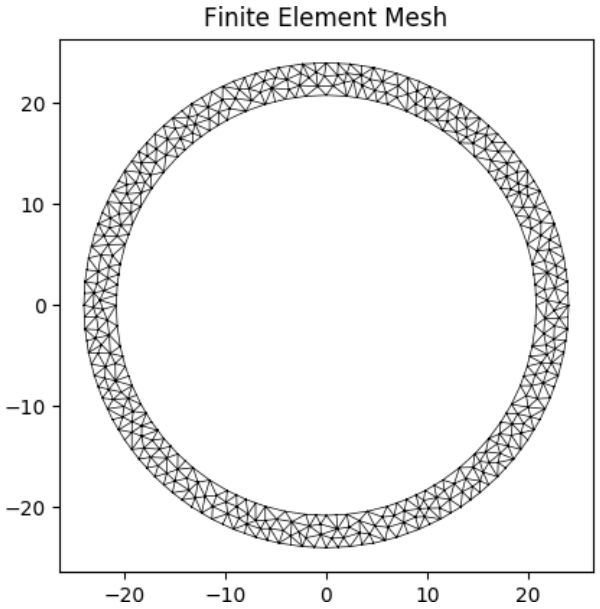


Fig. 18: Mesh generated from the above geometry.

Constructs an elliptical hollow section (EHS) centered at the origin *(0, 0)*, with outer vertical diameter *d_y*, outer horizontal diameter *d_x*, and thickness *t*, using *n* points to construct the inner and outer ellipses.

Parameters

- **d_y** (*float*) – Diameter of the ellipse in the y-dimension
- **d_x** (*float*) – Diameter of the ellipse in the x-dimension
- **t** (*float*) – Thickness of the EHS
- **n** (*int*) – Number of points discretising the inner and outer ellipses
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a EHS discretised with 30 points, with a outer vertical diameter of 25, outer horizontal diameter of 50, and thickness of 2.0, and generates a mesh with a maximum triangular area of 0.5:

```
from sectionproperties.pre.library.steel_sections import elliptical_hollow_section

geometry = elliptical_hollow_section(d_y=25, d_x=50, t=2.0, n=64)
geometry.create_mesh(mesh_sizes=[0.5])
```

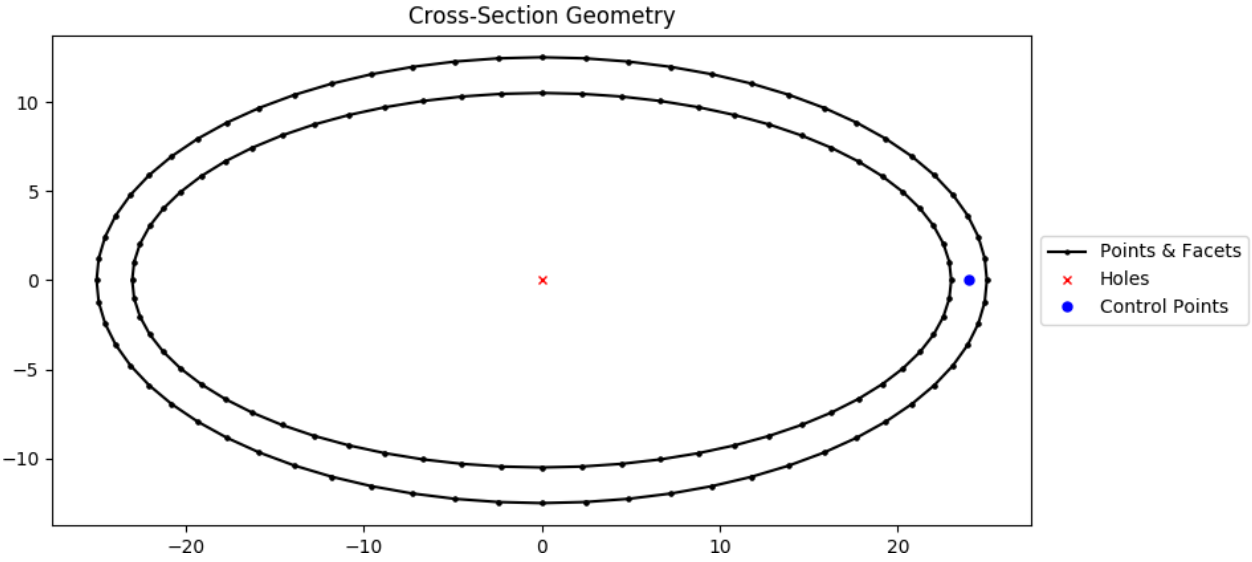


Fig. 19: EHS geometry.

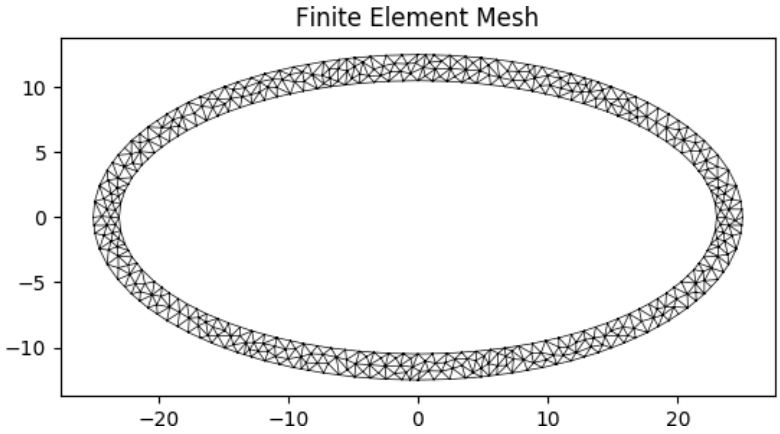


Fig. 20: Mesh generated from the above geometry.

rectangular_hollow_section

sectionproperties.pre.library.steel_sections.**rectangular_hollow_section**(*b: float*, *d: float*, *t: float*, *r_out: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a rectangular hollow section (RHS) centered at *(b/2, d/2)*, with depth *d*, width *b*, thickness *t* and outer radius *r_out*, using *n_r* points to construct the inner and outer radii. If the outer radius is less than the thickness of the RHS, the inner radius is set to zero.

Parameters

- **d** (*float*) – Depth of the RHS
- **b** (*float*) – Width of the RHS
- **t** (*float*) – Thickness of the RHS
- **r_out** (*float*) – Outer radius of the RHS
- **n_r** (*int*) – Number of points discretising the inner and outer radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an RHS with a depth of 100, a width of 50, a thickness of 6 and an outer radius of 9, using 8 points to discretise the inner and outer radii. A mesh is generated with a maximum triangular area of 2.0:

```
from sectionproperties.pre.library.steel_sections import rectangular_hollow_section

geometry = rectangular_hollow_section(d=100, b=50, t=6, r_out=9, n_r=8)
geometry.create_mesh(mesh_sizes=[2.0])
```

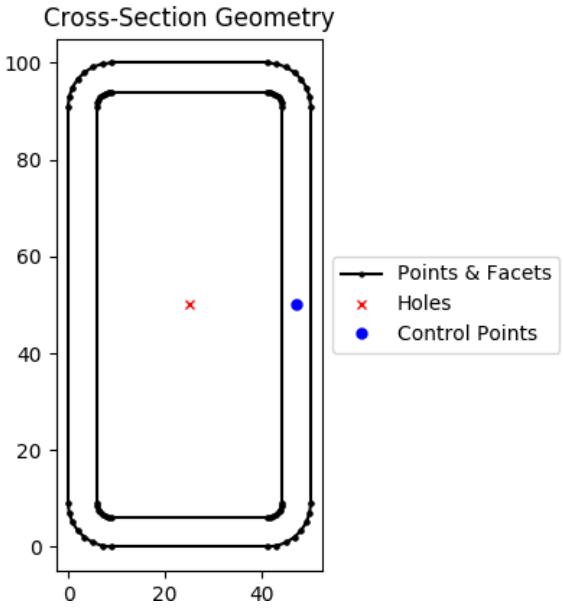


Fig. 21: RHS geometry.

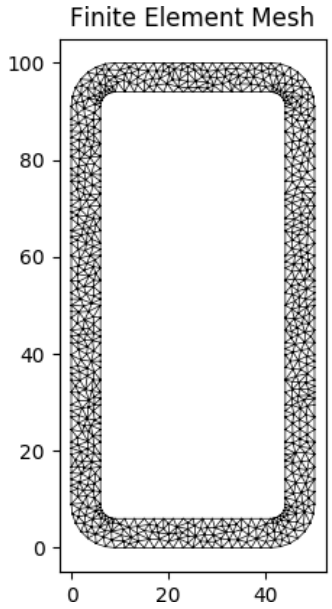


Fig. 22: Mesh generated from the above geometry.

polygon_hollow_section

sectionproperties.pre.library.steel_sections.**polygon_hollow_section**(*d: float*, *t: float*, *n_sides: int*, *r_in: float = 0*, *n_r: int = 1*, *rot: float = 0*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a regular hollow polygon section centered at *(0, 0)*, with a pitch circle diameter of bounding polygon *d*, thickness *t*, number of sides *n_sides* and an optional inner radius *r_in*, using *n_r* points to construct the inner and outer radii (if radii is specified).

Parameters

- **d** (*float*) – Pitch circle diameter of the outer bounding polygon (i.e. diameter of circle that passes through all vertices of the outer polygon)
- **t** (*float*) – Thickness of the polygon section wall
- **r_in** (*float*) – Inner radius of the polygon corners. By default, if not specified, a polygon with no corner radii is generated.
- **n_r** (*int*) – Number of points discretising the inner and outer radii, ignored if no inner radii is specified
- **rot** (*float*) – Initial counterclockwise rotation in degrees. By default bottom face is aligned with x axis.
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

Raises

Exception – Number of sides in polygon must be greater than or equal to 3

The following example creates an Octagonal section (8 sides) with a diameter of 200, a thickness of 6 and an inner radius of 20, using 12 points to discretise the inner and outer radii. A mesh is generated with a maximum triangular area of 5:

```
from sectionproperties.pre.library.steel_sections import polygon_hollow_
↪section

geometry = polygon_hollow_section(d=200, t=6, n_sides=8, r_in=20, n_r=12)
geometry.create_mesh(mesh_sizes=[5])
```

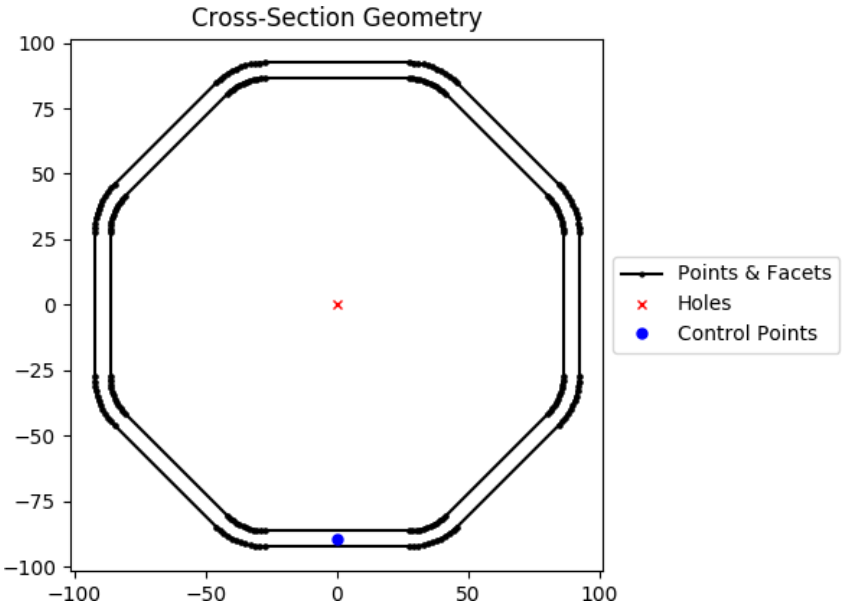


Fig. 23: Octagonal section geometry.

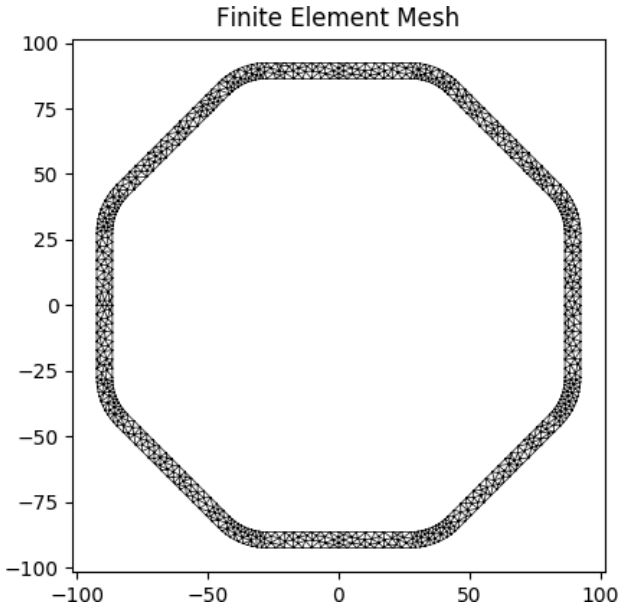


Fig. 24: Mesh generated from the above geometry.

i_section

sectionproperties.pre.library.steel_sections.**i_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs an I Section centered at *(b/2, d/2)*, with depth *d*, width *b*, flange thickness *t_f*, web thickness *t_w*, and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the I Section
- **b** (*float*) – Width of the I Section
- **t_f** (*float*) – Flange thickness of the I Section
- **t_w** (*float*) – Web thickness of the I Section
- **r** (*float*) – Root radius of the I Section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an I Section with a depth of 203, a width of 133, a flange thickness of 7.8, a web thickness of 5.8 and a root radius of 8.9, using 16 points to discretise the root radius. A mesh is generated with a maximum triangular area of 3.0:

```
from sectionproperties.pre.library.steel_sections import i_section

geometry = i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=16)
geometry.create_mesh(mesh_sizes=[3.0])
```

mono_i_section

sectionproperties.pre.library.steel_sections.**mono_i_section**(*d: float*, *b_t: float*, *b_b: float*, *t_ft: float*, *t_fb: float*, *t_w: float*, *r: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a monosymmetric I Section centered at *(max(b_t, b_b)/2, d/2)*, with depth *d*, top flange width *b_t*, bottom flange width *b_b*, top flange thickness *t_ft*, top flange thickness *t_fb*, web thickness *t_w*, and root radius *r*, using *n_r* points to construct the root radius.

Parameters

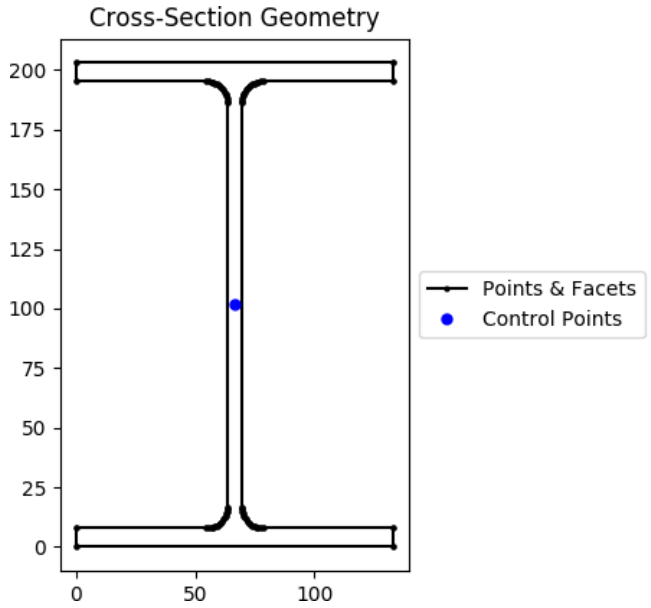


Fig. 25: I Section geometry.

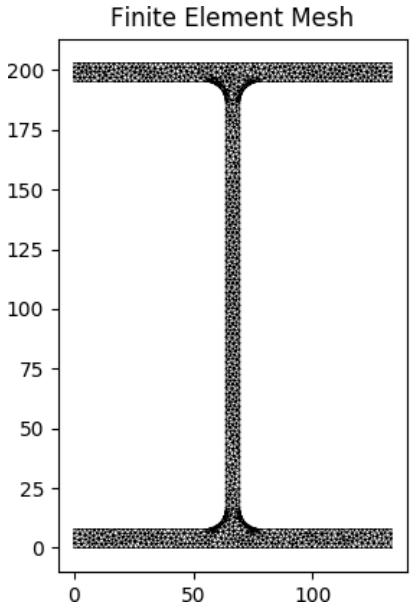


Fig. 26: Mesh generated from the above geometry.

- **d** (*float*) – Depth of the I Section
- **b_t** (*float*) – Top flange width
- **b_b** (*float*) – Bottom flange width
- **t_ft** (*float*) – Top flange thickness of the I Section
- **t_fb** (*float*) – Bottom flange thickness of the I Section
- **t_w** (*float*) – Web thickness of the I Section
- **r** (*float*) – Root radius of the I Section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a monosymmetric I Section with a depth of 200, a top flange width of 50, a top flange thickness of 12, a bottom flange width of 130, a bottom flange thickness of 8, a web thickness of 6 and a root radius of 8, using 16 points to discretise the root radius. A mesh is generated with a maximum triangular area of 3.0:

```
from sectionproperties.pre.library.steel_sections import mono_i_section

geometry = mono_i_section(
    d=200, b_t=50, b_b=130, t_ft=12, t_fb=8, t_w=6, r=8, n_r=16
)
geometry.create_mesh(mesh_sizes=[3.0])
```

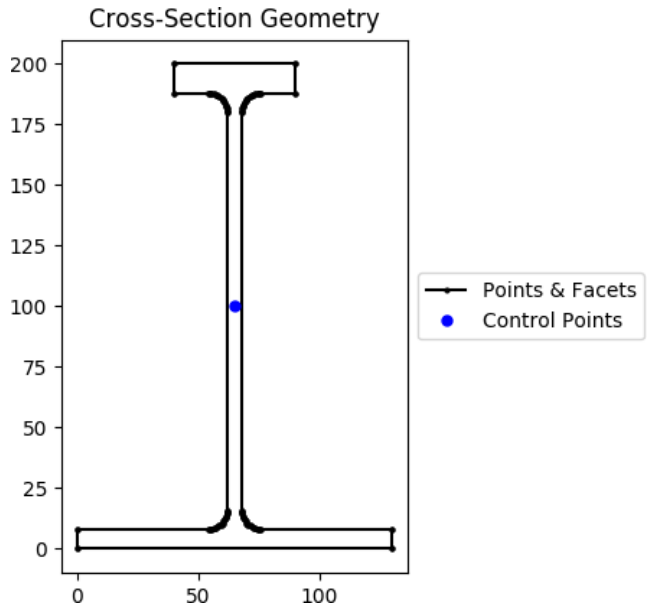


Fig. 27: I Section geometry.

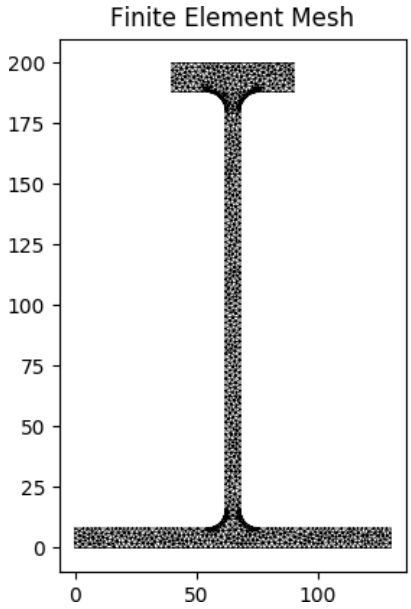


Fig. 28: Mesh generated from the above geometry.

tapered_flange_i_section

sectionproperties.pre.library.steel_sections.**tapered_flange_i_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r_r: float*, *r_f: float*, *alpha: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Tapered Flange I Section centered at *(b/2, d/2)*, with depth *d*, width *b*, mid-flange thickness *t_f*, web thickness *t_w*, root radius *r_r*, flange radius *r_f* and flange angle *alpha*, using *n_r* points to construct the radii.

Parameters

- **d** (*float*) – Depth of the Tapered Flange I Section
- **b** (*float*) – Width of the Tapered Flange I Section
- **t_f** (*float*) – Mid-flange thickness of the Tapered Flange I Section (measured at

the point equidistant from the face of the web to the edge of the flange)

- **t_w** (*float*) – Web thickness of the Tapered Flange I Section
- **r_r** (*float*) – Root radius of the Tapered Flange I Section
- **r_f** (*float*) – Flange radius of the Tapered Flange I Section
- **alpha** (*float*) – Flange angle of the Tapered Flange I Section (degrees)
- **n_r** (*int*) – Number of points discretising the radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a Tapered Flange I Section with a depth of 588, a width of 191, a mid-flange thickness of 27.2, a web thickness of 15.2, a root radius of 17.8, a flange radius of 8.9 and a flange angle of 8°, using 16 points to discretise the radii. A mesh is generated with a maximum triangular area of 20.0:

```
from sectionproperties.pre.library.steel_sections import tapered_flange_i_
↪section

geometry = tapered_flange_i_section(
    d=588, b=191, t_f=27.2, t_w=15.2, r_r=17.8, r_f=8.9, alpha=8, n_r=16
)
geometry.create_mesh(mesh_sizes=[20.0])
```

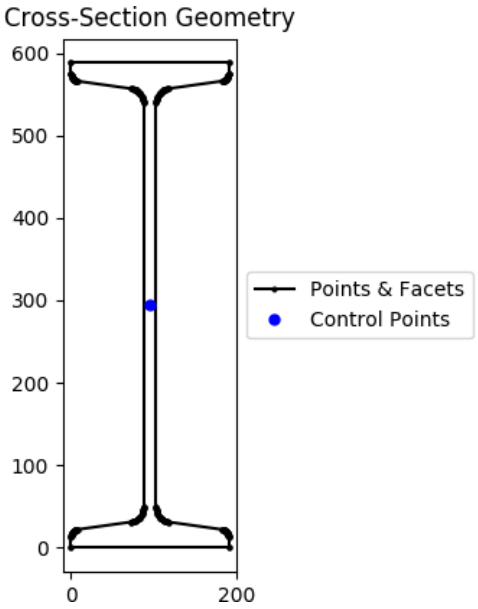


Fig. 29: I Section geometry.

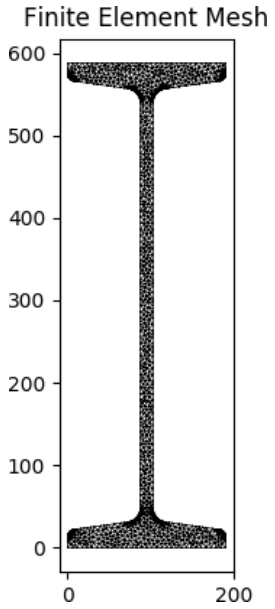


Fig. 30: Mesh generated from the above geometry.

channel_section

sectionproperties.pre.library.steel_sections.**channel_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a parallel-flange channel (PFC) section with the bottom left corner at the origin *(0, 0)*, with depth *d*, width *b*, flange thickness *t_f*, web thickness *t_w* and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the PFC section
- **b** (*float*) – Width of the PFC section
- **t_f** (*float*) – Flange thickness of the PFC section
- **t_w** (*float*) – Web thickness of the PFC section
- **r** (*float*) – Root radius of the PFC section
- **n_r** (*int*) – Number of points discretising the root radius
- **shift** (*list[float, float]*) – Vector that shifts the cross-section by *(x, y)*
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a PFC section with a depth of 250, a width of 90, a flange thickness of 15, a web thickness of 8 and a root radius of 12, using 8 points to discretise the root radius. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.steel_sections import channel_section

geometry = channel_section(d=250, b=90, t_f=15, t_w=8, r=12, n_r=8)
geometry.create_mesh(mesh_sizes=[5.0])
```

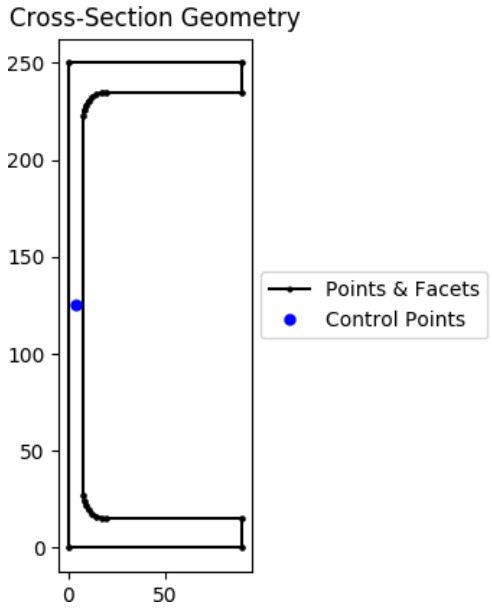


Fig. 31: PFC geometry.

tapered_flange_channel

sectionproperties.pre.library.steel_sections.**tapered_flange_channel**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r_r: float*, *r_f: float*, *alpha: float*, *n_r: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Tapered Flange Channel section with the bottom left corner at the origin *(0, 0)*, with

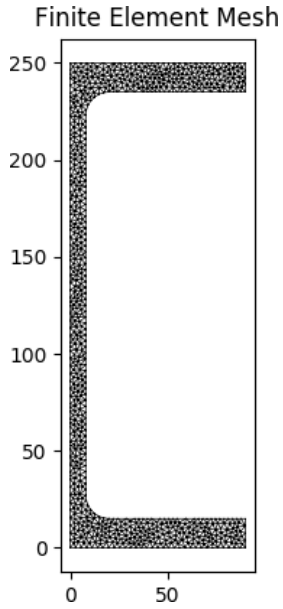


Fig. 32: Mesh generated from the above geometry.

depth *d*, width *b*, mid-flange thickness *t_f*, web thickness *t_w*, root radius *r_r*, flange radius *r_f* and flange angle *alpha*, using *n_r* points to construct the radii.

Parameters

- **d** (*float*) – Depth of the Tapered Flange Channel section
- **b** (*float*) – Width of the Tapered Flange Channel section
- **t_f** (*float*) – Mid-flange thickness of the Tapered Flange Channel section (measured at the point equidistant from the face of the web to the edge of the flange)
- **t_w** (*float*) – Web thickness of the Tapered Flange Channel section
- **r_r** (*float*) – Root radius of the Tapered Flange Channel section
- **r_f** (*float*) – Flange radius of the Tapered Flange Channel section
- **alpha** (*float*) – Flange angle of the Tapered Flange Channel section (degrees)
- **n_r** (*int*) – Number of points discretising the radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a Tapered Flange Channel section with a depth of 10, a width of 3.5, a mid-flange thickness of 0.575, a web thickness of 0.475, a root radius of 0.575, a flange radius of 0.4 and a flange angle of 8°, using 16 points to discretise the radii. A mesh is generated with a maximum triangular area of 0.02:

```
from sectionproperties.pre.library.steel_sections import tapered_flange_
↪channel

geometry = tapered_flange_channel(
    d=10, b=3.5, t_f=0.575, t_w=0.475, r_r=0.575, r_f=0.4, alpha=8, n_r=16
)
geometry.create_mesh(mesh_sizes=[0.02])
```

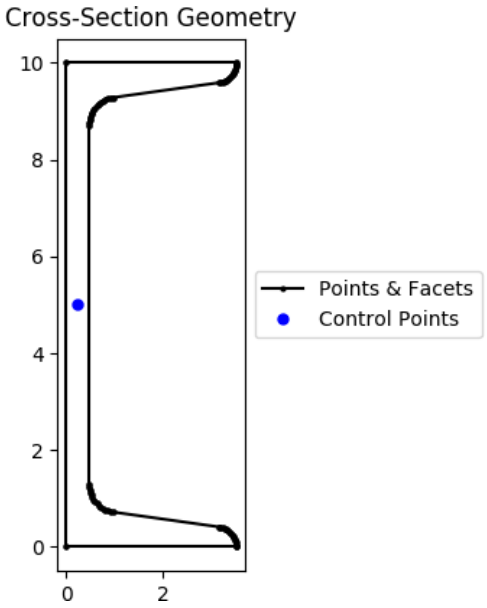


Fig. 33: Tapered flange channel geometry.

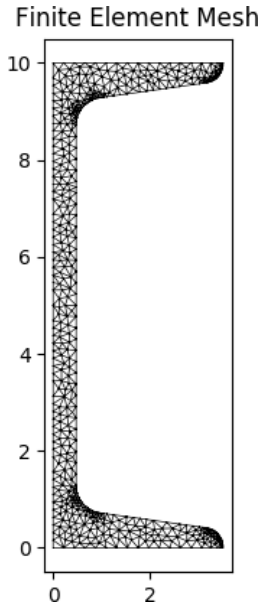


Fig. 34: Mesh generated from the above geometry.

tee_section

sectionproperties.pre.library.steel_sections.**tee_section**(*d: float*, *b: float*, *t_f: float*, *t_w: float*, *r: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Tee section with the top left corner at *(0, d)*, with depth *d*, width *b*, flange thickness *t_f*, web thickness *t_w* and root radius *r*, using *n_r* points to construct the root radius.

Parameters

- **d** (*float*) – Depth of the Tee section
- **b** (*float*) – Width of the Tee section
- **t_f** (*float*) – Flange thickness of the Tee section
- **t_w** (*float*) – Web thickness of the Tee section
- **r** (*float*) – Root radius of the Tee section
- **n_r** (*int*) – Number of points discretising the root radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a Tee section with a depth of 200, a width of 100, a flange thickness of 12, a web thickness of 6 and a root radius of 8, using 8 points to discretise the root radius. A mesh is generated with a maximum triangular area of 3.0:

```
from sectionproperties.pre.library.steel_sections import tee_section

geometry = tee_section(d=200, b=100, t_f=12, t_w=6, r=8, n_r=8)
geometry.create_mesh(mesh_sizes=[3.0])
```

angle_section

sectionproperties.pre.library.steel_sections.**angle_section**(*d: float*, *b: float*, *t: float*, *r_r: float*, *r_t: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs an angle section with the bottom left corner at the origin *(0, 0)*, with depth *d*, width *b*, thickness *t*, root radius *r_r* and toe radius *r_t*, using *n_r* points to construct the radii.

Parameters

- **d** (*float*) – Depth of the angle section
- **b** (*float*) – Width of the angle section

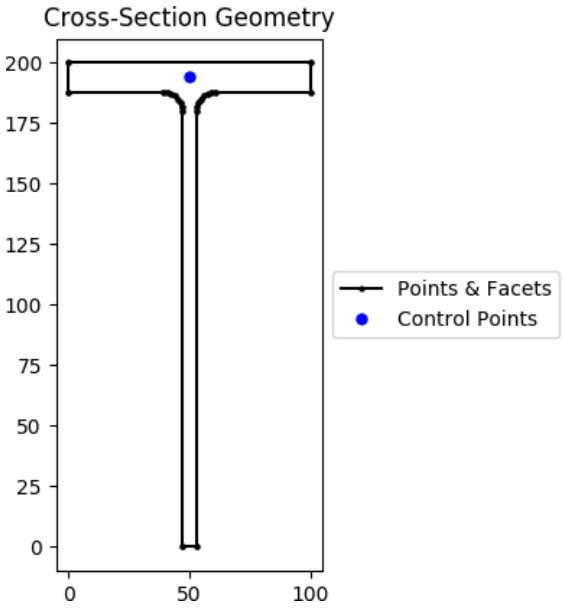


Fig. 35: Tee section geometry.

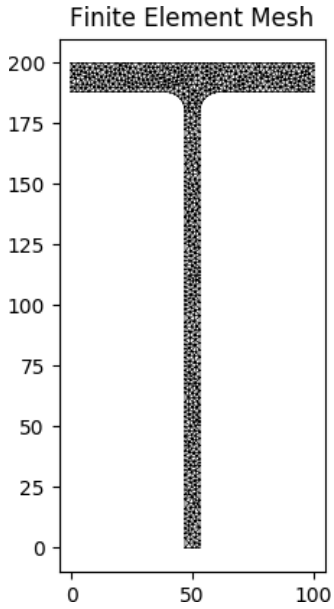


Fig. 36: Mesh generated from the above geometry.

- **t** (*float*) – Thickness of the angle section
- **r_r** (*float*) – Root radius of the angle section
- **r_t** (*float*) – Toe radius of the angle section
- **n_r** (*int*) – Number of points discretising the radii
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates an angle section with a depth of 150, a width of 100, a thickness of 8, a root radius of 12 and a toe radius of 5, using 16 points to discretise the radii. A mesh is generated with a maximum triangular area of 2.0:

```
from sectionproperties.pre.library.steel_sections import angle_section

geometry = angle_section(d=150, b=100, t=8, r_r=12, r_t=5, n_r=16)
geometry.create_mesh(mesh_sizes=[2.0])
```

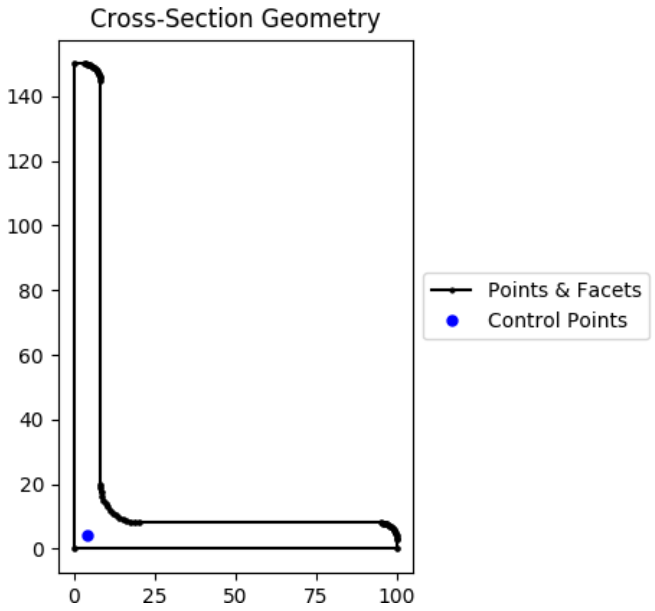
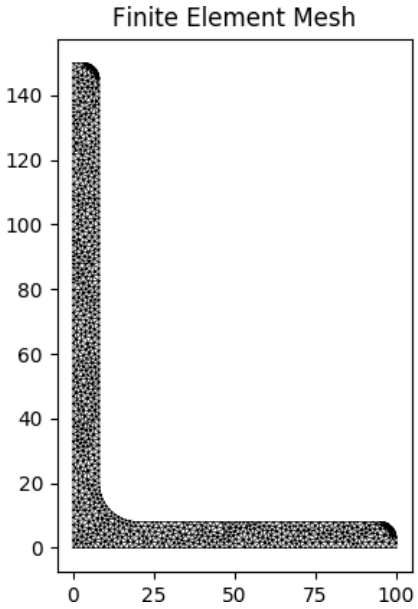


Fig. 37: Angle section geometry.

cee_section

sectionproperties.pre.library.steel_sections.**cee_section**(*d: float*, *b: float*, *l: float*, *t: float*, *r_out: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Cee section (typical of cold-formed steel) with the bottom left corner at the origin *(0, 0)*, with depth *d*, width *b*, lip *l*, thickness *t* and outer radius *r_out*, using *n_r* points to construct the radius. If the outer radius is less than the thickness of the Cee Section, the inner radius is set to zero.



Parameters

- **d** (*float*) – Depth of the Cee section
- **b** (*float*) – Width of the Cee section
- **l** (*float*) – Lip of the Cee section
- **t** (*float*) – Thickness of the Cee section
- **r_out** (*float*) – Outer radius of the Cee section
- **n_r** (*int*) – Number of points discretising the outer radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

Raises

Exception – Lip length must be greater than the outer radius

The following example creates a Cee section with a depth of 125, a width of 50, a lip of 30, a thickness of 1.5 and an outer radius of 6, using 8 points to discretise the radius. A mesh is generated with a maximum triangular area of 0.25:

```
from sectionproperties.pre.library.steel_sections import cee_section

geometry = cee_section(d=125, b=50, l=30, t=1.5, r_out=6, n_r=8)
geometry.create_mesh(mesh_sizes=[0.25])
```

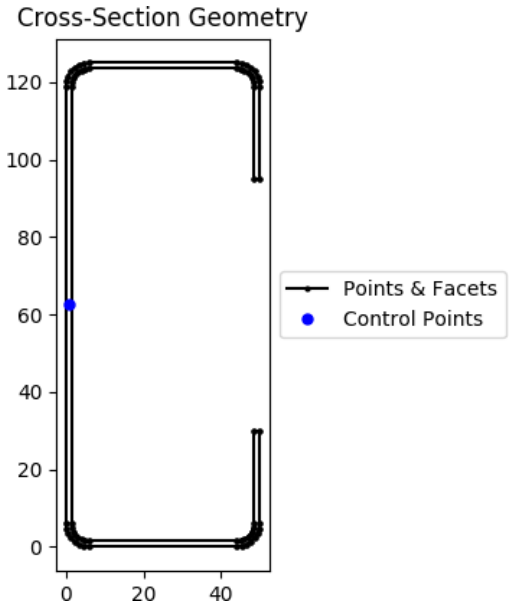
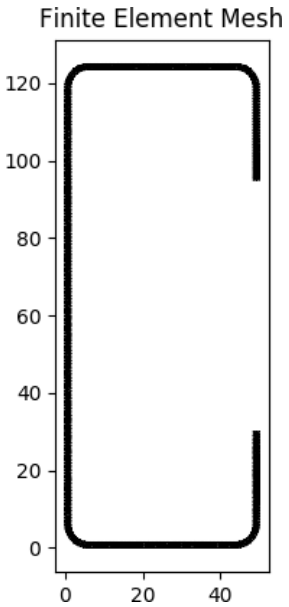


Fig. 38: Cee section geometry.



zed_section

sectionproperties.pre.library.steel_sections.**zed_section**(*d: float*, *b_l: float*, *b_r: float*, *l: float*, *t: float*, *r_out: float*, *n_r: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a zed section with the bottom left corner at the origin *(0, 0)*, with depth *d*, left flange width *b_l*, right flange width *b_r*, lip *l*, thickness *t* and outer radius *r_out*, using *n_r* points to construct the radius. If the outer radius is less than the thickness of the Zed Section, the inner radius is set to zero.

Parameters

- **d** (*float*) – Depth of the zed section
- **b_l** (*float*) – Left flange width of the Zed section
- **b_r** (*float*) – Right flange width of the Zed section
- **l** (*float*) – Lip of the Zed section
- **t** (*float*) – Thickness of the Zed section
- **r_out** (*float*) – Outer radius of the Zed section
- **n_r** (*int*) – Number of points discretising the outer radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a zed section with a depth of 100, a left flange width of 40, a right flange width of 50, a lip of 20, a thickness of 1.2 and an outer radius of 5, using 8 points to discretise the radius. A mesh is generated with a maximum triangular area of 0.15:

```
from sectionproperties.pre.library.steel_sections import zed_section

geometry = zed_section(d=100, b_l=40, b_r=50, l=20, t=1.2, r_out=5, n_r=8)
geometry.create_mesh(mesh_sizes=[0.15])
```

box_girder_section

sectionproperties.pre.library.steel_sections.**box_girder_section**(*d: float*, *b_t: float*, *b_b: float*, *t_ft: float*, *t_fb: float*, *t_w: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*)

Constructs a box girder section centered at at *(max(b_t, b_b)/2, d/2)*, with depth *d*, top width *b_t*, bottom width *b_b*, top flange thickness *t_ft*, bottom flange thickness *t_fb* and web thickness *t_w*.

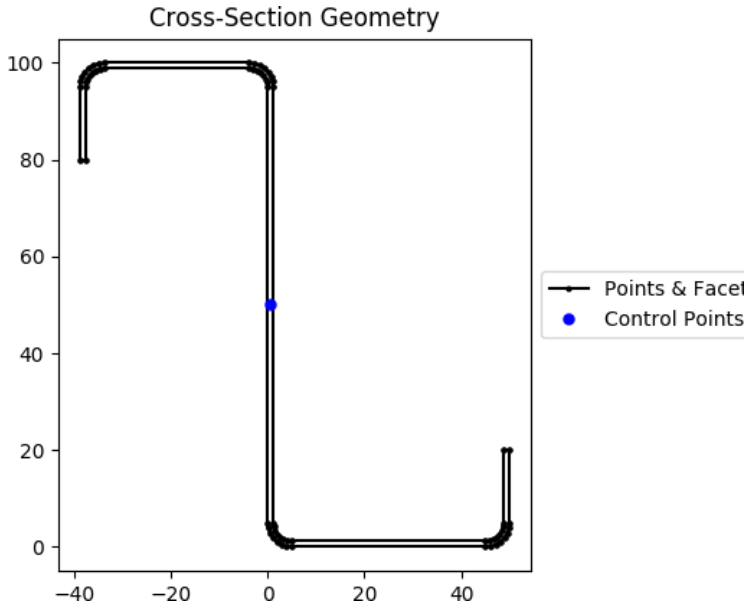
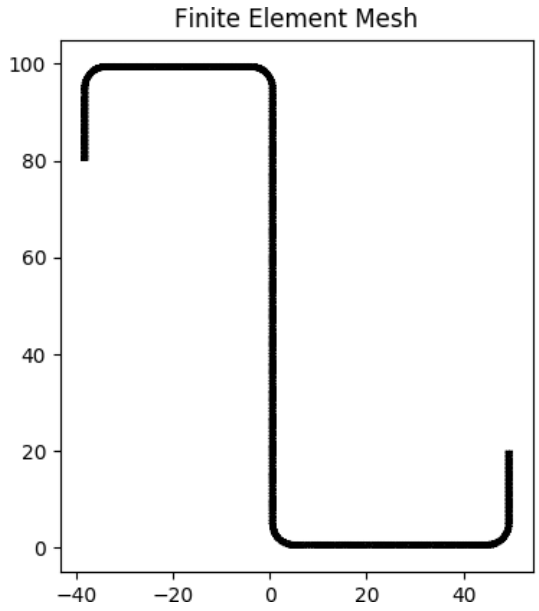


Fig. 39: zed section geometry.



Parameters

- **d** (*float*) – Depth of the Box Girder section
- **b_t** (*float*) – Top width of the Box Girder section
- **b_b** (*float*) – Bottom width of the Box Girder section
- **t_ft** (*float*) – Top flange thickness of the Box Girder section
- **t_fb** (*float*) – Bottom flange thickness of the Box Girder section
- **t_w** (*float*) – Web thickness of the Box Girder section

The following example creates a Box Girder section with a depth of 1200, a top width of 1200, a bottom width of 400, a top flange thickness of 16, a bottom flange thickness of 12 and a web thickness of 8. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.steel_sections import box_girder_section

geometry = box_girder_section(d=1200, b_t=1200, b_b=400, t_ft=100, t_fb=80,
↪ t_w=50)
geometry.create_mesh(mesh_sizes=[200.0])
```

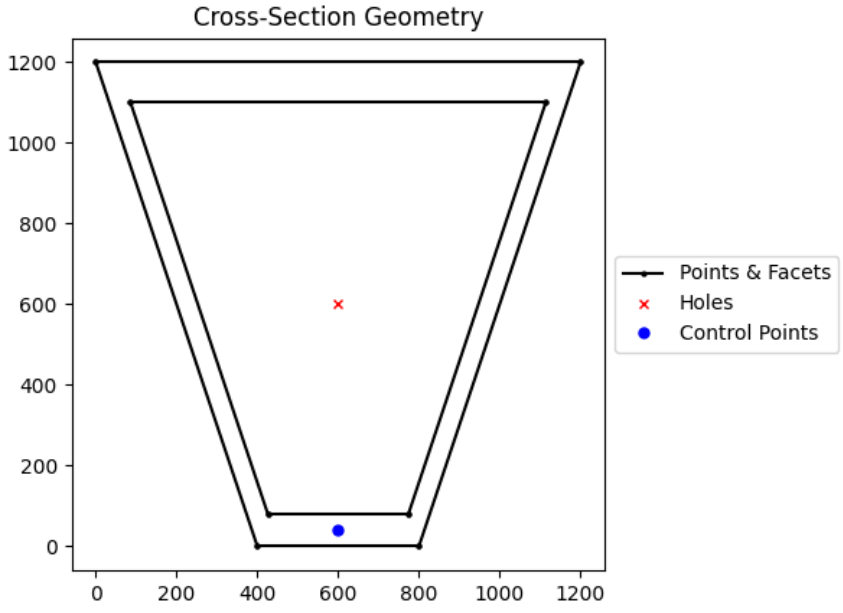


Fig. 40: Box Girder geometry.

bulb_section

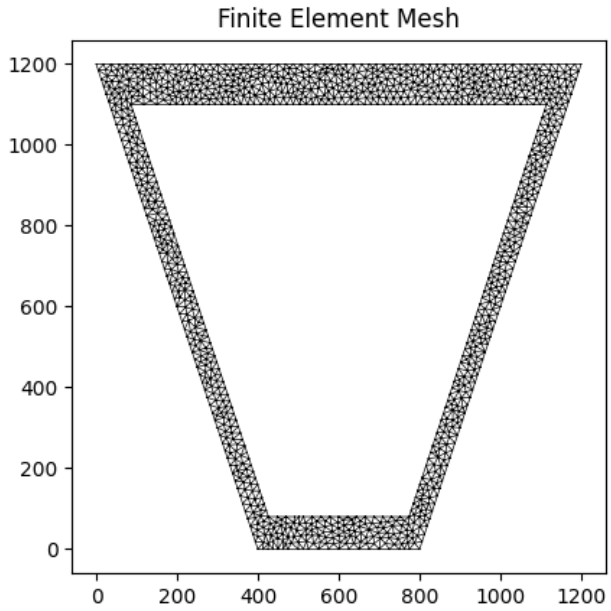


Fig. 41: Mesh generated from the above geometry.

sectionproperties.pre.library.steel_sections.**bulb_section**(*d: float*, *b: float*, *t: float*, *r: float*, *n_r: int*, *d_b: Optional[float] = None*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a bulb section with the bottom left corner at the point *(-t / 2, 0)*, with depth *d*, bulb depth *d_b*, bulb width *b*, web thickness *t* and radius *r*, using *n_r* points to construct the radius.

Parameters

- **d** (*float*) – Depth of the section
- **b** (*float*) – Bulb width
- **t** (*float*) – Web thickness
- **r** (*float*) – Bulb radius
- **d_b** (*float*) – Depth of the bulb (automatically calculated for standard sections, if provided the section may have sharp edges)
- **n_r** (*int*) – Number of points discretising the radius
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a bulb section with a depth of 240, a width of 34, a web thickness of 12 and a bulb radius of 16, using 16 points to discretise the radius. A mesh is generated with a maximum triangular area of 5.0:

```
from sectionproperties.pre.library.steel_sections import bulb_section

geometry = bulb_section(d=240, b=34, t=12, r=10, n_r=16)
geometry.create_mesh(mesh_sizes=[5.0])
```

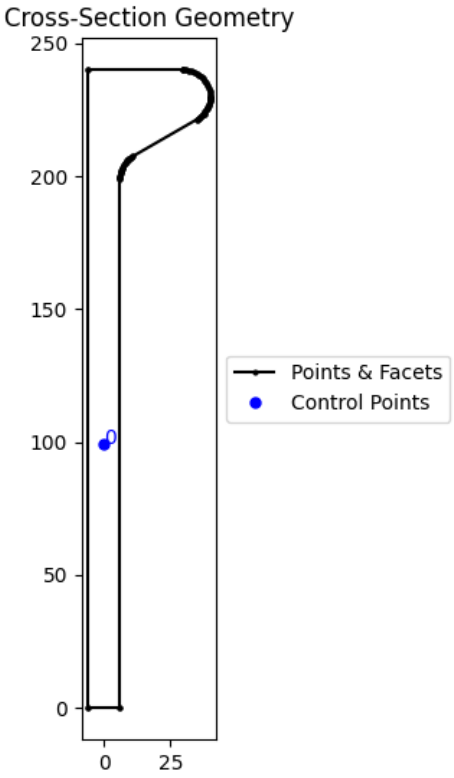


Fig. 42: Bulb section geometry.

9.1.7 *concrete_sections* Module

concrete_rectangular_section

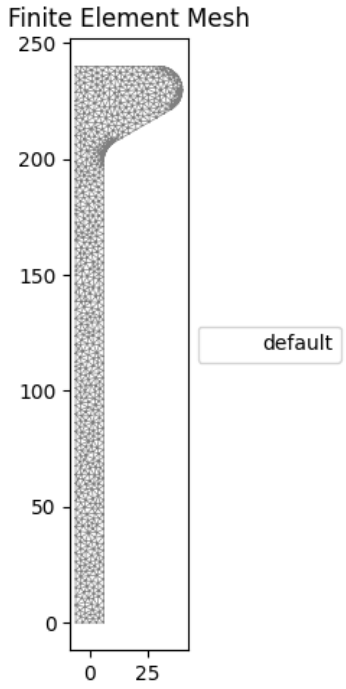


Fig. 43: Mesh generated from the above geometry.

sectionproperties.pre.library.concrete_sections.**concrete_rectangular_section**(*b: float*, *d: float*, *dia_top: float*, *n_top: int*, *dia_bot: float*, *n_bot: int*, *n_circle: int*, *cover: float*, *dia_side: Optional[float] = None*, *n_side: int = 0*, *area_top: Optional[float] = None*, *area_bot: Optional[float] = None*, *area_side: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Mate-*

Constructs a concrete rectangular section of width *b* and depth *d*, with *n_top* top steel bars of diameter *dia_top*, *n_bot* bottom steel bars of diameter *dia_bot*, *n_side* left & right side steel bars of diameter *dia_side* discretised with *n_circle* points with equal side and top/bottom *cover* to the steel.

Parameters

- **b** (*float*) – Concrete section width
- **d** (*float*) – Concrete section depth
- **dia_top** (*float*) – Diameter of the top steel reinforcing bars
- **n_top** (*int*) – Number of top steel reinforcing bars
- **dia_bot** (*float*) – Diameter of the bottom steel reinforcing bars
- **n_bot** (*int*) – Number of bottom steel reinforcing bars
- **n_circle** (*int*) – Number of points discretising the steel reinforcing bars
- **cover** (*float*) – Side and bottom cover to the steel reinforcing bars
- **dia_side** (*float*) – If provided, diameter of the side steel reinforcing bars
- **n_side** (*int*) – If provided, number of side bars either side of the section
- **area_top** (*float*) – If provided, constructs top reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **area_bot** (*float*) – If provided, constructs bottom reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **area_side** (*float*) – If provided, constructs side reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **conc_mat** – Material to associate with the concrete
- **steel_mat** – Material to associate with the steel

Raises

ValueError – If the number of bars is not greater than or equal to 2 in an active layer

The following example creates a 600D x 300W concrete beam with 3N20 bottom steel reinforcing bars and 30 mm cover:

```
from sectionproperties.pre.library.concrete_sections import concrete_rectangular_
↪section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_rectangular_section(
    b=300, d=600, dia_top=20, n_top=0, dia_bot=20, n_bot=3, n_circle=24, cover=30,
    conc_mat=concrete, steel_mat=steel
```

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```
)
geometry.create_mesh(mesh_sizes=[500])
```

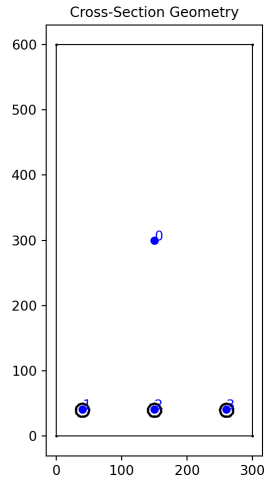

Fig. 44: Concrete rectangular section geometry.

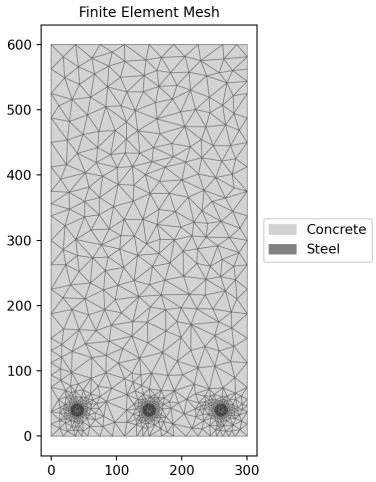

Fig. 45: Mesh generated from the above geometry.

concrete_column_section

sectionproperties.pre.library.concrete_sections.**concrete_column_section**(*b: float*, *d: float*, *cover: float*, *n_bars_b: int*, *n_bars_d: int*, *dia_bar: float*, *bar_area: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *filled: bool = False*, *n_circle: int = 4*) → *CompoundGeometry*

Constructs a concrete rectangular section of width *b* and depth *d*, with steel bar reinforcing organized as an *n_bars_b* by *n_bars_d* array, discretised with *n_circle* points with equal sides and top/bottom *cover* to the steel which is taken as the clear cover (edge of bar to edge of concrete).

Parameters

- **b** (*float*) – Concrete section width, parallel to the x-axis
- **d** (*float*) – Concrete section depth, parallel to the y-axis
- **cover** (*float*) – Clear cover, calculated as distance from edge of reinforcing bar to edge of section.
- **n_bars_b** (*int*) – Number of bars placed across the width of the section, minimum 2.
- **n_bars_d** (*int*) – Number of bars placed across the depth of the section, minimum 2.
- **dia_bar** (*float*) – Diameter of reinforcing bars. Used for calculating bar placement and, optionally, for calculating the bar area for section capacity calculations.
- **bar_area** (*float*) – Area of reinforcing bars. Used for section capacity calculations. If not provided, then dia_bar will be used to calculate the bar area.
- **conc_mat** (sectionproperties.pre.pre.Material) – Material to associate with the concrete
- **steel_mat** (sectionproperties.pre.pre.Material) – Material to associate with the reinforcing steel
- **filled** (*bool*) – When True, will populate the concrete section with an equally spaced 2D array of reinforcing bars numbering 'n_bars_b' by 'n_bars_d'. When False, only the bars around the perimeter of the array will be present.
- **n_circle** (*int*) – The number of points used to discretize the circle of the reinforcing bars. The bars themselves will have an exact area of 'bar_area' regardless of the number of points

used in the circle. Useful for making the reinforcing bars look more circular when plotting the concrete section.

Raises

ValueError – If the number of bars in either 'n_bars_b' or 'n_bars_d' is not greater than or equal to 2.

The following example creates a 600D x 300W concrete column with 25 mm diameter reinforcing bars each with 500 mm**2 area and 35 mm cover in a 3x6 array without the interior bars being filled:

```
from sectionproperties.pre.library.concrete_sections import concrete_column_section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_column_section(
    b=300, d=600, dia_bar=25, bar_area=500, cover=35, n_bars_b=3, n_bars_d=6,
    conc_mat=concrete, steel_mat=steel, filled=False, n_circle=4
)
geometry.create_mesh(mesh_sizes=[500])
```

concrete_tee_section

sectionproperties.pre.library.concrete_sections.**concrete_tee_section**(*b: float*, *d: float*, *b_f: float*, *d_f: float*, *dia_top: float*, *n_top: int*, *dia_bot: float*, *n_bot: int*, *n_circle: int*, *cover: float*, *area_top: Optional[float] = None*, *area_bot: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *CompoundGeometry*

Constructs a concrete tee section of width *b*, depth *d*, flange width *b_f* and flange depth *d_f*, with *n_top* top steel bars of diameter *dia_top*, *n_bot* bottom steel bars of diameter *dia_bot*, discretised with *n_circle* points with equal side and top/bottom *cover* to the steel.

Parameters

- **b** (*float*) – Concrete section width
- **d** (*float*) – Concrete section depth
- **b_f** (*float*) – Concrete section flange width
- **d_f** (*float*) – Concrete section flange depth
- **dia_top** (*float*) – Diameter of the top steel reinforcing bars
- **n_top** (*int*) – Number of top steel reinforcing bars
- **dia_bot** (*float*) – Diameter of the bottom steel reinforcing bars
- **n_bot** (*int*) – Number of bottom steel reinforcing bars
- **n_circle** (*int*) – Number of points discretising the steel reinforcing bars
- **cover** (*float*) – Side and bottom cover to the steel reinforcing bars
- **area_top** (*float*) – If provided, constructs top reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **area_bot** (*float*) – If provided, constructs bottom reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to circle discretisation)
- **conc_mat** – Material to associatewith the concrete
- **steel_mat** – Material toassociate with the steel

Raises

ValueErorr – If the number of bars is not greater than or equal to 2 in an active layer

The following example creates a 900D x 450W concrete beam with a 1200W x 250D flange, with 5N24 steel reinforcing bars and 30 mm cover:

```
from sectionproperties.pre.library.concrete_sections import concrete_tee_section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_tee_section(
    b=450, d=900, b_f=1200, d_f=250, dia_top=24, n_top=0, dia_bot=24, n_bot=5,
    n_circle=24, cover=30, conc_mat=concrete, steel_mat=steel
)
geometry.create_mesh(mesh_sizes=[500])
```

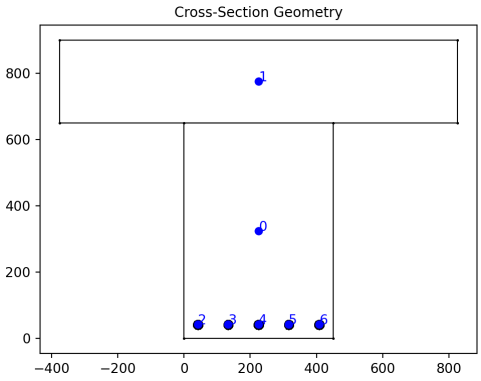

Fig. 46: Concrete tee section geometry.

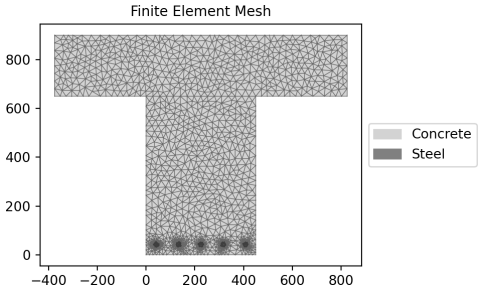

Fig. 47: Mesh generated from the above geometry.

concrete_circular_section

sectionproperties.pre.library.concrete_sections.**concrete_circular_section**(*d: float*, *n: int*, *dia: float*, *n_bar: int*, *n_circle: int*, *cover: float*, *area_conc: Optional[float] = None*, *area_bar: Optional[float] = None*, *conc_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *steel_mat: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *CompoundGeometry*

Constructs a concrete circular section of diameter *d* discretised with *n* points, with *n_bar* steel bars of diameter *dia*, discretised with *n_circle* points with equal side and bottom *cover* to the steel.

Parameters

- **d** (*float*) – Concrete diameter
- **n** (*float*) – Number of points discretising the concrete section
- **dia** (*float*) – Diameter of the steel reinforcing bars
- **n_bar** (*int*) – Number of steel reinforcing bars
- **n_circle** (*int*) – Number of points discretising the steel reinforcing bars
- **cover** (*float*) – Side and bottom cover to the steel reinforcing bars
- **area_conc** (*float*) – If provided, constructs the concrete based on its area rather than diameter (prevents the underestimation of concrete area due to circle discretisation)
- **area_bar** (*float*) – If provided, constructs reinforcing bars based on their area rather than diameter (prevents the underestimation of steel area due to
- **conc_mat** – Material to associate with the concrete
- **steel_mat** – Material to associate with the steel

Raises

ValueErorr – If the number of bars is not greater than or equal to 2

The following example creates a 450DIA concrete column with with 6N20 steel reinforcing bars and 45 mm cover:

```
from sectionproperties.pre.library.concrete_sections import concrete_circular_
↪section
from sectionproperties.pre.pre import Material

concrete = Material(
    name='Concrete', elastic_modulus=30.1e3, poissons_ratio=0.2, yield_strength=32,
    density=2.4e-6, color='lightgrey'
)
steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, yield_strength=500,
    density=7.85e-6, color='grey'
)

geometry = concrete_circular_section(
    d=450, n=64, dia=20, n_bar=6, n_circle=24, cover=45, conc_mat=concrete, steel_
↪mat=steel
)
geometry.create_mesh(mesh_sizes=[500])
```

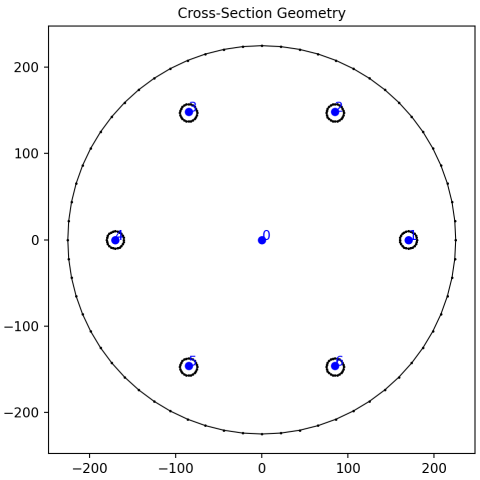

Fig. 48: Concrete circular section geometry.

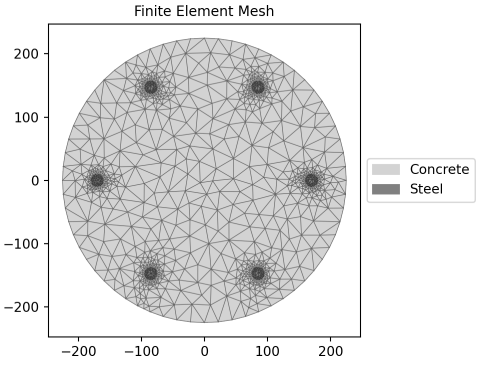

Fig. 49: Mesh generated from the above geometry.

add_bar

sectionproperties.pre.library.concrete_sections.**add_bar**(*geometry: Union[Geometry, CompoundGeometry]*, *area: float*, *material: Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*, *x: float*, *y: float*, *n: int = 4*) → *CompoundGeometry*

Adds a reinforcing bar to a *sectionproperties* geometry.

Bars are discretised by four points by default.

Parameters

- **geometry** – Reinforced concrete geometry to which the new bar will be added
- **area** – Bar cross-sectional area
- **material** – Material object for the bar
- **x** – x-position of the bar
- **y** – y-position of the bar
- **n** – Number of points to discretise the bar circle

Returns

Reinforced concrete geometry with added bar

9.1.8 *bridge_sections* Module

super_t_girder_section

sectionproperties.pre.library.bridge_sections.**super_t_girder_section**(*girder_type: int*, *girder_subtype: int = 2*, *w: float = 2100*, *t_w: Optional[float] = None*, *t_f: float = 75*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Super T Girder section to AS5100.5.

Parameters

- **girder_type** (*int*) – Type of Super T (1 to 5)
- **girder_subtype** (*int*) – Era Super T (1: pre-2001, 2:contemporary)
- **w** (*float*) – Overall width of top flange
- **t_w** (*float*) – Web thickness of the Super-T section (defaults to those of AS5100.5 Tb D3(B))
- **t_f** (*float*) – Thickness of top flange (VIC (default) = 75 mm; NSW = 90 mm)

- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a T5 Super-T section with a 180 mm overlay slab and assigns the different material properties:

```
import sectionproperties.pre.library.bridge_sections as bridge_sections
import sectionproperties.pre.library.primitive_sections as primitive_sections
from sectionproperties.pre.pre import Material
from sectionproperties.analysis.section import Section

Dslab, w, t_f = 180, 2100, 75

precast = Material(
    name="65 MPa",
    elastic_modulus=37.4e3,
    poissons_ratio=0.2,
    yield_strength=65,
    density=2.4e-6,
    color="grey",
)
insitu = Material(
    name="40 MPa",
    elastic_modulus=32.8e3,
    poissons_ratio=0.2,
    yield_strength=40,
    density=2.4e-6,
    color="lightgrey",
)

super_t = bridge_sections.super_t_girder_section(girder_type=5, w=w,␣
↪material=precast)
slab = primitive_sections.rectangular_section(
    d=Dslab, b=w, material=insitu
).shift_section(-w / 2, t_f)

geom = super_t + slab
geom.plot_geometry()
geom.create_mesh(mesh_sizes=[500])

sec = Section(geom)
sec.plot_mesh()

sec.calculate_geometric_properties()
sec.calculate_warping_properties()
sec.display_results(fmt=".3f")
```

Note that the properties are reported as `modulus weighted` properties (e.g. E.A) and can be normalized to the reference material by dividing by that elastic modulus:

```
A_65 = section.get_ea() / precast.elastic_modulus
```

The reported section centroids are already weighted.

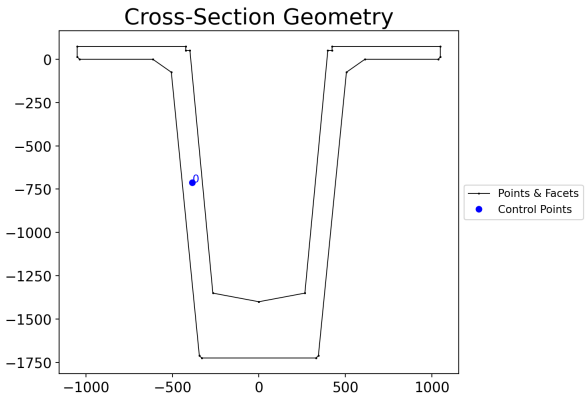

Fig. 50: Super Tee Girder.

i_girder_section

sectionproperties.pre.library.bridge_sections.**i_girder_section**(*girder_type: int*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a precast I girder section to AS5100.5.

Parameters

- **girder_type** (*int*) – Type of I Girder (1 to 4)
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

As an example, replicate the table shown in AS5100.5 Fig. D1(A):

```
import pandas as pd
import sectionproperties.pre.library.bridge_sections as bridge_sections
from sectionproperties.analysis.section import Section

df = pd.DataFrame(columns=["Ag", "Zt", "Zb", "I", "dy", "th"])

for i in range(4):
    geom = bridge_sections.i_girder_section(girder_type=i + 1)
    dims = bridge_sections.get_i_girder_dims(girder_type=i + 1)
    d = sum(dims[-5:])
    geom.create_mesh(mesh_sizes=[200])
    geom.plot_geometry()
    sec = Section(geom)
    sec.plot_mesh()
    sec.calculate_geometric_properties()
    sec.calculate_warping_properties()

    A = sec.get_area()
```

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```
    th = A / (sec.get_perimeter() / 2)

    df.loc[i] = [
        A,
        *(sec.get_z()[:2]),
        sec.get_ic()[0],
        d + sec.get_c()[1],
        th,
    ]

print(df)
```

Note that the section depth is obtained by summing the heights from the section dictionary in `get_i_girder_dims()`.

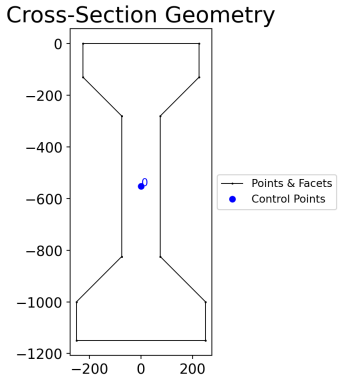

Fig. 51: I Girder.

get_super_t_girder_dims

sectionproperties.pre.library.bridge_sections.**get_super_t_girder_dims**(*girder_type*)

Returns a dictionary of Super-T dimensions, refer to AS5100.5, Appendix D

Parameters

girder_type (*int*) – Type of Super T (1 to 5)

get_i_girder_dims

sectionproperties.pre.library.bridge_sections.**get_i_girder_dims**(*girder_type*)

Returns a dictionary of I girder dimensions, refer to AS5100.5, Appendix D

Parameters

girder_type (*int*) – Type of I Girder (1 to 4)

9.1.9 *nastran_sections* Module

This module contains sections as defined by Nastran and Nastran-based programs, such as MYSTRAN and ASTROS.

nastran_bar

sectionproperties.pre.library.nastran_sections.**nastran_bar**(*DIM1: float*, *DIM2: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a BAR section with the center at the origin *(0, 0)*, with two parameters defining dimensions. See Nastran documentation[1][2][3][4][5] for definition of parameters. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of bar
- **DIM2** (*float*) – Depth (y) of bar
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a BAR cross-section with a depth of 1.5 and width of 2.0, and generates a mesh with a maximum triangular area of 0.001:

```
from sectionproperties.pre.library.nastran_sections import nastran_bar

geom = nastran_bar(DIM1=2.0, DIM2=1.5)
mesh = geometry.create_mesh(mesh_sizes=[0.001])
```

nastran_box

sectionproperties.pre.library.nastran_sections.**nastran_box**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a BOX section with the center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[1][2][3][4][5] for definition of parameters. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of box
- **DIM2** (*float*) – Depth (y) of box

[1] MSC Nastran Quick Reference Guide 2012, PBEAML - Simple Beam Cross-Section Property, pp. 2890-2894 https://simcompanion.mscsoftware.com/infocenter/index?page=content&id=DOC10351

[2] Siemens NX Nastran 12 Quick Reference Guide, PBEAML, pp. 16-59 - 16-62 https://docs.plm.automation.siemens.com/data_services/resources/nxnastran/12/help/tdoc/en_US/pdf/QRG.pdf

[3] AutoDesk Nastran Online Documentation, Nastran Reference Guide, Section 4 - Bulk Data, PBEAML http://help.autodesk.com/view/NSTRN/2018/ENU/?guid=GUID-B7044BA7-3C26-49DA-9EE7-DA7505FD4B2C

[4] Users Reference Manual for the MYSTRAN General Purpose Finite Element Structural Analysis Computer Program, Jan. 2019, Section 6.4.1.53 - PBARL, pp. 154-156 https://www.mystran.com/Executable/MYSTRAN-Users-Manual.pdf

[5] Astros Enhancements - Volume III - Astros Theoretical Manual, Section 5.1.3.2, pp. 56 https://apps.dtic.mil/dtic/tr/fulltext/u2/a308134.pdf

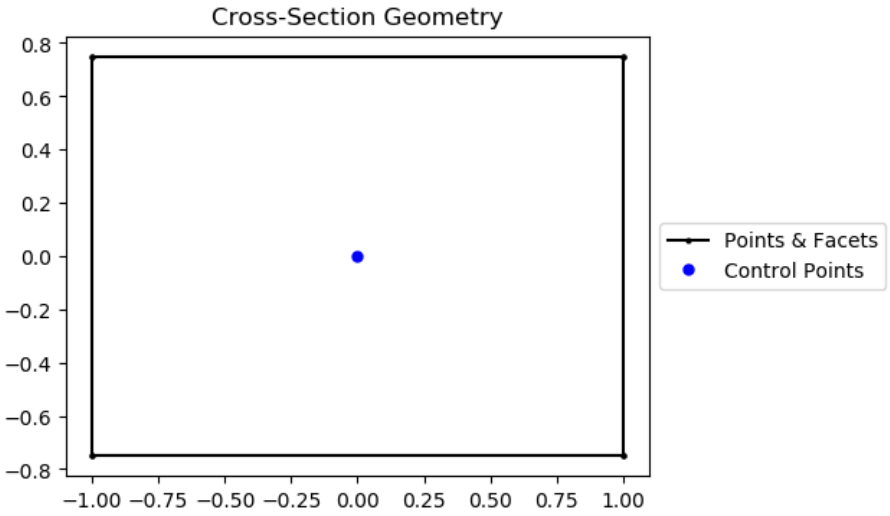

Fig. 52: BAR section geometry.

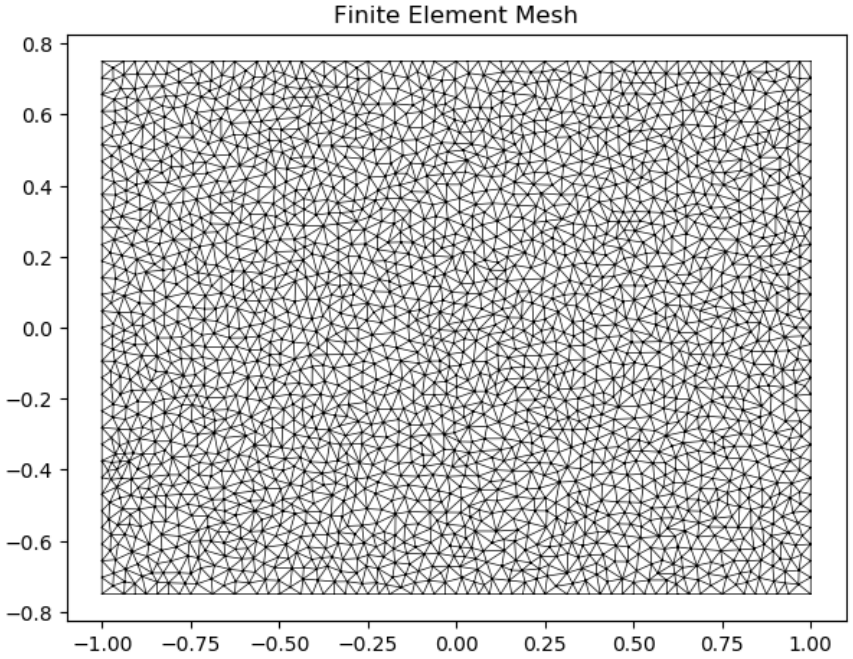

Fig. 53: Mesh generated from the above geometry.

- **DIM3** (*float*) – Thickness of box in y direction
- **DIM4** (*float*) – Thickness of box in x direction
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a BOX cross-section with a depth of 3.0 and width of 4.0, and generates a mesh with a maximum triangular area of 0.001:

```
from sectionproperties.pre.library.nastran_sections import nastran_box

geom = nastran_box(DIM1=4.0, DIM2=3.0, DIM3=0.375, DIM4=0.5)
mesh = geometry.create_mesh(mesh_sizes=[0.001])
```

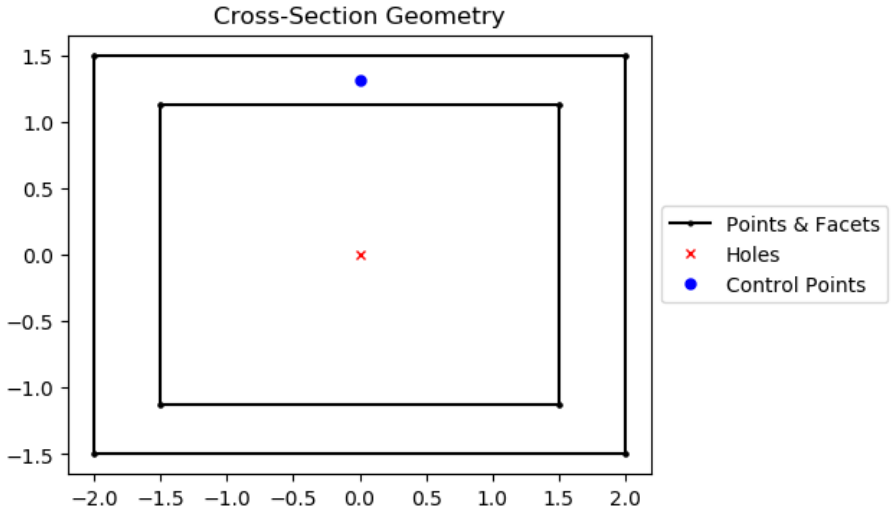

Fig. 54: BOX section geometry.

nastran_box1

sectionproperties.pre.library.nastran_sections.**nastran_box1**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *DIM5: float*, *DIM6: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a BOX1 section with the center at the origin *(0, 0)*, with six parameters defining dimensions. See Nastran documentation[1 Page 290, 2 Page 290, 3 Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of box
- **DIM2** (*float*) – Depth (y) of box
- **DIM3** (*float*) – Thickness of top wall
- **DIM4** (*float*) – Thickness of bottom wall
- **DIM5** (*float*) – Thickness of left wall

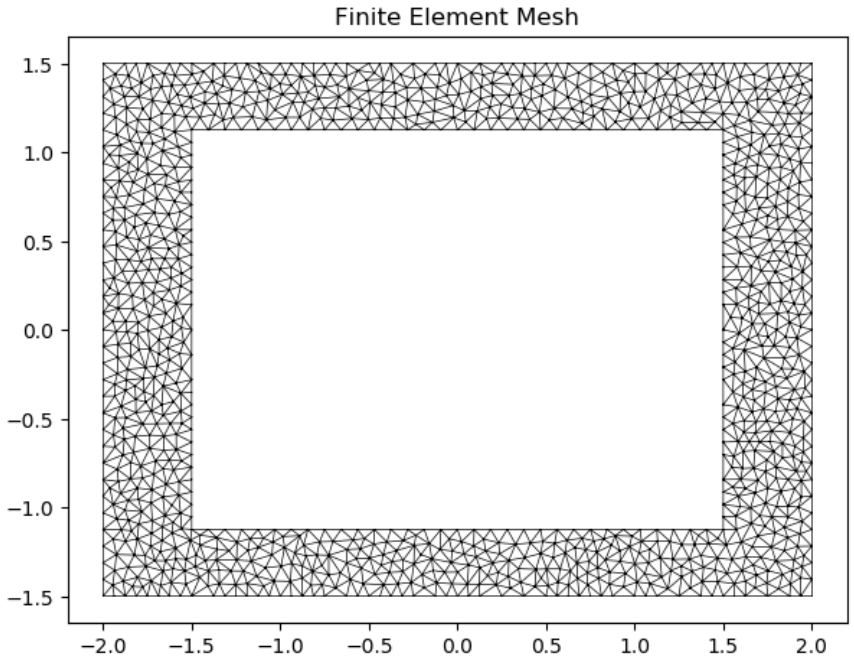

Fig. 55: Mesh generated from the above geometry.

- **DIM6** (*float*) – Thickness of right wall
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a BOX1 cross-section with a depth of 3.0 and width of 4.0, and generates a mesh with a maximum triangular area of 0.007:

```
from sectionproperties.pre.library.nastran_sections import nastran_box1

geom = nastran_box1(
    DIM1=4.0, DIM2=3.0, DIM3=0.375, DIM4=0.5, DIM5=0.25, DIM6=0.75
)
mesh = geometry.create_mesh(mesh_sizes=[0.007])
```

nastran_chan

sectionproperties.pre.library.nastran_sections.**nastran_chan**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a CHAN (C-Channel) section with the web's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of the CHAN-section

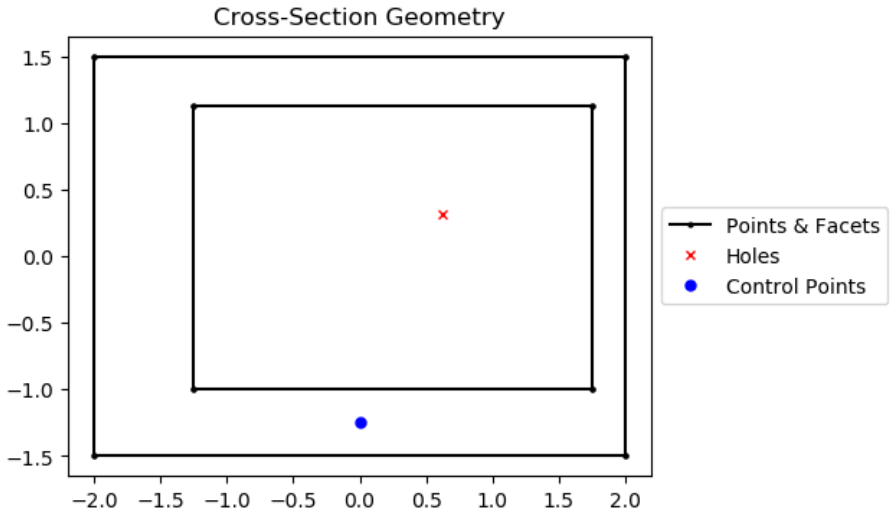

Fig. 56: BOX1 section geometry.

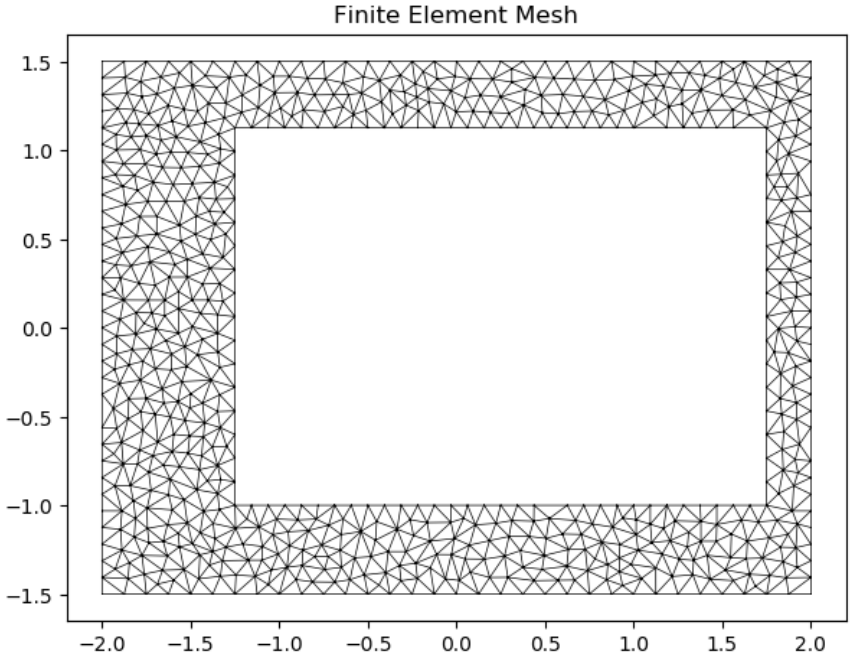

Fig. 57: Mesh generated from the above geometry.

- **DIM2** (*float*) – Depth (y) of the CHAN-section
- **DIM3** (*float*) – Thickness of web (vertical portion)
- **DIM4** (*float*) – Thickness of flanges (top/bottom portion)
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a CHAN cross-section with a depth of 4.0 and width of 2.0, and generates a mesh with a maximum triangular area of 0.008:

```
from sectionproperties.pre.library.nastran_sections import nastran_chan

geom = nastran_chan(DIM1=2.0, DIM2=4.0, DIM3=0.25, DIM4=0.5)
mesh = geometry.create_mesh(mesh_sizes=[0.008])
```

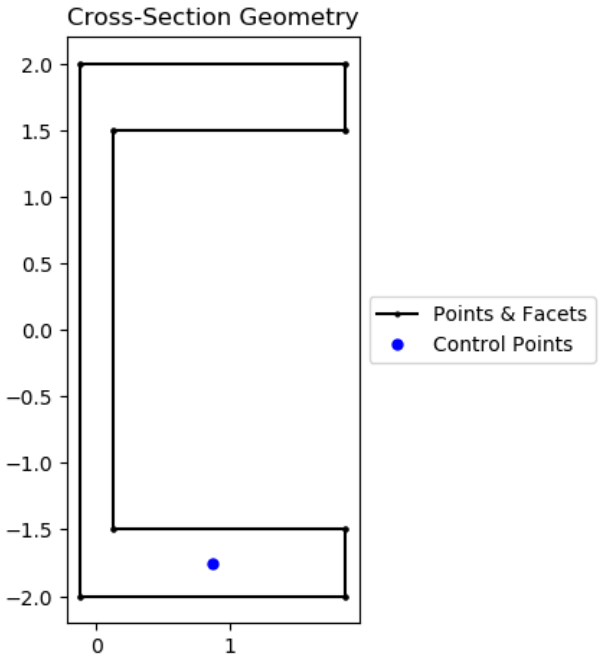

Fig. 58: CHAN section geometry.

nastran_chan1

sectionproperties.pre.library.nastran_sections.**nastran_chan1**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a CHAN1 (C-Channel) section with the web's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 4] for more details. Added by JohnDN90.

Parameters

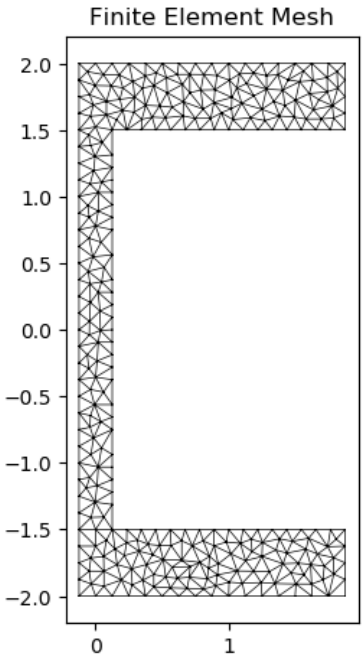

Fig. 59: Mesh generated from the above geometry.

- **DIM1** (*float*) – Width (x) of channels
- **DIM2** (*float*) – Thickness (x) of web
- **DIM3** (*float*) – Spacing between channels (length of web)
- **DIM4** (*float*) – Depth (y) of CHAN1-section
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a CHAN1 cross-section with a depth of 4.0 and width of 1.75, and generates a mesh with a maximum triangular area of 0.01:

```
from sectionproperties.pre.library.nastran_sections import nastran_chan1

geom = nastran_chan1(DIM1=0.75, DIM2=1.0, DIM3=3.5, DIM4=4.0)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

nastran_chan2

sectionproperties.pre.library.nastran_sections.**nastran_chan2**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a CHAN2 (C-Channel) section with the bottom web's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 4] for more details. Added by JohnDN90.

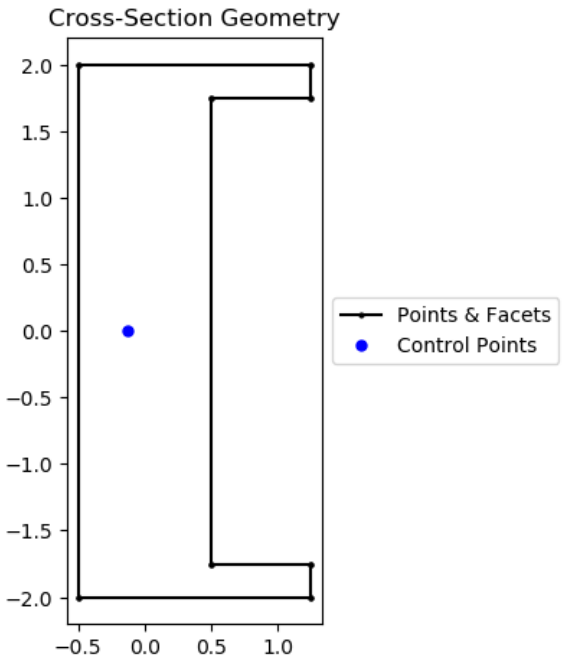

Fig. 60: CHAN1 section geometry.

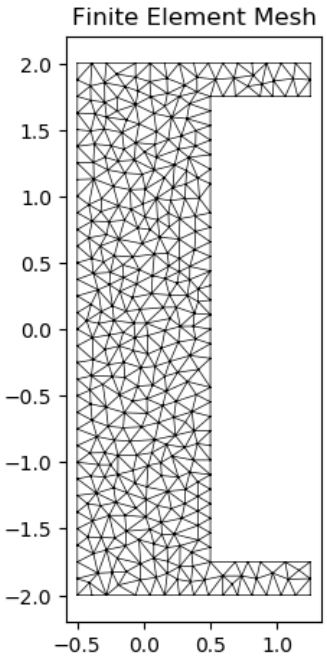

Fig. 61: Mesh generated from the above geometry.

Parameters

- **DIM1** (*float*) – Thickness of channels
- **DIM2** (*float*) – Thickness of web
- **DIM3** (*float*) – Depth (y) of CHAN2-section
- **DIM4** (*float*) – Width (x) of CHAN2-section
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a CHAN2 cross-section with a depth of 2.0 and width of 4.0, and generates a mesh with a maximum triangular area of 0.01:

```
from sectionproperties.pre.library.nastran_sections import nastran_chan2

geom = nastran_chan2(DIM1=0.375, DIM2=0.5, DIM3=2.0, DIM4=4.0)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

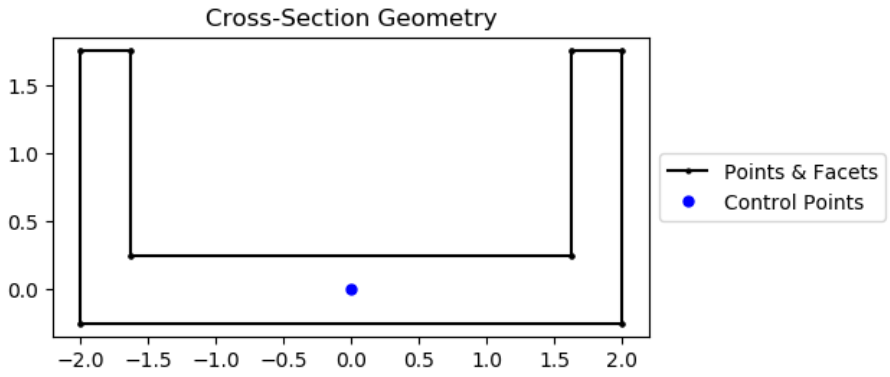

Fig. 62: CHAN2 section geometry.

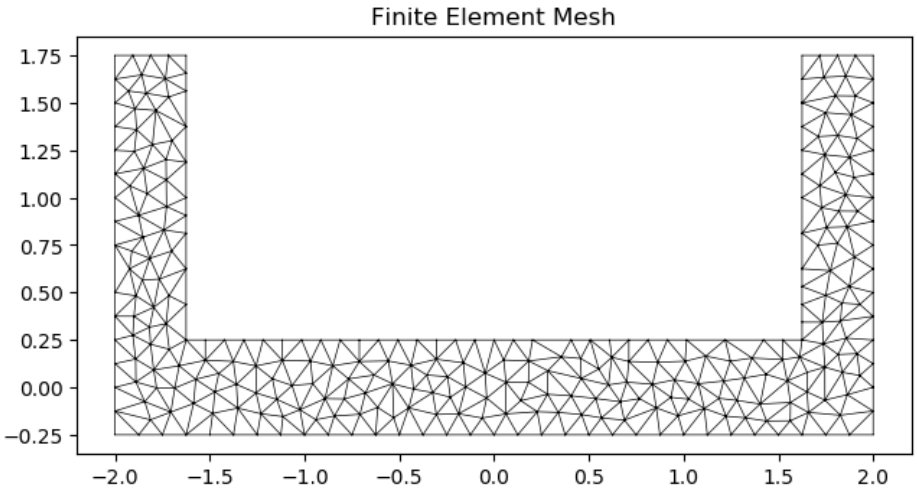

Fig. 63: Mesh generated from the above geometry.

nastran_cross

sectionproperties.pre.library.nastran_sections.**nastran_cross**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material: Material = Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs Nastran's cruciform/cross section with the intersection's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Twice the width of horizontal member protruding from the vertical center member
- **DIM2** (*float*) – Thickness of the vertical member
- **DIM3** (*float*) – Depth (y) of the CROSS-section
- **DIM4** (*float*) – Thickness of the horizontal members
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a rectangular cross-section with a depth of 3.0 and width of 1.875, and generates a mesh with a maximum triangular area of 0.008:

```
from sectionproperties.pre.library.nastran_sections import nastran_cross

geom = nastran_cross(DIM1=1.5, DIM2=0.375, DIM3=3.0, DIM4=0.25)
mesh = geometry.create_mesh(mesh_sizes=[0.008])
```

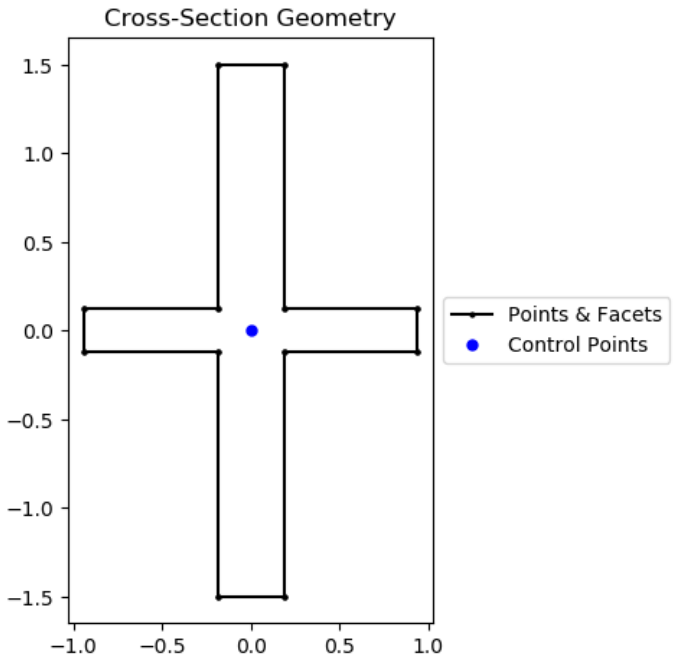

Fig. 64: Cruciform/cross section geometry.

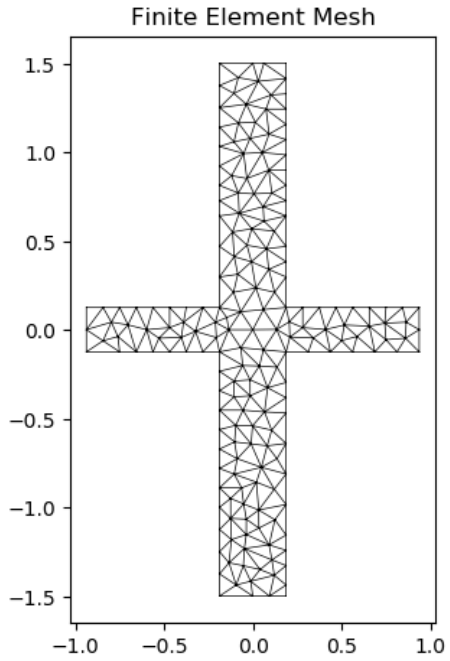

Fig. 65: Mesh generated from the above geometry.

nastran_dbox

sectionproperties.pre.library.nastran_sections.**nastran_dbox**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *DIM5: float*, *DIM6: float*, *DIM7: float*, *DIM8: float*, *DIM9: float*, *DIM10: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a DBOX section with the center at the origin *(0, 0)*, with ten parameters defining dimensions. See MSC Nastran documentation[Page 290, 1] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of the DBOX-section
- **DIM2** (*float*) – Depth (y) of the DBOX-section
- **DIM3** (*float*) – Width (x) of left-side box
- **DIM4** (*float*) – Thickness of left wall
- **DIM5** (*float*) – Thickness of center wall
- **DIM6** (*float*) – Thickness of right wall
- **DIM7** (*float*) – Thickness of top left wall
- **DIM8** (*float*) – Thickness of bottom left wall
- **DIM9** (*float*) – Thickness of top right wall
- **DIM10** (*float*) – Thickness of bottom right wall

- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a DBOX cross-section with a depth of 3.0 and width of 8.0, and generates a mesh with a maximum triangular area of 0.01:

```
from sectionproperties.pre.library.nastran_sections import nastran_dbox

geom = nastran_dbox(
    DIM1=8.0, DIM2=3.0, DIM3=3.0, DIM4=0.5, DIM5=0.625, DIM6=0.75, DIM7=0.375,␣
↪DIM8=0.25,
    DIM9=0.5, DIM10=0.375
)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

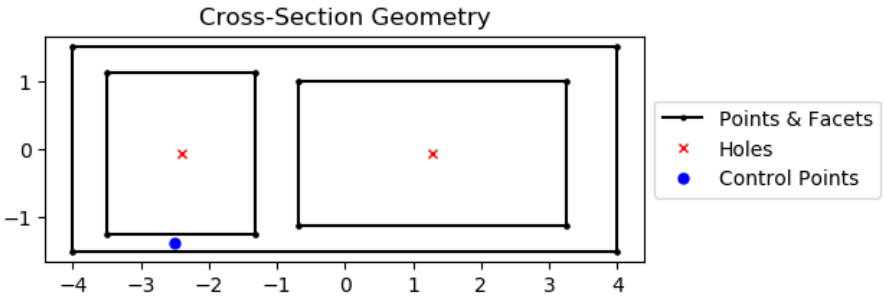

Fig. 66: DBOX section geometry.

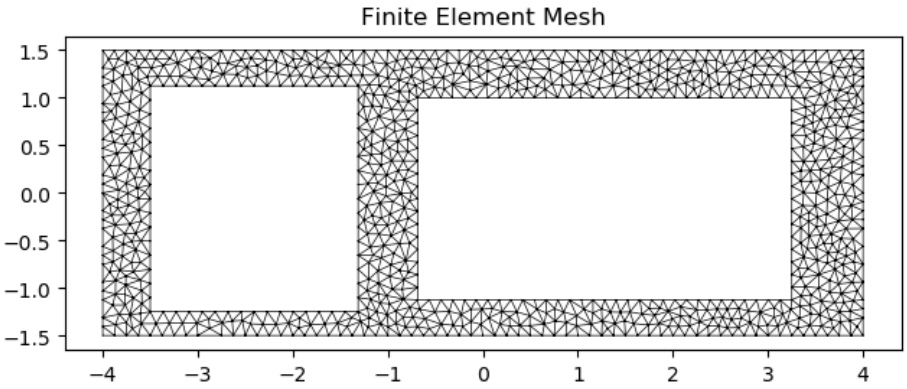

Fig. 67: Mesh generated from the above geometry.

nastran_fcross

sectionproperties.pre.library.nastran_sections.**nastran_fcross**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *DIM5: float*, *DIM6: float*, *DIM7: float*, *DIM8: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a flanged cruciform/cross section with the intersection's middle center at the origin *(0, 0)*, with eight parameters defining dimensions. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Depth (y) of flanged cruciform
- **DIM2** (*float*) – Width (x) of flanged cruciform
- **DIM3** (*float*) – Thickness of vertical web
- **DIM4** (*float*) – Thickness of horizontal web
- **DIM5** (*float*) – Length of flange attached to vertical web
- **DIM6** (*float*) – Thickness of flange attached to vertical web
- **DIM7** (*float*) – Length of flange attached to horizontal web
- **DIM8** (*float*) – Thickness of flange attached to horizontal web
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example demonstrates the creation of a flanged cross section:

```
from sectionproperties.pre.library.nastran_sections import nastran_fcross

geom = nastran_fcross(
    DIM1=9.0, DIM2=6.0, DIM3=0.75, DIM4=0.625, DIM5=2.1, DIM6=0.375, DIM7=4.5,␣
↪DIM8=0.564
)
mesh = geometry.create_mesh(mesh_sizes=[0.03])
```

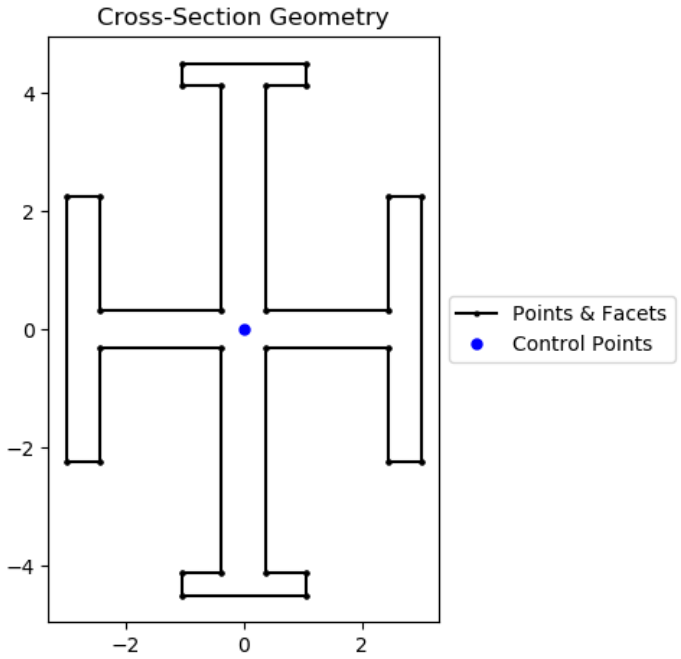

Fig. 68: Flanged Cruciform/cross section geometry.

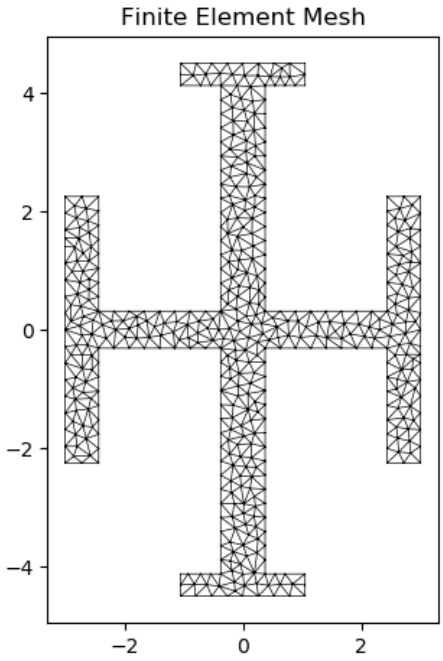

Fig. 69: Mesh generated from the above geometry.

nastran_gbox

sectionproperties.pre.library.nastran_sections.**nastran_gbox**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *DIM5: float*, *DIM6: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a GBOX section with the center at the origin *(0, 0)*, with six parameters defining dimensions. See ASTROS documentation[Page 290, 5] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of the GBOX-section
- **DIM2** (*float*) – Depth (y) of the GBOX-section
- **DIM3** (*float*) – Thickness of top flange
- **DIM4** (*float*) – Thickness of bottom flange
- **DIM5** (*float*) – Thickness of webs
- **DIM6** (*float*) – Spacing between webs
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a GBOX cross-section with a depth of 2.5 and width of 6.0, and generates a mesh with a maximum triangular area of 0.01:

```
from sectionproperties.pre.library.nastran_sections import nastran_gbox

geom = nastran_gbox(
    DIM1=6.0, DIM2=2.5, DIM3=0.375, DIM4=0.25, DIM5=0.625, DIM6=1.0
)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

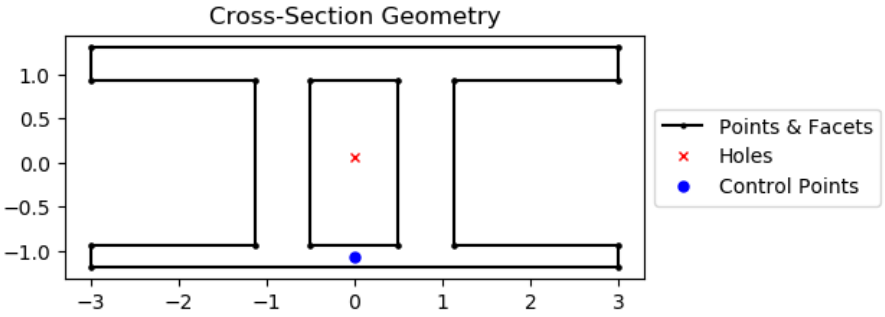

Fig. 70: GBOX section geometry.

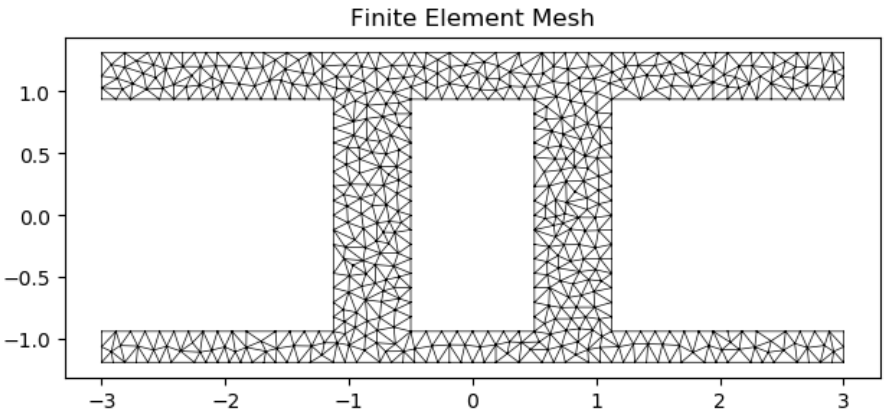

Fig. 71: Mesh generated from the above geometry.

nastran_h

sectionproperties.pre.library.nastran_sections.**nastran_h**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a H section with the middle web's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Spacing between vertical flanges (length of web)
- **DIM2** (*float*) – Twice the thickness of the vertical flanges

- **DIM3** (*float*) – Depth (y) of the H-section
- **DIM4** (*float*) – Thickness of the middle web
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a H cross-section with a depth of 3.5 and width of 2.75, and generates a mesh with a maximum triangular area of 0.005:

```
from sectionproperties.pre.library.nastran_sections import nastran_h

geom = nastran_h(DIM1=2.0, DIM2=0.75, DIM3=3.5, DIM4=0.25)
mesh = geometry.create_mesh(mesh_sizes=[0.005])
```

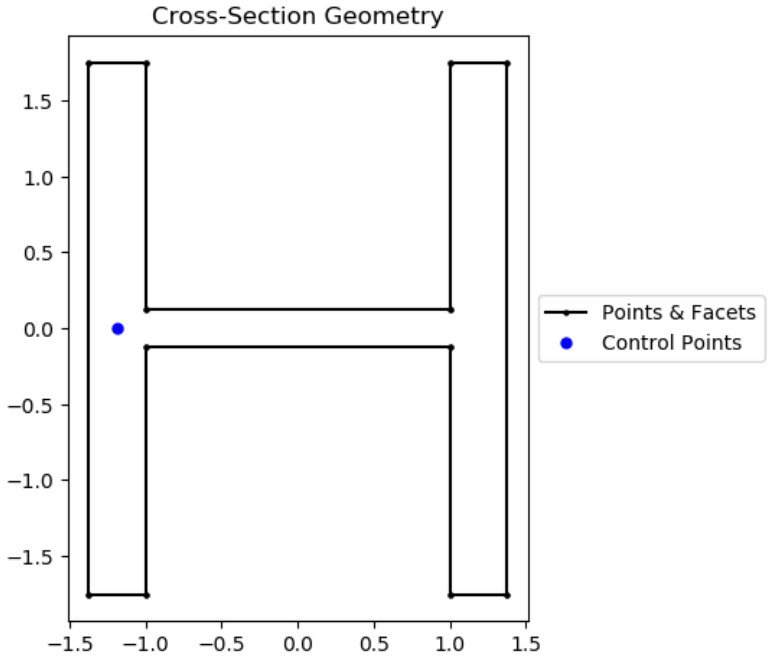

Fig. 72: H section geometry.

nastran_hat

sectionproperties.pre.library.nastran_sections.**nastran_hat**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Hat section with the top most section's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for more details. Note that HAT in ASTROS is actually HAT1 in this code. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Depth (y) of HAT-section

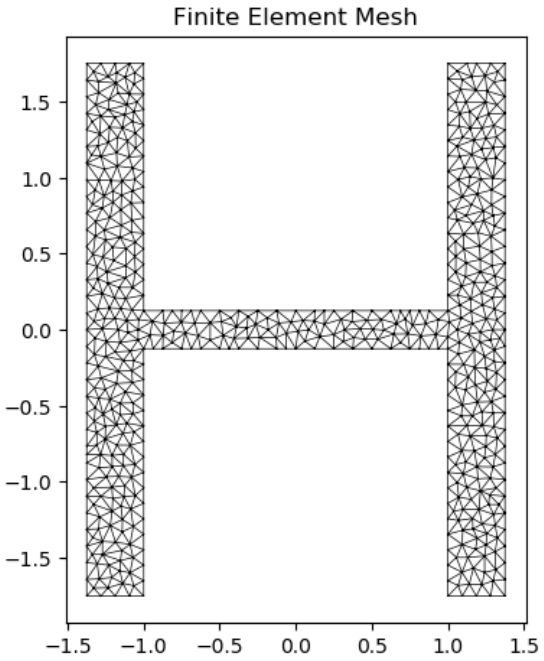

Fig. 73: Mesh generated from the above geometry.

- **DIM2** (*float*) – Thickness of HAT-section
- **DIM3** (*float*) – Width (x) of top most section
- **DIM4** (*float*) – Width (x) of bottom sections
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a HAT cross-section with a depth of 1.25 and width of 2.5, and generates a mesh with a maximum triangular area of 0.001:

```
from sectionproperties.pre.library.nastran_sections import nastran_hat

geom = nastran_hat(DIM1=1.25, DIM2=0.25, DIM3=1.5, DIM4=0.5)
mesh = geometry.create_mesh(mesh_sizes=[0.001])
```

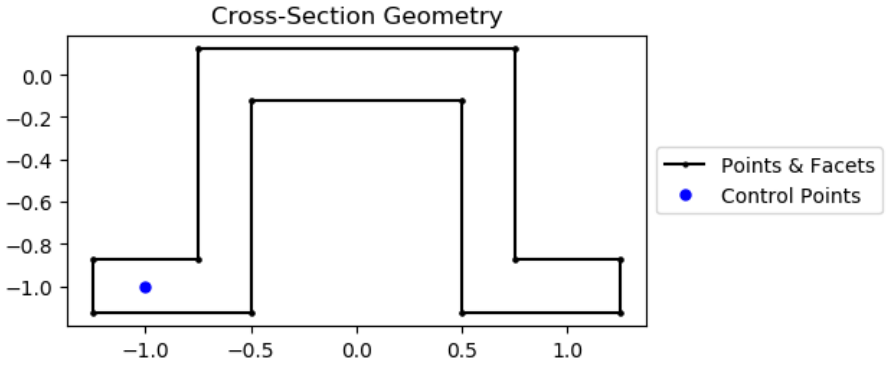

Fig. 74: HAT section geometry.

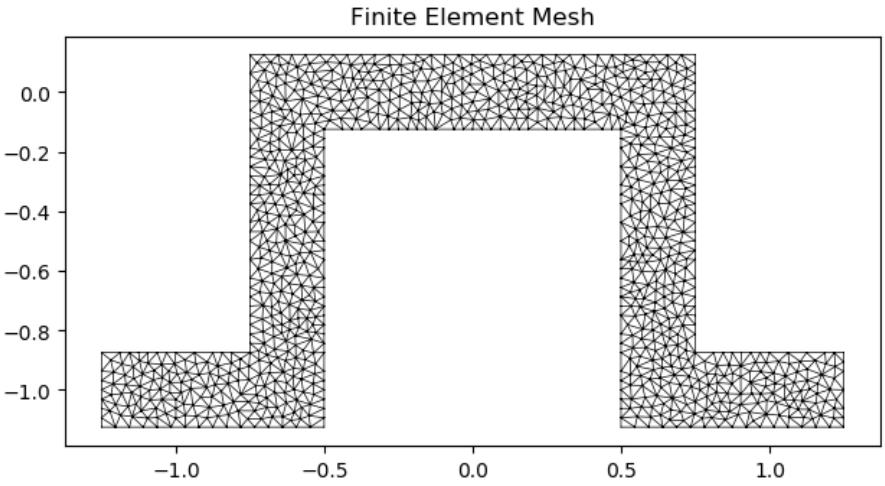

Fig. 75: Mesh generated from the above geometry.

nastran_hat1

sectionproperties.pre.library.nastran_sections.**nastran_hat1**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *DIM5: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a HAT1 section with the bottom plate's bottom center at the origin *(0, 0)*, with five parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 5] for definition of parameters. Note that in ASTROS, HAT1 is called HAT. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width(x) of the HAT1-section
- **DIM2** (*float*) – Depth (y) of the HAT1-section
- **DIM3** (*float*) – Width (x) of hat's top flange
- **DIM4** (*float*) – Thickness of hat stiffener
- **DIM5** (*float*) – Thicknesss of bottom plate
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a HAT1 cross-section with a depth of 2.0 and width of 4.0, and generates a mesh with a maximum triangular area of 0.005:

```
from sectionproperties.pre.library.nastran_sections import nastran_hat1

geom = nastran_hat1(DIM1=4.0, DIM2=2.0, DIM3=1.5, DIM4=0.1875, DIM5=0.375)
mesh = geometry.create_mesh(mesh_sizes=[0.005])
```

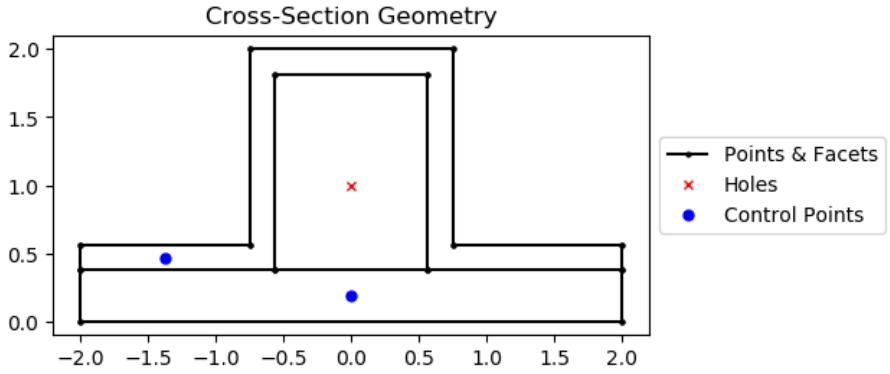

Fig. 76: HAT1 section geometry.

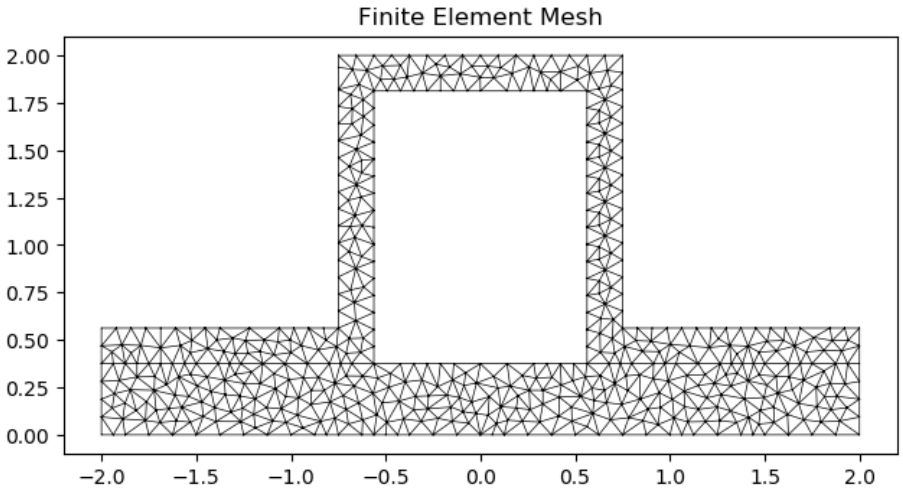

Fig. 77: Mesh generated from the above geometry.

nastran_hexa

sectionproperties.pre.library.nastran_sections.**nastran_hexa**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a HEXA (hexagon) section with the center at the origin *(0, 0)*, with three parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Spacing between bottom right point and right most point
- **DIM2** (*float*) – Width (x) of hexagon
- **DIM3** (*float*) – Depth (y) of hexagon
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a rectangular cross-section with a depth of 1.5 and width of 2.0, and generates a mesh with a maximum triangular area of 0.005:

```
from sectionproperties.pre.library.nastran_sections import nastran_hexa

geom = nastran_hexa(DIM1=0.5, DIM2=2.0, DIM3=1.5)
mesh = geometry.create_mesh(mesh_sizes=[0.005])
```

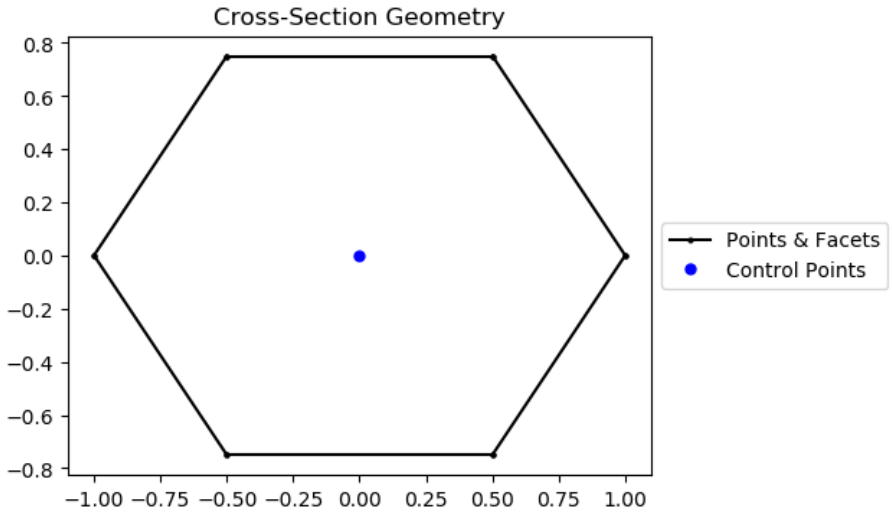

Fig. 78: HEXA section geometry.

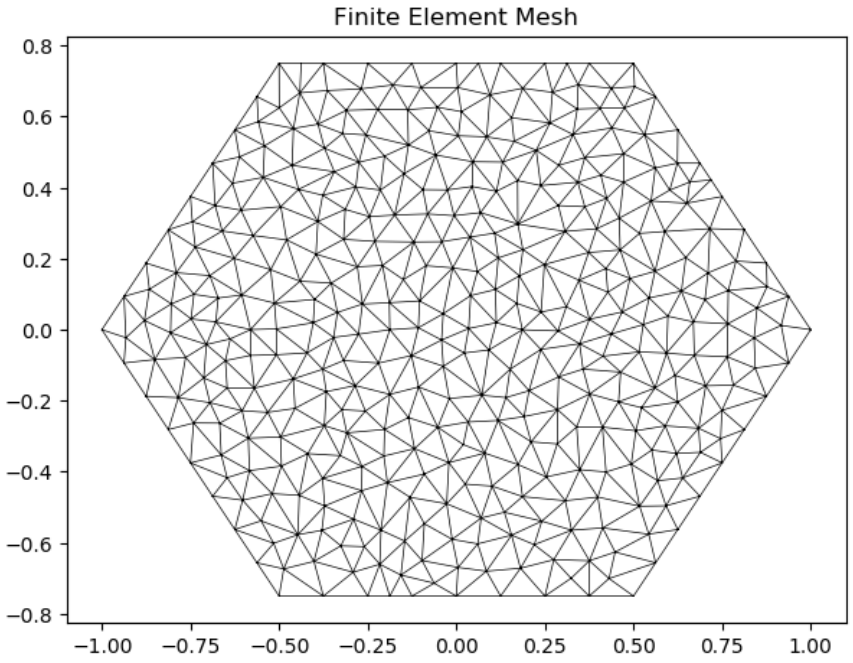

Fig. 79: Mesh generated from the above geometry.

nastran_i

sectionproperties.pre.library.nastran_sections.**nastran_i**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *DIM5: float*, *DIM6: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs Nastran's I section with the bottom flange's middle center at the origin *(0, 0)*, with six parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for definition of parameters. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Depth(y) of the I Section
- **DIM2** (*float*) – Width (x) of bottom flange
- **DIM3** (*float*) – Width (x) of top flange
- **DIM4** (*float*) – Thickness of web
- **DIM5** (*float*) – Thickness of bottom web
- **DIM6** (*float*) – Thickness of top web
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a Nastran I cross-section with a depth of 5.0, and generates a mesh with a maximum triangular area of 0.008:

```
from sectionproperties.pre.library.nastran_sections import nastran_i

geom = nastran_i(
    DIM1=5.0, DIM2=2.0, DIM3=3.0, DIM4=0.25, DIM5=0.375, DIM6=0.5
)
mesh = geometry.create_mesh(mesh_sizes=[0.008])
```

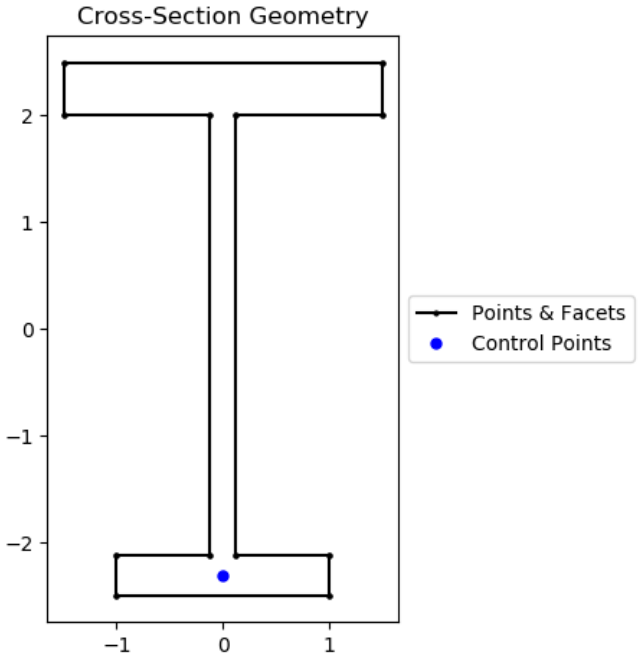

Fig. 80: Nastran's I section geometry.

nastran_i1

sectionproperties.pre.library.nastran_sections.**nastran_i1**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a I1 section with the web's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Twice distance from web end to flange end
- **DIM2** (*float*) – Thickness of web
- **DIM3** (*float*) – Length of web (spacing between flanges)
- **DIM4** (*float*) – Depth (y) of the I1-section
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

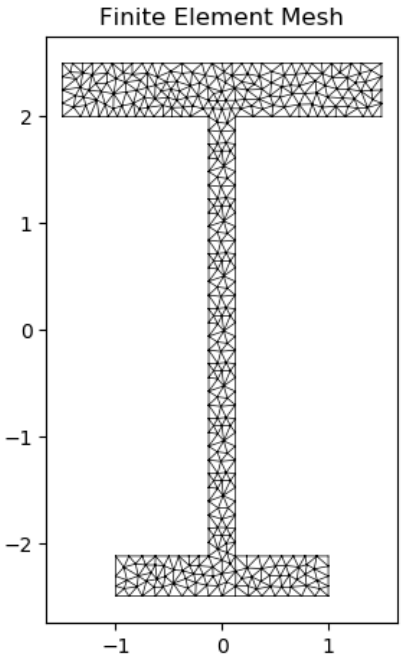

Fig. 81: Mesh generated from the above geometry.

The following example creates a I1 cross-section with a depth of 5.0 and width of 1.75, and generates a mesh with a maximum triangular area of 0.02:

```
from sectionproperties.pre.library.nastran_sections import nastran_i1

geom = nastran_i1(DIM1=1.0, DIM2=0.75, DIM3=4.0, DIM4=5.0)
mesh = geometry.create_mesh(mesh_sizes=[0.02])
```

nastran_l

sectionproperties.pre.library.nastran_sections.**nastran_l**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a L section with the intersection's center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of the L-section
- **DIM2** (*float*) – Depth (y) of the L-section
- **DIM3** (*float*) – Thickness of flange (horizontal portion)
- **DIM4** (*float*) – Thickness of web (vertical portion)
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

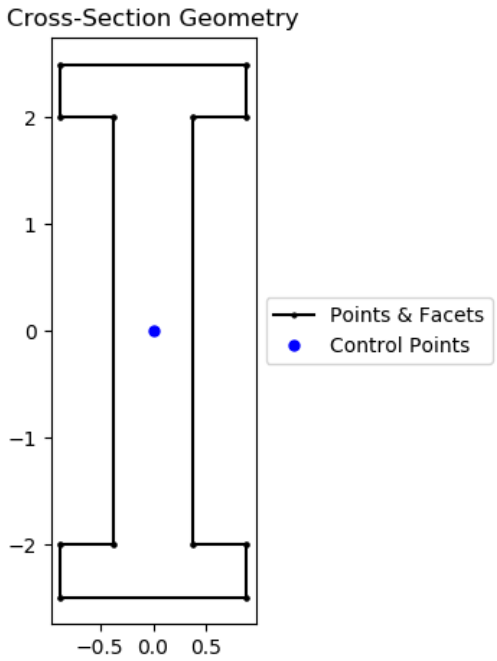

Fig. 82: I1 section geometry.

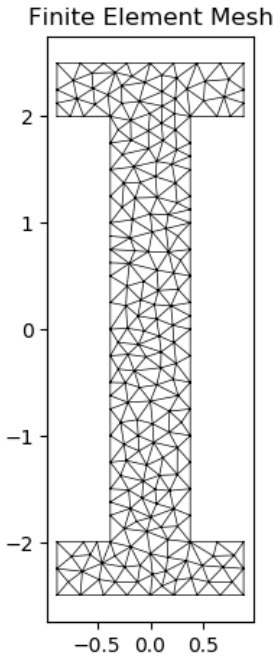

Fig. 83: Mesh generated from the above geometry.

The following example creates a L cross-section with a depth of 6.0 and width of 3.0, and generates a mesh with a maximum triangular area of 0.01:

```
from sectionproperties.pre.library.nastran_sections import nastran_l

geom = nastran_l(DIM1=3.0, DIM2=6.0, DIM3=0.375, DIM4=0.625)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

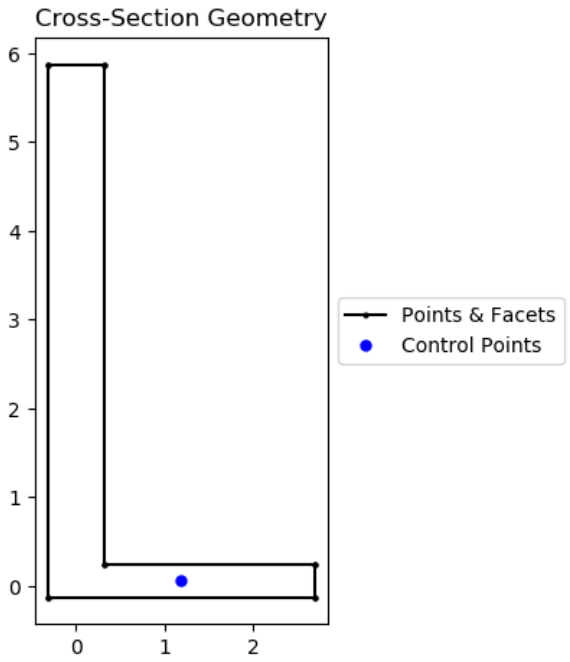

Fig. 84: L section geometry.

nastran_rod

sectionproperties.pre.library.nastran_sections.**nastran_rod**(*DIM1: float*, *n: int*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a circular rod section with the center at the origin *(0, 0)*, with one parameter defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Radius of the circular rod section
- **n** (*int*) – Number of points discretising the circle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a circular rod with a radius of 3.0 and 50 points discretising the boundary, and generates a mesh with a maximum triangular area of 0.01:

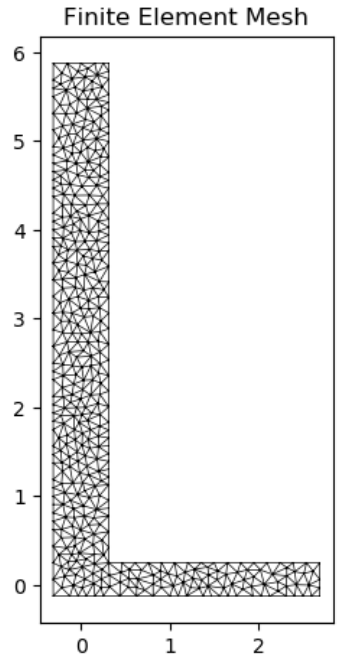

Fig. 85: Mesh generated from the above geometry.

```
from sectionproperties.pre.library.nastran_sections import nastran_rod

geom = nastran_rod(DIM1=3.0, n=50)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

nastran_tee

sectionproperties.pre.library.nastran_sections.**nastran_tee**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a T section with the top flange's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 4][Page 290, 5] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of top flange
- **DIM2** (*float*) – Depth (y) of the T-section
- **DIM3** (*float*) – Thickness of top flange
- **DIM4** (*float*) – Thickness of web
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

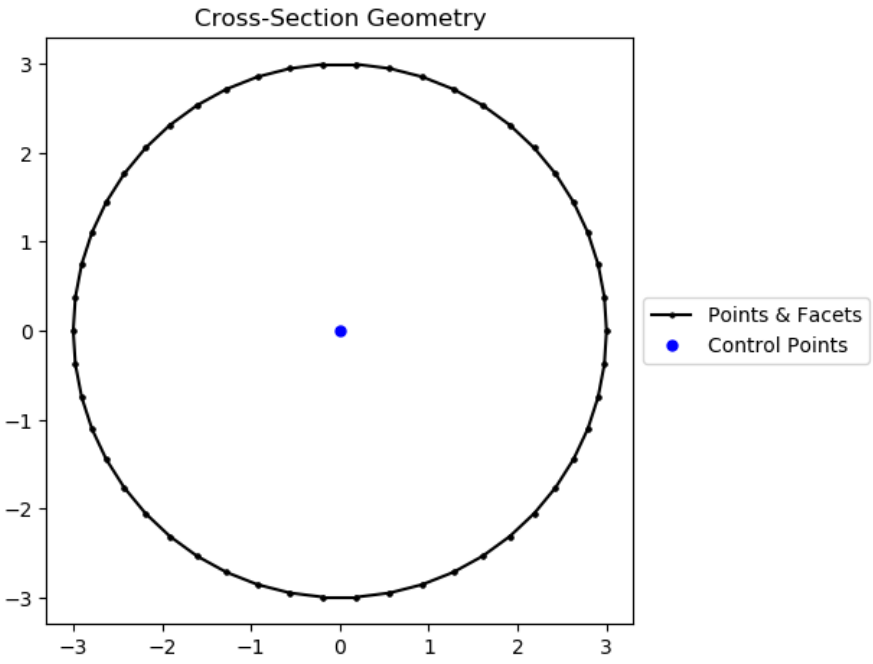

Fig. 86: Rod section geometry.

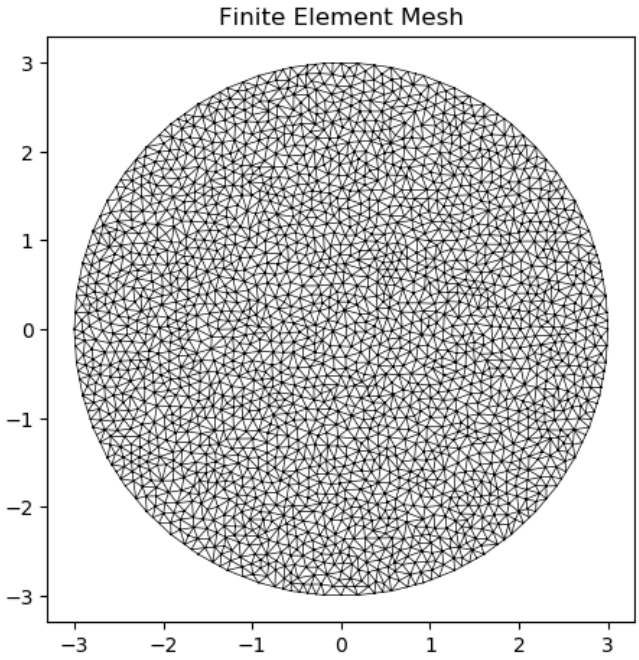

Fig. 87: Mesh generated from the above geometry.

The following example creates a T cross-section with a depth of 4.0 and width of 3.0, and generates a mesh with a maximum triangular area of 0.001:

```
from sectionproperties.pre.library.nastran_sections import nastran_tee

geom = nastran_tee(DIM1=3.0, DIM2=4.0, DIM3=0.375, DIM4=0.25)
mesh = geometry.create_mesh(mesh_sizes=[0.001])
```

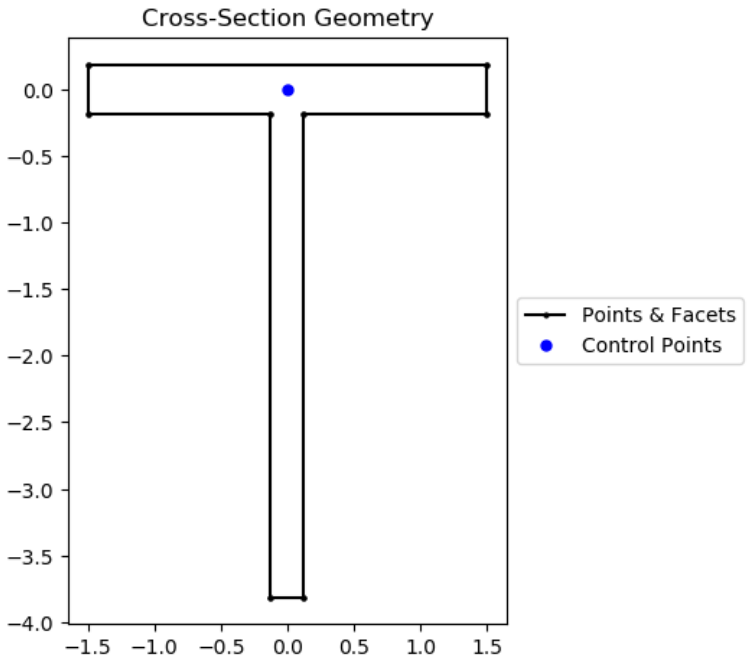

Fig. 88: T section geometry.

nastran_tee1

sectionproperties.pre.library.nastran_sections.**nastran_tee1**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a T1 section with the right flange's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Depth (y) of T1-section
- **DIM2** (*float*) – Length (x) of web
- **DIM3** (*float*) – Thickness of right flange
- **DIM4** (*float*) – Thickness of web
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

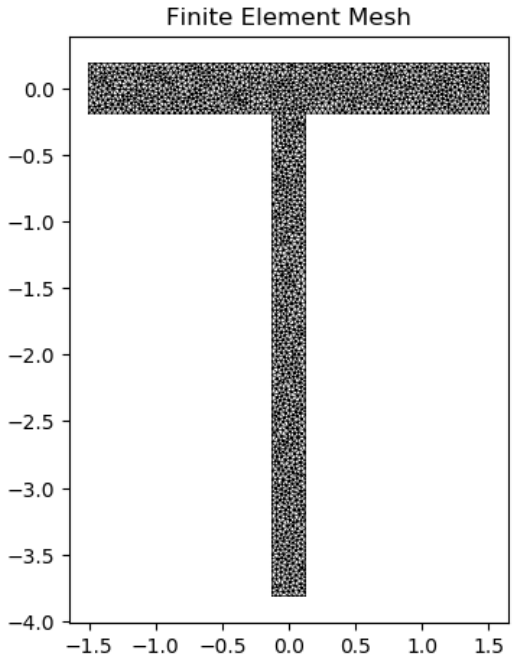

Fig. 89: Mesh generated from the above geometry.

The following example creates a T1 cross-section with a depth of 3.0 and width of 3.875, and generates a mesh with a maximum triangular area of 0.001:

```
from sectionproperties.pre.library.nastran_sections import nastran_tee1

geom = nastran_tee1(DIM1=3.0, DIM2=3.5, DIM3=0.375, DIM4=0.25)
mesh = geometry.create_mesh(mesh_sizes=[0.001])
```

nastran_tee2

sectionproperties.pre.library.nastran_sections.**nastran_tee2**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material *= Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a T2 section with the bottom flange's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of T2-section
- **DIM2** (*float*) – Depth (y) of T2-section
- **DIM3** (*float*) – Thickness of bottom flange
- **DIM4** (*float*) – Thickness of web

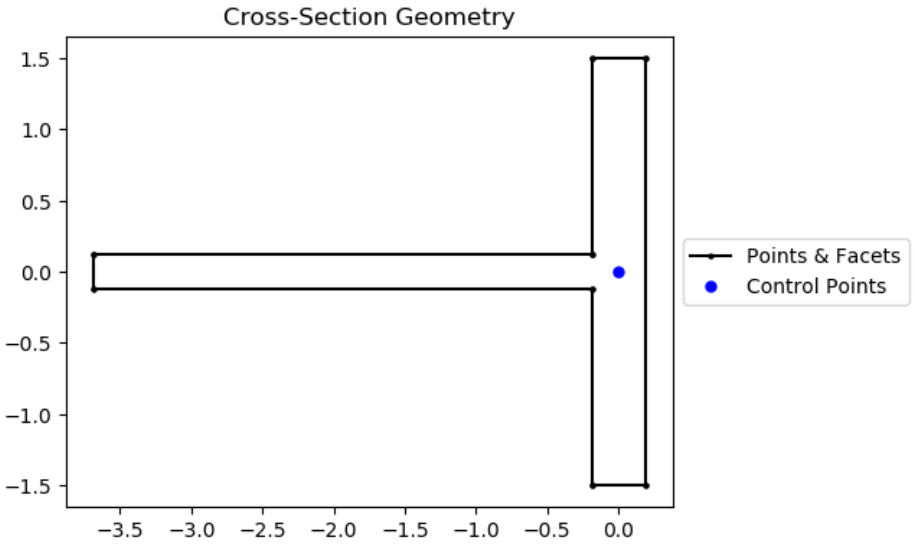

Fig. 90: T1 section geometry.

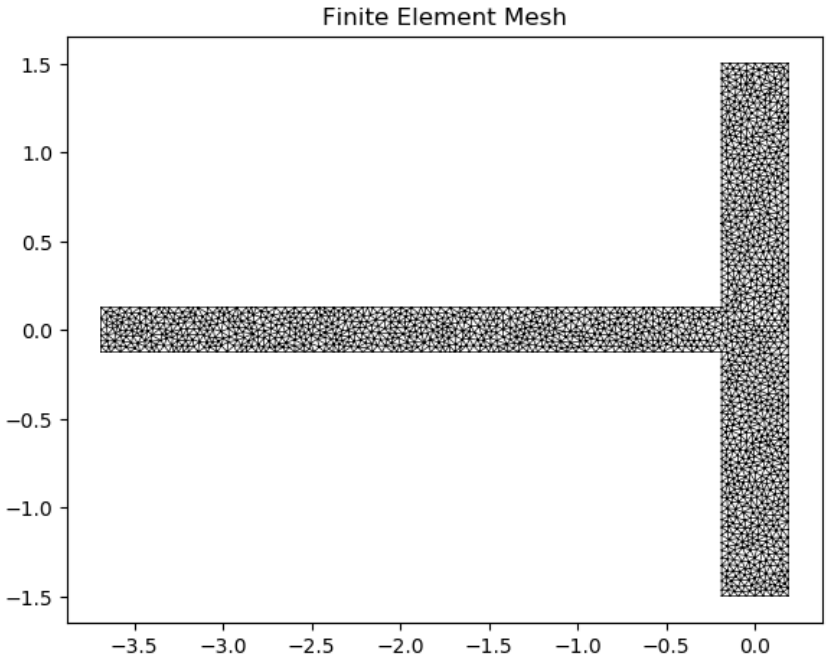

Fig. 91: Mesh generated from the above geometry.

- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

The following example creates a T2 cross-section with a depth of 4.0 and width of 3.0, and generates a mesh with a maximum triangular area of 0.005:

```
from sectionproperties.pre.library.nastran_sections import nastran_tee2

geom = nastran_tee2(DIM1=3.0, DIM2=4.0, DIM3=0.375, DIM4=0.5)
mesh = geometry.create_mesh(mesh_sizes=[0.005])
```

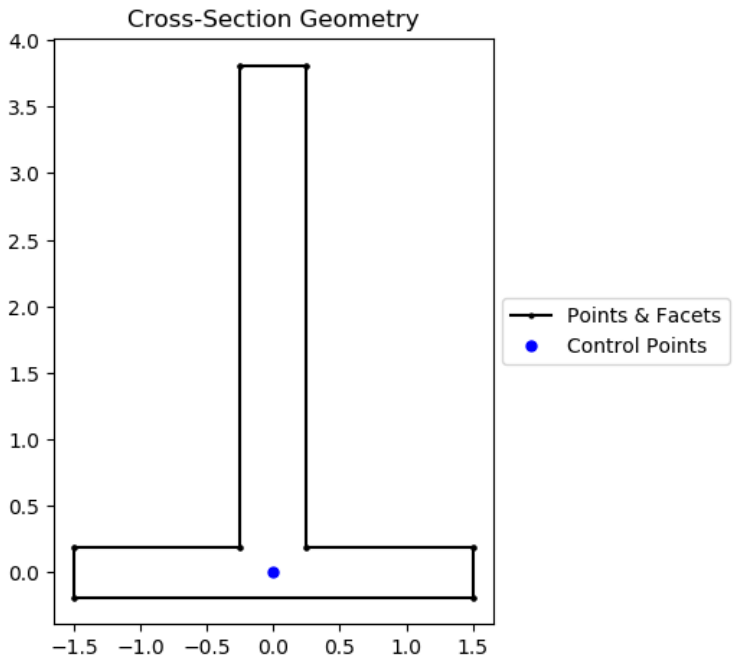

Fig. 92: T2 section geometry.

nastran_tube

sectionproperties.pre.library.nastran_sections.**nastran_tube**(*DIM1: float*, *DIM2: float*, *n: int*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a circular tube section with the center at the origin *(0, 0)*, with two parameters defining dimensions. See Nastran documentation[Page 290, 1][Page 290, 2][Page 290, 3][Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Outer radius of the circular tube section
- **DIM2** (*float*) – Inner radius of the circular tube section
- **n** (*int*) – Number of points discretising the circle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

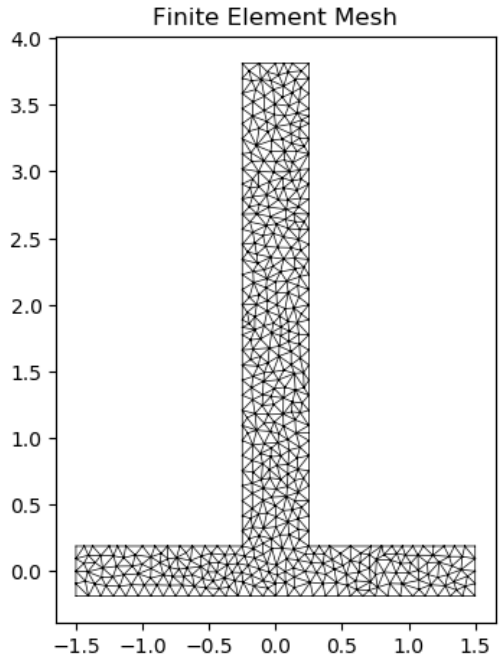

Fig. 93: Mesh generated from the above geometry.

The following example creates a circular tube cross-section with an outer radius of 3.0 and an inner radius of 2.5, and generates a mesh with 37 points discretising the boundaries and a maximum triangular area of 0.01:

```
from sectionproperties.pre.library.nastran_sections import nastran_tube

geom = nastran_tube(DIM1=3.0, DIM2=2.5, n=37)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

nastran_tube2

sectionproperties.pre.library.nastran_sections.**nastran_tube2**(*DIM1: float*, *DIM2: float*, *n: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a circular TUBE2 section with the center at the origin *(0, 0)*, with two parameters defining dimensions. See MSC Nastran documentation[Page 290, 1] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Outer radius of the circular tube section
- **DIM2** (*float*) – Thickness of wall
- **n** (*int*) – Number of points discretising the circle
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

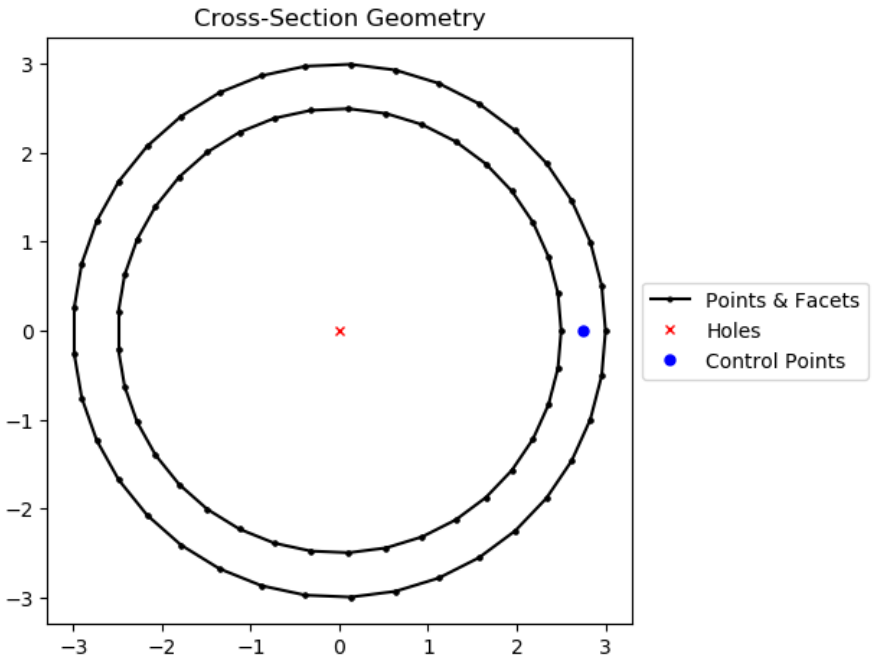

Fig. 94: TUBE section geometry.

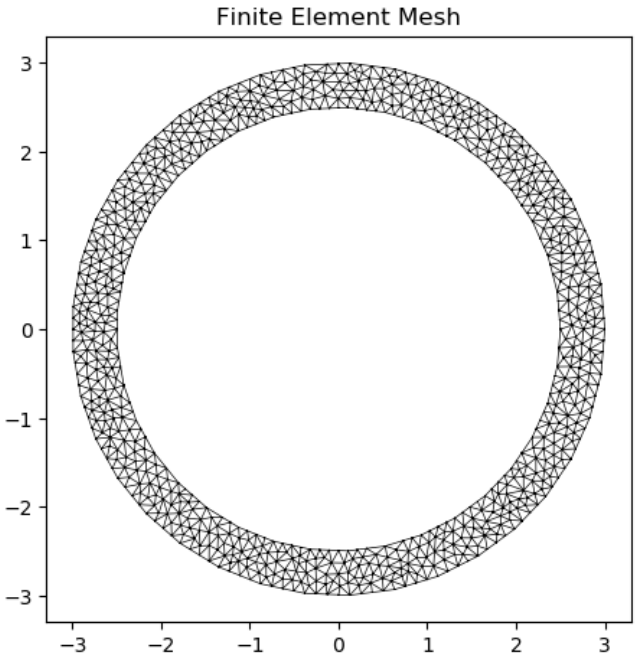

Fig. 95: Mesh generated from the above geometry.

The following example creates a circular TUBE2 cross-section with an outer radius of 3.0 and a wall thickness of 0.5, and generates a mesh with 37 point discretising the boundary and a maximum triangular area of 0.01:

```
from sectionproperties.pre.library.nastran_sections import nastran_tube2

geom = nastran_tube2(DIM1=3.0, DIM2=0.5, n=37)
mesh = geometry.create_mesh(mesh_sizes=[0.01])
```

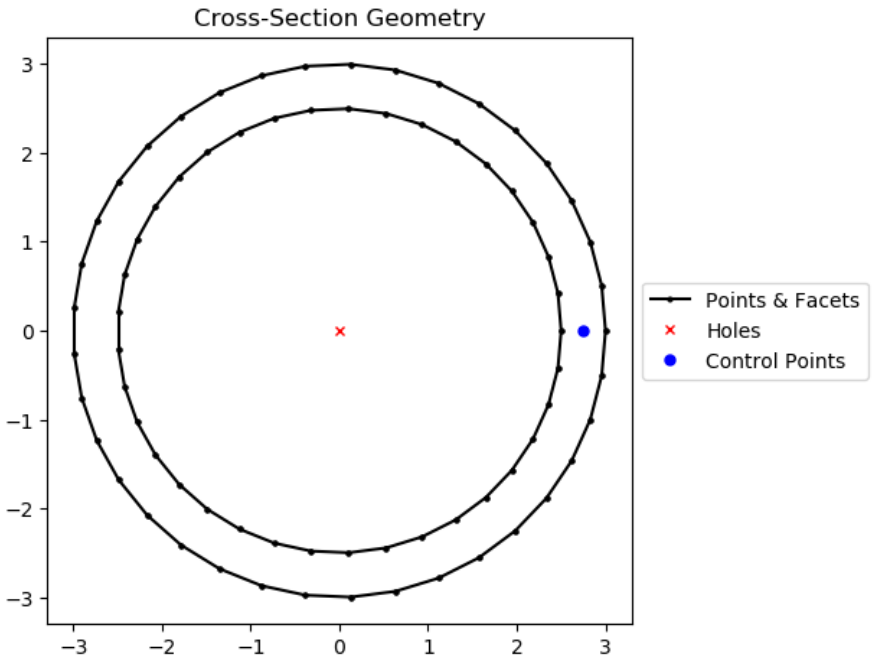

Fig. 96: TUBE2 section geometry.

nastran_zed

sectionproperties.pre.library.nastran_sections.**nastran_zed**(*DIM1: float*, *DIM2: float*, *DIM3: float*, *DIM4: float*, *material:* Material = *Material(name='default', elastic_modulus=1, poissons_ratio=0, yield_strength=1, density=1, color='w')*) → *Geometry*

Constructs a Z section with the web's middle center at the origin *(0, 0)*, with four parameters defining dimensions. See Nastran documentation[Page 290, 1Page 290, 2Page 290, 3Page 290, 4] for more details. Added by JohnDN90.

Parameters

- **DIM1** (*float*) – Width (x) of horizontal members
- **DIM2** (*float*) – Thickness of web
- **DIM3** (*float*) – Spacing between horizontal members (length of web)
- **DIM4** (*float*) – Depth (y) of Z-section
- **Optional[sectionproperties.pre.pre.Material]** – Material to associate with this geometry

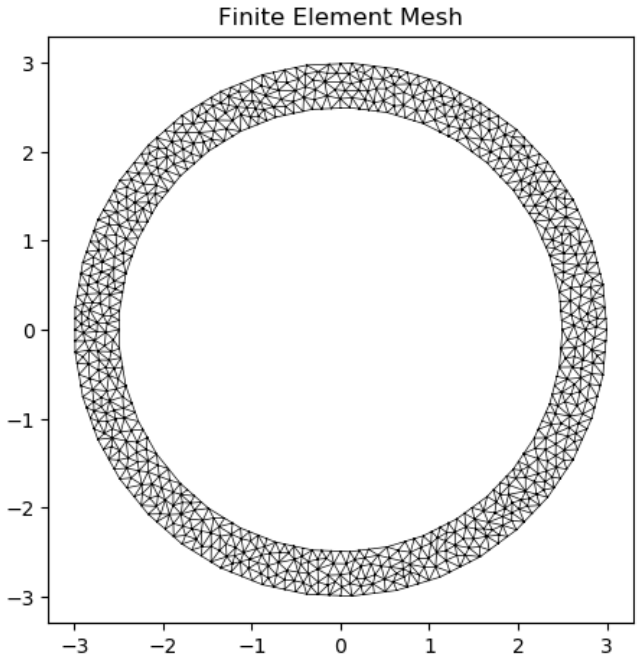

Fig. 97: Mesh generated from the above geometry.

The following example creates a rectangular cross-section with a depth of 4.0 and width of 2.75, and generates a mesh with a maximum triangular area of 0.005:

```
from sectionproperties.pre.library.nastran_sections import nastran_zed

geom = nastran_zed(DIM1=1.125, DIM2=0.5, DIM3=3.5, DIM4=4.0)
mesh = geometry.create_mesh(mesh_sizes=[0.005])
```

Nastran References

9.2 Analysis Package

9.2.1 *section* Module

Section Class

class sectionproperties.analysis.section.**Section**(*geometry: Union[Geometry, CompoundGeometry]*, *time_info: bool = False*)

Bases: `object`

Class for structural cross-sections.

Stores the finite element geometry, mesh and material information and provides methods to compute the cross-section properties. The element type used in this program is the six-noded quadratic triangular element.

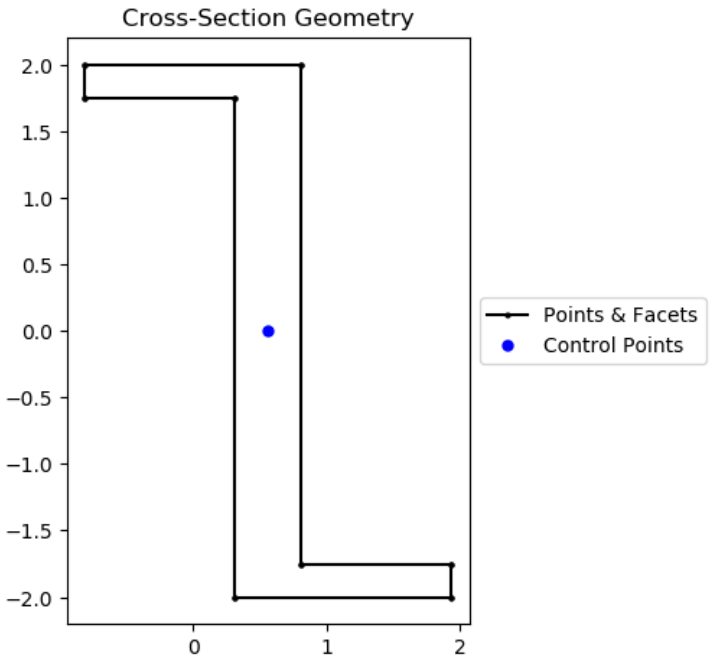

Fig. 98: Z section geometry.

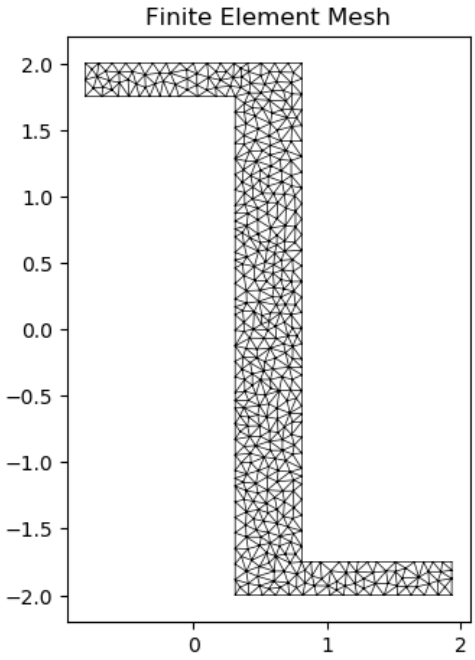

Fig. 99: Mesh generated from the above geometry.

The constructor extracts information from the provided mesh object and creates and stores the corresponding Tri6 finite element objects.

Parameters

- **geometry** (*Geometry*) – Cross-section geometry object used to generate the mesh
- **time_info** (*bool*) – If set to True, a detailed description of the computation and the time cost is printed to the terminal for every computation performed.

The following example creates a *Section* object of a 100D x 50W rectangle using a mesh size of 5:

```
import sectionproperties.pre.library.primitive_sections as primitive_
↪sections
from sectionproperties.analysis.section import Section

geometry = primitive_sections.rectangular_section(d=100, b=50)
geometry.create_mesh(mesh_sizes=[5])
section = Section(geometry)
```

Variables

- **elements** (list[*Tri6*]) – List of finite element objects describing the cross-section mesh
- **num_nodes** (*int*) – Number of nodes in the finite element mesh
- **geometry** (*Geometry*) – Cross-section geometry object used to generate the mesh
- **mesh** (*dict(mesh)*) – Mesh dict returned by triangle
- **mesh_nodes** (numpy.ndarray) – Array of node coordinates from the mesh
- **mesh_elements** (numpy.ndarray) – Array of connectivities from the mesh
- **mesh_attributes** (numpy.ndarray) – Array of attributes from the mesh
- **materials** – List of materials
- **material_groups** – List of objects containing the elements in each defined material
- **section_props** (*SectionProperties*) – Class to store calculated section properties

Raises

- **AssertionError** – If the number of materials does not equal the number of regions
- **ValueError** – If geometry does not contain a mesh

assemble_torsion(*progress=None*, *task=None*)

Assembles stiffness matrices to be used for the computation of warping properties and the torsion load vector (f_torsion). A Lagrangian multiplier (k_lg) stiffness matrix is returned. The stiffness matrix are assembled using the sparse COO format and returned in the sparse CSC format.

Returns

Lagrangian multiplier stiffness matrix and torsion load vector *(k_lg, f_torsion)*

Return type

tuple(scipy.sparse.csc_matrix, numpy.ndarray)

calculate_frame_properties(*solver_type='direct'*)

Calculates and returns the properties required for a frame analysis. The properties are also stored in the *SectionProperties* object contained in the section_props class variable.

Parameters
solver_type (*string*) – Solver used for solving systems of linear equations, either using the *'direct'* method or *'cgs'* iterative method
Returns
Cross-section properties to be used for a frame analysis *(area, ixx, iyy, ixy, j, phi)*
Return type
tuple(float, float, float, float, float, float)

The following section properties are calculated:

- Cross-sectional area *(area)*
- Second moments of area about the centroidal axis *(ixx, iyy, ixy)*
- Torsion constant *(j)*
- Principal axis angle *(phi)*

If materials are specified for the cross-section, the area, second moments of area and torsion constant are elastic modulus weighted.

The following example demonstrates the use of this method:

```
section = Section(geometry)
(area, ixx, iyy, ixy, j, phi) = section.calculate_frame_properties()
```

calculate_geometric_properties()

Calculates the geometric properties of the cross-section and stores them in the *SectionProperties* object contained in the section_props class variable.

The following geometric section properties are calculated:

- Cross-sectional area
- Cross-sectional perimeter
- Cross-sectional mass
- Area weighted material properties, composite only E_{eff}, G_{eff}, nu_{eff}
- Modulus weighted area (axial rigidity)
- First moments of area
- Second moments of area about the global axis
- Second moments of area about the centroidal axis
- Elastic centroid
- Centroidal section moduli
- Radii of gyration
- Principal axis properties

If materials are specified for the cross-section, the moments of area and section moduli are elastic modulus weighted.

The following example demonstrates the use of this method:

```
section = Section(geometry)
section.calculate_geometric_properties()
```

calculate_plastic_properties(*verbose=False*)

Calculates the plastic properties of the cross-section and stores them in the *SectionProperties* object contained in the section_props class variable.
Parameters
verbose (*bool*) – If set to True, the number of iterations required for each plastic axis is printed to the terminal.

The following warping section properties are calculated:

- Plastic centroid for bending about the centroidal and principal axes
- Plastic section moduli for bending about the centroidal and principal axes
- Shape factors for bending about the centroidal and principal axes

If materials are specified for the cross-section, the plastic section moduli are displayed as plastic moments (i.e $M_p = f_y S$) and the shape factors are not calculated.

Note that the geometric properties must be calculated before the plastic properties are calculated:

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
```

Raises

RuntimeError – If the geometric properties have not been calculated prior to calling this method

calculate_stress(*N=0*, *Vx=0*, *Vy=0*, *Mxx=0*, *Myy=0*, *M11=0*, *M22=0*, *Mzz=0*)

Calculates the cross-section stress resulting from design actions and returns a *StressPost* object allowing post-processing of the stress results.

Parameters

- **N** (*float*) – Axial force
- **Vx** (*float*) – Shear force acting in the x-direction
- **Vy** (*float*) – Shear force acting in the y-direction
- **Mxx** (*float*) – Bending moment about the centroidal xx-axis
- **Myy** (*float*) – Bending moment about the centroidal yy-axis
- **M11** (*float*) – Bending moment about the centroidal 11-axis
- **M22** (*float*) – Bending moment about the centroidal 22-axis
- **Mzz** (*float*) – Torsion moment about the centroidal zz-axis

Returns

Object for post-processing cross-section stresses

Return type

StressPost

Note that a geometric analysis must be performed prior to performing a stress analysis. Further, if the shear force or torsion is non-zero a warping analysis must also be performed:

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(N=1e3, Vy=3e3, Mxx=1e6)
```

Raises

RuntimeError – If a geometric and warping analysis (if required) have not been performed prior to calling this method

calculate_warping_properties(*solver_type='direct'*)

Calculates all the warping properties of the cross-section and stores them in the *SectionProperties* object contained in the `section_props` class variable.

Parameters

solver_type (*string*) – Solver used for solving systems of linear equations, either using the *'direct'* method or *'cgs'* iterative method

The following warping section properties are calculated:

- Torsion constant
- Shear centre
- Shear area
- Warping constant
- Monosymmetry constant

If materials are specified, the values calculated for the torsion constant, warping constant and shear area are elastic modulus weighted.

Note that the geometric properties must be calculated prior to the calculation of the warping properties:

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
```

Raises

RuntimeError – If the geometric properties have not been calculated prior to calling this method

display_mesh_info()

Prints mesh statistics (number of nodes, elements and regions) to the command window.

The following example displays the mesh statistics for a Tee section merged from two rectangles:

```
import sectionproperties.pre.library.primitive_sections as primitive_
↪sections
from sectionproperties.analysis.section import Section

rec1 = primitive_sections.rectangular_section(d=100, b=25)
rec2 = primitive_sections.rectangular_section(d=25, b=100)
rec1 = rec1.shift_section(x_offset=-12.5)
rec2 = rec2.shift_section(x_offset=-50, y_offset=100)

geometry = rec1 + rec2
geometry.create_mesh(mesh_sizes=[5, 2.5])
section = Section(geometry)
section.display_mesh_info()

>>>Mesh Statistics:
>>>--4920 nodes
>>>--2365 elements
>>>--2 regions
```

display_results(*fmt='8.6e'*)

Prints the results that have been calculated to the terminal.

Parameters

fmt (*string*) – Number formatting string

The following example displays the geometric section properties for a 100D x 50W rectangle with three digits after the decimal point:

```
import sectionproperties.pre.library.primitive_sections as primitive_
↪sections
from sectionproperties.analysis.section import Section

geometry = primitive_sections.rectangular_section(d=100, b=50)
geometry.create_mesh(mesh_sizes=[5])

section = Section(geometry)
section.calculate_geometric_properties()

section.display_results(fmt='.3f')
```

get_As()

Returns
Shear area for loading about the centroidal axis *(A_sx, A_sy)*
Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(A_sx, A_sy) = section.get_As()
```

get_As_p()

Returns
Shear area for loading about the principal bending axis *(A_s11, A_s22)*
Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(A_s11, A_s22) = section.get_As_p()
```

get_area()

Returns
Cross-section area
Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
area = section.get_area()
```

get_beta()

Returns
Monosymmetry constant for bending about both global axes *(beta_x_plus, beta_x_minus, beta_y_plus, beta_y_minus)*. The *plus* value relates to the top flange in compression and the *minus* value relates to the bottom flange in compression.
Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(beta_x_plus, beta_x_minus, beta_y_plus, beta_y_minus) = section.get_
↪beta()
```

get_beta_p()

Returns
Monosymmetry constant for bending about both principal axes *(beta_11_plus, beta_11_minus, beta_22_plus, beta_22_minus)*. The *plus* value relates to the top flange in compression and the *minus* value relates to the bottom flange in compression.
Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(beta_11_plus, beta_11_minus, beta_22_plus, beta_22_minus) = section.
↪get_beta_p()
```

get_c()

Returns
Elastic centroid *(cx, cy)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(cx, cy) = section.get_c()
```

get_e_eff()

Returns
Effective elastic modulus based on area

Return type
float

```
section = Section(geometry)
section.calculate_warping_properties()
e_eff = section.get_e_eff()
```

get_ea()

Returns
Modulus weighted area (axial rigidity)

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
ea = section.get_ea()
```

get_g_eff()

Returns
Effective shear modulus based on area

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
g_eff = section.get_g_eff()
```

get_gamma()

Returns
Warping constant

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
gamma = section.get_gamma()
```

get_ic()

Returns
Second moments of area centroidal axis *(ixx_c, iyy_c, ixy_c)*

Return type
tuple(float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(ixx_c, iyy_c, ixy_c) = section.get_ic()
```

get_ig()

Returns
Second moments of area about the global axis *(ixx_g, iyy_g, ixy_g)*

Return type
tuple(float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(ixx_g, iyy_g, ixy_g) = section.get_ig()
```

get_ip()

Returns
Second moments of area about the principal axis *(i11_c, i22_c)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(i11_c, i22_c) = section.get_ip()
```

get_j()

Returns
St. Venant torsion constant

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
j = section.get_j()
```

get_mass()

Returns
Cross-section mass

Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
perimeter = section.get_mass()
```

get_nu_eff()

Returns
Effective Poisson's ratio
Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
nu_eff = section.get_nu_eff()
```

get_pc()

Returns
Centroidal axis plastic centroid *(x_pc, y_pc)*
Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(x_pc, y_pc) = section.get_pc()
```

get_pc_p()

Returns
Principal bending axis plastic centroid *(x11_pc, y22_pc)*
Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(x11_pc, y22_pc) = section.get_pc_p()
```

get_perimeter()

Returns
Cross-section perimeter
Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
perimeter = section.get_perimeter()
```

get_phi()

Returns
Principal bending axis angle
Return type
float

```
section = Section(geometry)
section.calculate_geometric_properties()
phi = section.get_phi()
```

get_q()

Returns
First moments of area about the global axis *(qx, qy)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(qx, qy) = section.get_q()
```

get_rc()

Returns
Radii of gyration about the centroidal axis *(rx, ry)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(rx, ry) = section.get_rc()
```

get_rp()

Returns
Radii of gyration about the principal axis *(r11, r22)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(r11, r22) = section.get_rp()
```

get_s()

Returns
Plastic section moduli about the centroidal axis *(sxx, syy)*

Return type
tuple(float, float)

If material properties have been specified, returns the plastic moment $M_p = f_y S$.

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(sxx, syy) = section.get_s()
```

get_sc()

Returns
Centroidal axis shear centre (elasticity approach) *(x_se, y_se)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(x_se, y_se) = section.get_sc()
```

get_sc_p()

Returns
Principal axis shear centre (elasticity approach) *(x11_se, y22_se)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(x11_se, y22_se) = section.get_sc_p()
```

get_sc_t()

Returns
Centroidal axis shear centre (Trefftz's approach) *(x_st, y_st)*

Return type
tuple(float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
(x_st, y_st) = section.get_sc_t()
```

get_sf()

Returns
Centroidal axis shape factors with respect to the top and bottom fibres *(sf_xx_plus, sf_xx_minus, sf_yy_plus, sf_yy_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(sf_xx_plus, sf_xx_minus, sf_yy_plus, sf_yy_minus) = section.get_sf()
```

get_sf_p()

Returns
Principal bending axis shape factors with respect to the top and bottom fibres *(sf_11_plus, sf_11_minus, sf_22_plus, sf_22_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(sf_11_plus, sf_11_minus, sf_22_plus, sf_22_minus) = section.get_sf_p()
```

get_sp()

Returns
Plastic section moduli about the principal bending axis *(s11, s22)*
Return type
tuple(float, float)

If material properties have been specified, returns the plastic moment $M_p = f_y S$.

```
section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_plastic_properties()
(s11, s22) = section.get_sp()
```

get_stress_at_point(*pt: ~typing.List[float], N=0, Mxx=0, Myy=0, M11=0, M22=0, Mzz=0, Vx=0, Vy=0, agg_func=<function average>*) → Tuple[float]

Calculates the stress at a point within an element for given design actions and returns *(sigma_zz, tau_xz, tau_yz)*

Parameters
- **pt** (*list[float, float]*) – The point. A list of the x and y coordinate
- **N** (*float*) – Axial force
- **Vx** (*float*) – Shear force acting in the x-direction
- **Vy** (*float*) – Shear force acting in the y-direction
- **Mxx** (*float*) – Bending moment about the centroidal xx-axis
- **Myy** (*float*) – Bending moment about the centroidal yy-axis
- **M11** (*float*) – Bending moment about the centroidal 11-axis
- **M22** (*float*) – Bending moment about the centroidal 22-axis
- **Mzz** (*float*) – Torsion moment about the centroidal zz-axis
- **agg_function** (*function, optional*) – A function that aggregates the stresses if the point is shared by several elements. If the point, pt, is shared by several elements (e.g. if it is a node or on an edge), the stress (sigma_zz, tau_xz, tau_yz) are retrieved from each element and combined according to this function. By default, *numpy.average* is used.

Returns
Resultant normal and shear stresses list[(sigma_zz, tau_xz, tau_yz)]. If a point it not in the section then None is returned.
Return type
Union[Tuple[float, float, float], None]

get_stress_at_points(*pts: ~typing.List[~typing.List[float]], N=0, Mxx=0, Myy=0, M11=0, M22=0, Mzz=0, Vx=0, Vy=0, agg_func=<function average>*) → List[Tuple]

Calculates the stress at a set of points within an element for given design actions and returns *(sigma_zz, tau_xz, tau_yz)*

Parameters
- **pts** (*list[list[float, float]]*) – The points. A list of several x and y coordinates
- **N** (*float*) – Axial force
- **Vx** (*float*) – Shear force acting in the x-direction
- **Vy** (*float*) – Shear force acting in the y-direction
- **Mxx** (*float*) – Bending moment about the centroidal xx-axis
- **Myy** (*float*) – Bending moment about the centroidal yy-axis
- **M11** (*float*) – Bending moment about the centroidal 11-axis
- **M22** (*float*) – Bending moment about the centroidal 22-axis
- **Mzz** (*float*) – Torsion moment about the centroidal zz-axis
- **agg_function** (*function, optional*) – A function that aggregates the stresses if the point is shared by several elements. If the point, pt, is shared by several ele-

ments (e.g. if it is a node or on an edge), the stress (sigma_zz, tau_xz, tau_yz) are retrieved from each element and combined according to this function. By default, *numpy.average* is used.

Returns
Resultant normal and shear stresses list[(sigma_zz, tau_xz, tau_yz)]. If a point it not in the section then None is returned for that element in the list.

Return type
List[Union[Tuple[float, float, float], None]]

`get_z()`

Returns
Elastic section moduli about the centroidal axis with respect to the top and bottom fibres *(zxx_plus, zxx_minus, zyy_plus, zyy_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(zxx_plus, zxx_minus, zyy_plus, zyy_minus) = section.get_z()
```

`get_zp()`

Returns
Elastic section moduli about the principal axis with respect to the top and bottom fibres *(z11_plus, z11_minus, z22_plus, z22_minus)*

Return type
tuple(float, float, float, float)

```
section = Section(geometry)
section.calculate_geometric_properties()
(z11_plus, z11_minus, z22_plus, z22_minus) = section.get_zp()
```

`plot_centroids`(*title='Centroids'*, ***kwargs*)

Plots the elastic centroid, the shear centre, the plastic centroids and the principal axis, if they have been calculated, on top of the finite element mesh.

Parameters
- **title** (*string*) – Plot title
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example analyses a 200 PFC section and displays a plot of the centroids:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.channel_section(d=200, b=75, t_f=12, t_w=6,␣
↪r=12, n_r=8)
geometry.create_mesh(mesh_sizes=[20])

section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties()
```

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```
section.plot_centroids()
```

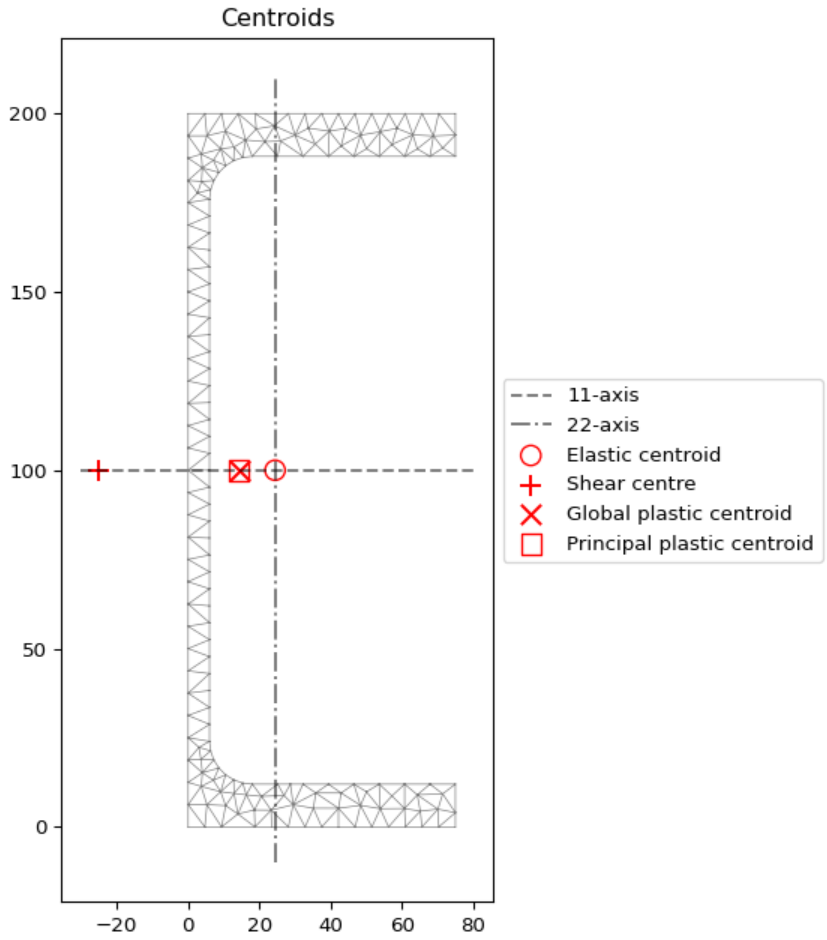

Fig. 100: Plot of the centroids generated by the above example.

The following example analyses a 150x90x12 UA section and displays a plot of the centroids:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_
↪t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])

section = Section(geometry)
section.calculate_geometric_properties()
section.calculate_warping_properties()
section.calculate_plastic_properties()

section.plot_centroids()
```

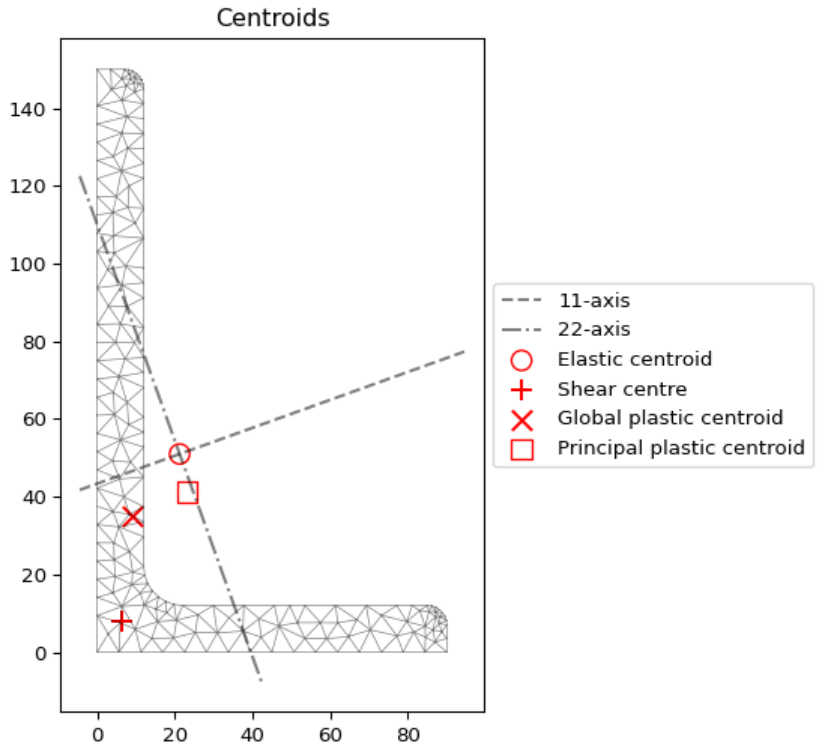

Fig. 101: Plot of the centroids generated by the above example.

plot_mesh(*alpha=0.5*, *materials=True*, *mask=None*, *title='Finite Element Mesh'*, ***kwargs*)

Plots the finite element mesh.

Parameters

- **alpha** (*float*) – Transparency of the mesh outlines: $0 \leq \alpha \leq 1$
- **materials** (*bool*) – If set to true shades the elements with the specified material colours
- **mask** (*list[bool]*) – Mask array, of length num_nodes, to mask out triangles
- **title** (*string*) – Plot title
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the mesh generated for the second example listed under the *Section* object definition:

```
import sectionproperties.pre.library.primitive_sections as primitive_
↪sections
from sectionproperties.pre.pre import Material
from sectionproperties.analysis.section import Section

steel = Material(
    name='Steel', elastic_modulus=200e3, poissons_ratio=0.3, density=7.
↪85e-6,
    yield_strength=250, color='grey'
```

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```
)
timber = Material(
    name='Timber', elastic_modulus=8e3, poissons_ratio=0.35, density=6.
↪5e-7,
    yield_strength=20, color='burlywood'
)

geom_steel = primitive_sections.rectangular_section(d=50, b=50,␣
↪material=steel)
geom_timber = primitive_sections.rectangular_section(d=50, b=50,␣
↪material=timber)
geometry = geom_timber.align_to(geom_steel, on="right") + geom_steel

geometry.create_mesh(mesh_sizes=[10, 5])

section = Section(geometry)
section.plot_mesh(materials=True, alpha=0.5)
```

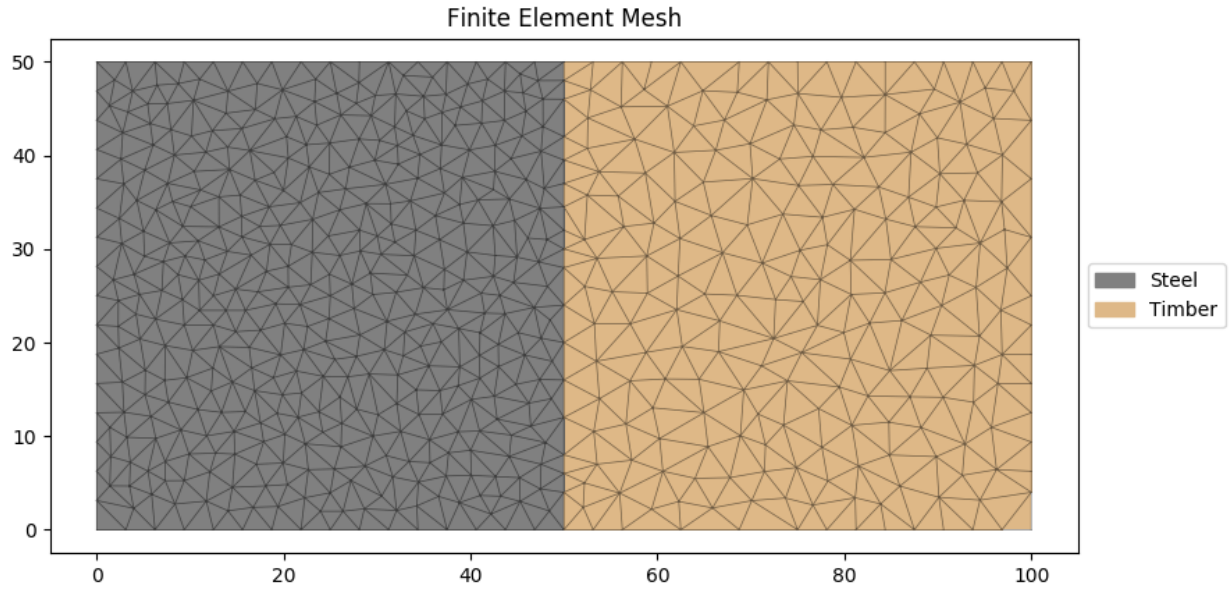

Fig. 102: Finite element mesh generated by the above example.

PlasticSection Class

class sectionproperties.analysis.section.**PlasticSection**(*geom: Union[*Geometry, CompoundGeometry*]*)

Bases: `object`

Class for the plastic analysis of cross-sections.

Stores the finite element geometry and material information and provides methods to compute the plastic section properties.

Parameters

section (*Section*) – Section object

Variables

- **geometry** (*Geometry*) – Deep copy of the Section geometry object provided to the constructor
- **materials** (list[*Material*]) – A list of material properties corresponding to various regions in the geometry and mesh.
- **mesh** (*dict(mesh)*) – Mesh dict returned by triangle
- **mesh_nodes** (numpy.ndarray) – Array of node coordinates from the mesh
- **mesh_elements** (numpy.ndarray) – Array of connectivities from the mesh
- **elements** (list[*Tri6*]) – List of finite element objects describing the cross-section mesh
- **f_top** – Current force in the top region
- **c_top** – Centroid of the force in the top region *(c_top_x, c_top_y)*
- **c_bot** – Centroid of the force in the bottom region *(c_bot_x, c_bot_y)*

calculate_centroid(*elements*)

Calculates the elastic centroid from a list of finite elements.

Parameters
elements (list[*Tri6*]) – A list of Tri6 finite elements.

Returns
A tuple containing the x and y location of the elastic centroid.

Return type
tuple(float, float)

calculate_extreme_fibres(*angle*)

Calculates the locations of the extreme fibres along and perpendicular to the axis specified by 'angle' using the elements stored in *self.elements*.

Parameters
angle (*float*) – Angle (in degrees) along which to calculate the extreme fibre locations

Returns
The location of the extreme fibres parallel (u) and perpendicular (v) to the axis *(u_min, u_max, v_min, v_max)*

Return type
tuple(float, float, float, float)

calculate_plastic_force(*u: ndarray*, *p: ndarray*) → Tuple[float, float]

Sums the forces above and below the axis defined by unit vector *u* and point *p*. Also returns the force centroid of the forces above and below the axis.

Parameters
- **elements** (list[*Tri6*]) – A list of Tri6 finite elements.
- **u** (numpy.ndarray) – Unit vector defining the direction of the axis
- **p** (numpy.ndarray) – Point on the axis

Returns
Force in the top and bottom areas *(f_top, f_bot)*

Return type
tuple(float, float)

calculate_plastic_properties(*section*, *verbose*, *progress=None*)

Calculates the location of the plastic centroid with respect to the centroidal and principal bending axes, the plastic section moduli and shape factors and stores the results to the supplied *Section* object.

Parameters

- **section** (*Section*) – Cross section object that uses the same geometry and materials specified in the class constructor
- **verbose** (*bool*) – If set to True, the number of iterations required for each plastic axis is printed to the terminal.

check_convergence(*root_result*, *axis*)

Checks that the function solver converged and if not, raises a helpful error.

Parameters

- **root_result** (scipy.optimize.RootResults) – Result object from the root finder
- **axis** (*string*) – Axis being considered by the function solver

Raises

RuntimeError – If the function solver did not converge

evaluate_force_eq(*d*, *u*, *u_p*, *verbose*)

Given a distance *d* from the centroid to an axis (defined by unit vector *u*), creates a mesh including the new and axis and calculates the force equilibrium. The resultant force, as a ratio of the total force, is returned.

Parameters

- **d** (*float*) – Distance from the centroid to current axis
- **u** (numpy.ndarray) – Unit vector defining the direction of the axis
- **u_p** (numpy.ndarray) – Unit vector perpendicular to the direction of the axis
- **verbose** (*bool*) – If set to True, the number of iterations required for each plastic axis is printed to the terminal.

Returns

The force equilibrium norm

Return type

float

pc_algorithm(*u*, *dlim*, *axis*, *verbose*)

An algorithm used for solving for the location of the plastic centroid. The algorithm searches for the location of the axis, defined by unit vector *u* and within the section depth, that satisfies force equilibrium.

Parameters

- **u** (numpy.ndarray) – Unit vector defining the direction of the axis
- **dlim** (*list[float, float]*) – List [dmax, dmin] containing the distances from the centroid to the extreme fibres perpendicular to the axis
- **axis** (*int*) – The current axis direction: 1 (e.g. x or 11) or 2 (e.g. y or 22)
- **verbose** (*bool*) – If set to True, the number of iterations required for each plastic axis is printed to the terminal.

Returns

The distance to the plastic centroid axis *d*, the result object *r*, the force in the top of the section *f_top* and the location of the centroids of the top and bottom areas *c_top* and *c_bottom*

Return type

tuple(float, scipy.optimize.RootResults, float, list[float, float], list[float, float])

print_verbose(*d*, *root_result*, *axis*)

Prints information related to the function solver convergence to the terminal.

Parameters

- **d** (*float*) – Location of the plastic centroid axis
- **root_result** (scipy.optimize.RootResults) – Result object from the root finder
- **axis** (*string*) – Axis being considered by the function solver

StressPost Class

class sectionproperties.analysis.section.**StressPost**(*section*)

Bases: object

Class for post-processing finite element stress results.

A StressPost object is created when a stress analysis is carried out and is returned as an object to allow post-processing of the results. The StressPost object creates a deep copy of the MaterialGroups within the cross-section to allow the calculation of stresses for each material. Methods for post-processing the calculated stresses are provided.

Parameters
section (*Section*) – Cross section object for stress calculation

Variables

- **section** (*Section*) – Cross section object for stress calculation
- **material_groups** (list[MaterialGroup]) – A deep copy of the *section* material groups to allow a new stress analysis

get_stress()

Returns the stresses within each material belonging to the current *StressPost* object.

Returns
A list of dictionaries containing the cross-section stresses for each material.

Return type
list[dict]

A dictionary is returned for each material in the cross-section, containing the following keys and values:

- *'Material'*: Material name
- *'sig_zz_n'*: Normal stress $\sigma_{zz,N}$ resulting from the axial load N
- *'sig_zz_mxx'*: Normal stress $\sigma_{zz,Mxx}$ resulting from the bending moment M_{xx}
- *'sig_zz_myy'*: Normal stress $\sigma_{zz,Myy}$ resulting from the bending moment M_{yy}
- *'sig_zz_m11'*: Normal stress $\sigma_{zz,M11}$ resulting from the bending moment M_{11}
- *'sig_zz_m22'*: Normal stress $\sigma_{zz,M22}$ resulting from the bending moment M_{22}
- *'sig_zz_m'*: Normal stress $\sigma_{zz,\Sigma M}$ resulting from all bending moments
- *'sig_zx_mzz'*: *x*-component of the shear stress $\sigma_{zx,Mzz}$ resulting from the torsion moment
- *'sig_zy_mzz'*: *y*-component of the shear stress $\sigma_{zy,Mzz}$ resulting from the torsion moment
- *'sig_zxy_mzz'*: Resultant shear stress $\sigma_{zxy,Mzz}$ resulting from the torsion moment
- *'sig_zx_vx'*: *x*-component of the shear stress $\sigma_{zx,Vx}$ resulting from the shear force V_x
- *'sig_zy_vx'*: *y*-component of the shear stress $\sigma_{zy,Vx}$ resulting from the shear force V_x
- *'sig_zxy_vx'*: Resultant shear stress $\sigma_{zxy,Vx}$ resulting from the shear force V_x
- *'sig_zx_vy'*: *x*-component of the shear stress $\sigma_{zx,Vy}$ resulting from the shear force V_y
- *'sig_zy_vy'*: *y*-component of the shear stress $\sigma_{zy,Vy}$ resulting from the shear force V_y
- *'sig_zxy_vy'*: Resultant shear stress $\sigma_{zxy,Vy}$ resulting from the shear force V_y
- *'sig_zx_v'*: *x*-component of the shear stress $\sigma_{zx,\Sigma V}$ resulting from all shear forces

- *'sig_zy_v'*: y-component of the shear stress $\sigma_{zy,\Sigma V}$ resulting from all shear forces
- *'sig_zxy_v'*: Resultant shear stress $\sigma_{zxy,\Sigma V}$ resulting from all shear forces
- *'sig_zz'*: Combined normal stress σ_{zz} resulting from all actions
- *'sig_zx'*: x-component of the shear stress σ_{zx} resulting from all actions
- *'sig_zy'*: y-component of the shear stress σ_{zy} resulting from all actions
- *'sig_zxy'*: Resultant shear stress σ_{zxy} resulting from all actions
- *'sig_1'*: Major principal stress σ_1 resulting from all actions
- *'sig_3'*: Minor principal stress σ_3 resulting from all actions
- *'sig_vm'*: von Mises stress σ_{vM} resulting from all actions

The following example returns stresses for each material within a composite section, note that a result is generated for each node in the mesh for all materials irrespective of whether the materials exists at that point or not.

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)
stresses = stress_post.get_stress()

print("Number of nodes: {0}".format(section.num_nodes))

for stress in stresses:
    print('Material: {0}'.format(stress['Material']))
    print('List Size: {0}'.format(len(stress['sig_zz_n'])))
    print('Normal Stresses: {0}'.format(stress['sig_zz_n']))
    print('von Mises Stresses: {0}'.format(stress['sig_vm']))
```

```
$ Number of nodes: 2465

$ Material: Timber
$ List Size: 2465
$ Normal Stresses: [0.76923077 0.76923077 0.76923077 ... 0.76923077 0.76923077␣
↪0.76923077]
$ von Mises Stresses: [7.6394625  5.38571866 3.84784964 ... 3.09532948 3.
↪66992556 2.81976647]

$ Material: Steel
$ List Size: 2465
$ Normal Stresses: [19.23076923 0. 0. ... 0. 0. 0.]
$ von Mises Stresses: [134.78886419 0. 0. ... 0. 0. 0.]
```

plot_mohrs_circles(*x*, *y*, *title=None*, ***kwargs*)

Plots Mohr's Circles of the 3D stress state at position x, y

Parameters

- **x** (*float*) – x-coordinate of the point to draw Mohr's Circle
- **y** (*float*) – y-coordinate of the point to draw Mohr's Circle
- **title** (*string*) – Plot title
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the Mohr's Circles for the 3D stress state within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN
- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

at the point (10, 88.9).

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
mesh = geometry.create_mesh(mesh_sizes=[2.5])
section = Section(geometry, mesh)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_mohrs_circles(10, 88.9)
```

plot_stress_1(*title='Stress Contour Plot - $\\sigma_{1}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the major principal stress σ_1 resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

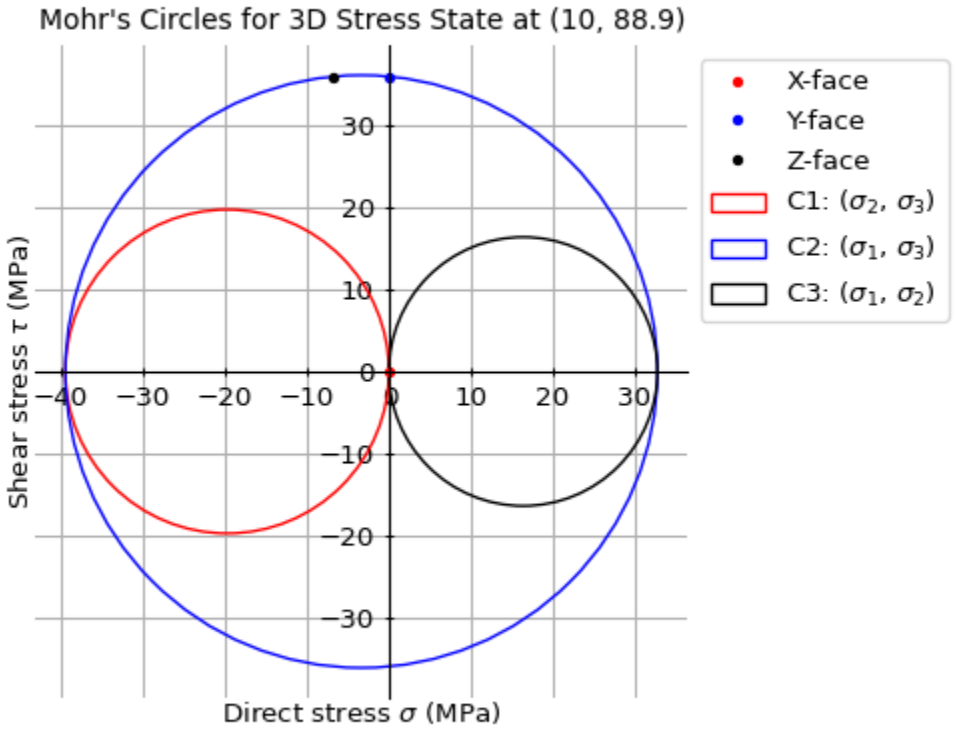

Fig. 103: Mohr's Circles of the 3D stress state at (10, 88.9).

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots a contour of the major principal stress within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN
- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
mesh = geometry.create_mesh(mesh_sizes=[2.5])
section = CrossSection(geometry, mesh)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_stress_1()
```

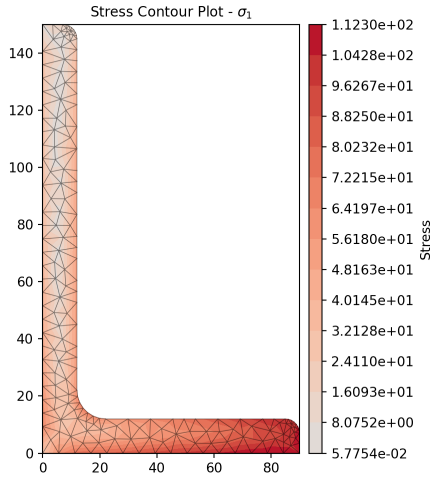

Fig. 104: Contour plot of the major principal stress.

plot_stress_3(*title='Stress Contour Plot - $\\sigma_{3}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the Minor principal stress σ_3 resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots a contour of the Minor principal stress within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN
- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
mesh = geometry.create_mesh(mesh_sizes=[2.5])
section = CrossSection(geometry, mesh)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_stress_3()
```

plot_stress_contour(*sigs*, *title*, *cmap*, *normalize*, ***kwargs*)

Plots filled stress contours over the finite element mesh.

Parameters

- **sigs** (list[numpy.ndarray]) – List of nodal stress values for each material
- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

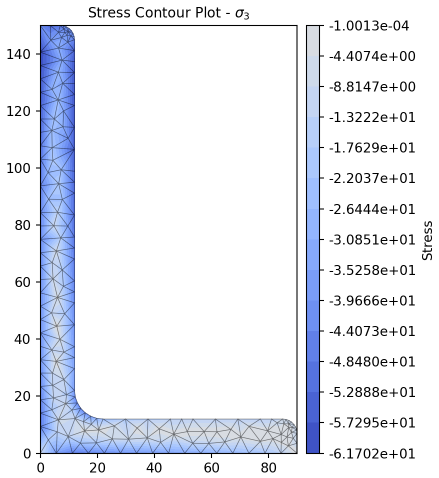

Fig. 105: Contour plot of the minor principal stress.

Returns
Matplotlib axe object

Return type
`matplotlib.axes`

plot_stress_m11_zz(*title='Stress Contour Plot - $\\sigma_{zz,M11}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,M11}$ resulting from the bending moment M_{11}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the 11-axis of 5 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
```

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```
stress_post = section.calculate_stress(M11=5e6)

stress_post.plot_stress_m11_zz()
```

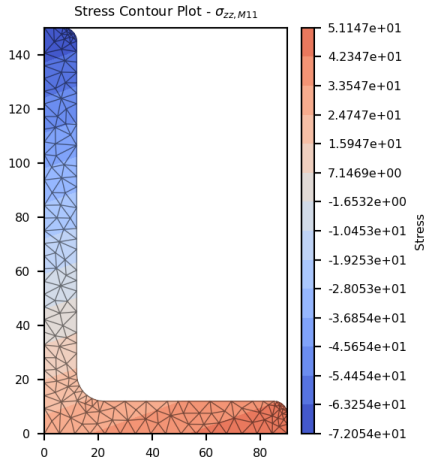

Fig. 106: Contour plot of the bending stress.

plot_stress_m22_zz(*title='Stress Contour Plot - $\\sigma_{zz,M22}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,M22}$ resulting from the bending moment M_{22}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the 22-axis of 2 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
```

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```
stress_post = section.calculate_stress(M22=5e6)

stress_post.plot_stress_m22_zz()
```

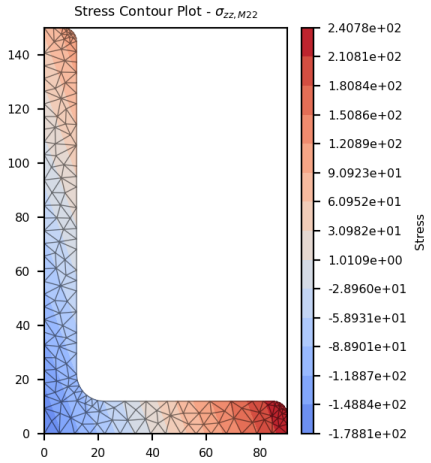

Fig. 107: Contour plot of the bending stress.

plot_stress_m_zz(*title='Stress Contour Plot - $\\sigma_{zz,\\Sigma M}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,\Sigma M}$ resulting from all bending moments $M_{xx} + M_{yy} + M_{11} + M_{22}$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the x-axis of 5 kN.m, a bending moment about the y-axis of 2 kN.m and a bending moment of 3 kN.m about the 11-axis:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)
```

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```
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mxx=5e6, Myy=2e6, M11=3e6)

stress_post.plot_stress_m_zz()
```

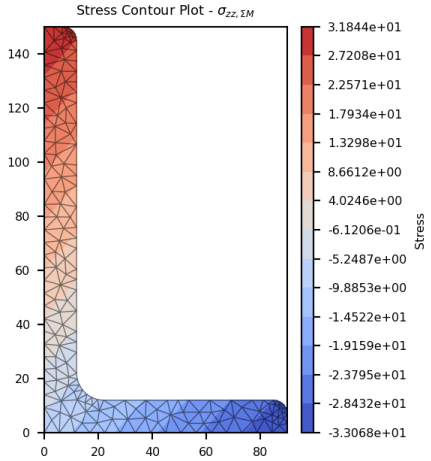

Fig. 108: Contour plot of the bending stress.

plot_stress_mxx_zz(*title='Stress Contour Plot - $\\sigma_{zz,Mxx}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,Mxx}$ resulting from the bending moment M_{xx}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the x-axis of 5 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)
```

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```
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mxx=5e6)

stress_post.plot_stress_mxx_zz()
```

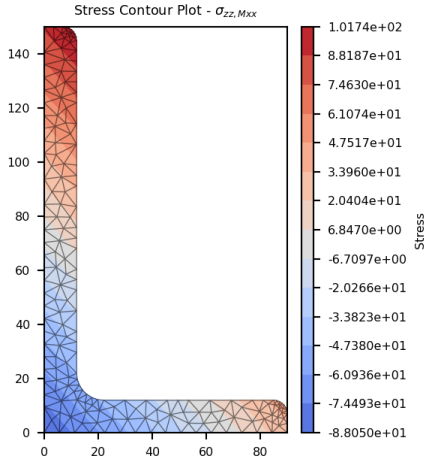

Fig. 109: Contour plot of the bending stress.

plot_stress_myy_zz(*title='Stress Contour Plot - $\\sigma_{zz,Myy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,Myy}$ resulting from the bending moment M_{yy}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from a bending moment about the y-axis of 2 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)
```

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```
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Myy=2e6)

stress_post.plot_stress_myy_zz()
```

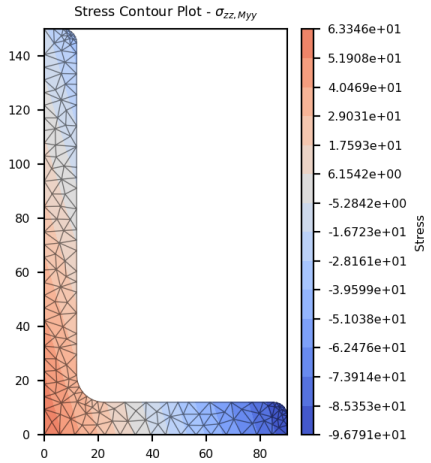

Fig. 110: Contour plot of the bending stress.

plot_stress_mzz_zx(*title='Stress Contour Plot - $\\sigma_{zx,Mzz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)
```

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```
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_stress_mzz_zx()
```

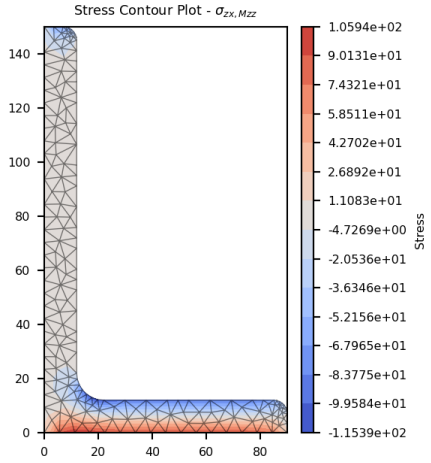

Fig. 111: Contour plot of the shear stress.

plot_stress_mzz_zxy(*title='Stress Contour Plot - $\\sigma_{zxy,Mzz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)
```

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```
section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_stress_mzz_zxy()
```

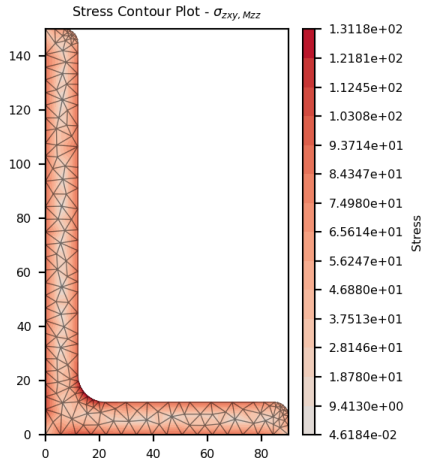

Fig. 112: Contour plot of the shear stress.

plot_stress_mzz_zy(*title='Stress Contour Plot - $\\sigma_{zy,Mzz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress $\sigma_{zy,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
```

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```
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_stress_mzz_zy()
```

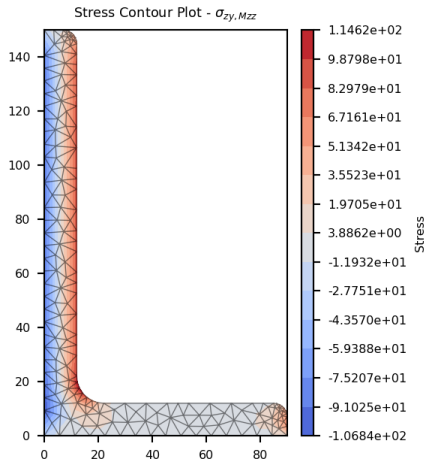

Fig. 113: Contour plot of the shear stress.

plot_stress_n_zz(*title='Stress Contour Plot - $\\sigma_{zz,N}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the normal stress $\sigma_{zz,N}$ resulting from the axial load N.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from an axial force of 10 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
```

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```
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(N=10e3)

stress_post.plot_stress_n_zz()
```

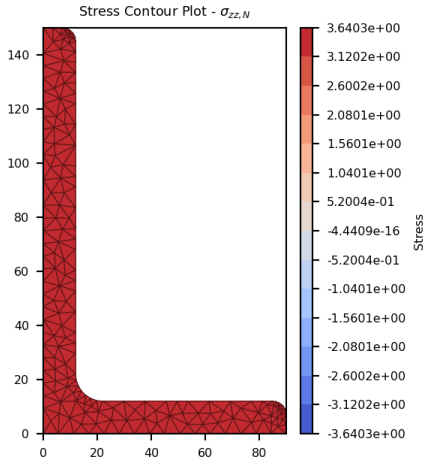

Fig. 114: Contour plot of the axial stress.

plot_stress_v_zx(*title='Stress Contour Plot - $\\sigma_{zx,\\Sigma V}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
```

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```
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_stress_v_zx()
```

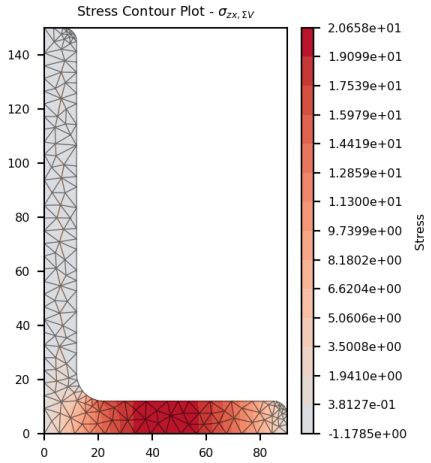

Fig. 115: Contour plot of the shear stress.

plot_stress_v_zxy(*title='Stress Contour Plot - $\\sigma_{zxy,\\Sigma V}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
```

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```
geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_stress_v_zxy()
```

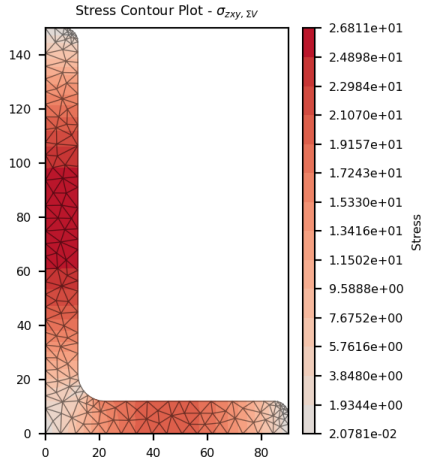

Fig. 116: Contour plot of the shear stress.

plot_stress_v_zy(*title='Stress Contour Plot - $\\sigma_{zy,\\Sigma V}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress $\sigma_{zy,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
```

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```
geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_stress_v_zy()
```

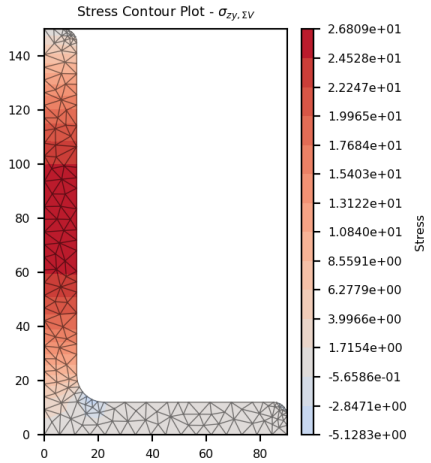

Fig. 117: Contour plot of the shear stress.

plot_stress_vector(*sigxs*, *sigys*, *title*, *cmap*, *normalize*, ***kwargs*)

Plots stress vectors over the finite element mesh.

Parameters

- **sigxs** (list[numpy.ndarray]) – List of x-components of the nodal stress values for each material
- **sigys** (list[numpy.ndarray]) – List of y-components of the nodal stress values for each material
- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

plot_stress_vm(*title='Stress Contour Plot - $\\sigma_{vM}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the von Mises stress σ_{vM} resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots a contour of the von Mises stress within a 150x90x12 UA section resulting from the following actions:

- $N = 50$ kN
- $M_{xx} = -5$ kN.m
- $M_{22} = 2.5$ kN.m
- $M_{zz} = 1.5$ kN.m
- $V_x = 10$ kN
- $V_y = 5$ kN

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(
    N=50e3, Mxx=-5e6, M22=2.5e6, Mzz=0.5e6, Vx=10e3, Vy=5e3
)

stress_post.plot_stress_vm()
```

plot_stress_vx_zx(*title='Stress Contour Plot - $\\sigma_{zx,Vx}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.

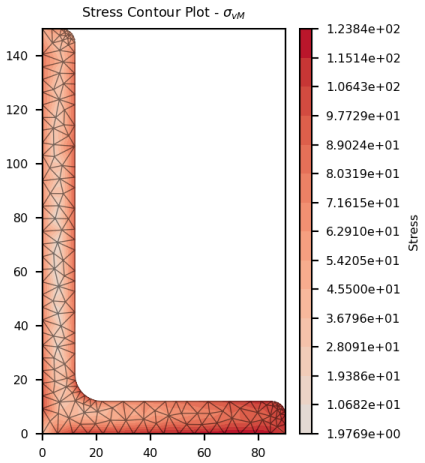

Fig. 118: Contour plot of the von Mises stress.

- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_stress_vx_zx()
```

plot_stress_vx_zxy(*title='Stress Contour Plot - $\\sigma_{zz,Myy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

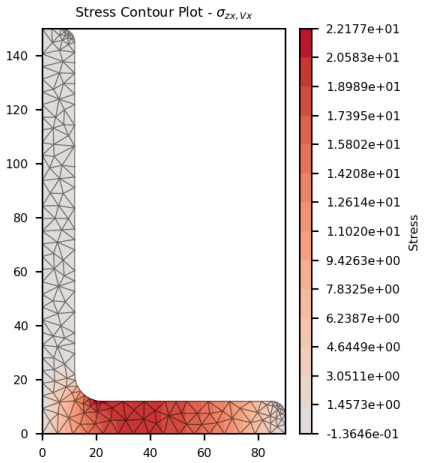

Fig. 119: Contour plot of the shear stress.

Returns
Matplotlib axes object

Return type
`matplotlib.axes`

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_stress_vx_zxy()
```

plot_stress_vx_zy(*title='Stress Contour Plot - $\\sigma_{zy,Vx}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress $\sigma_{zy,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

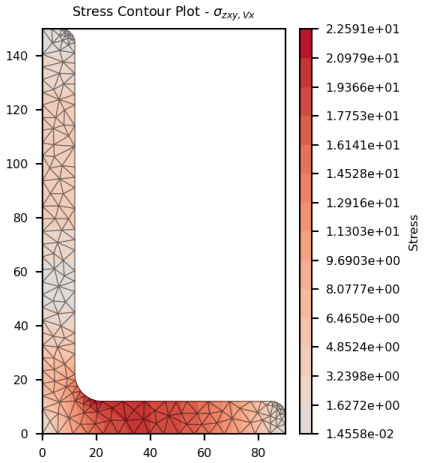

Fig. 120: Contour plot of the shear stress.

Return type
matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_stress_vx_zy()
```

plot_stress_vy_zx(*title='Stress Contour Plot - $\\sigma_{zx,Vy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress $\sigma_{zx,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

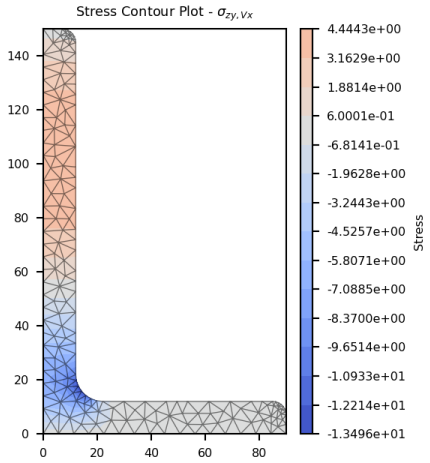

Fig. 121: Contour plot of the shear stress.

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_stress_vy_zx()
```

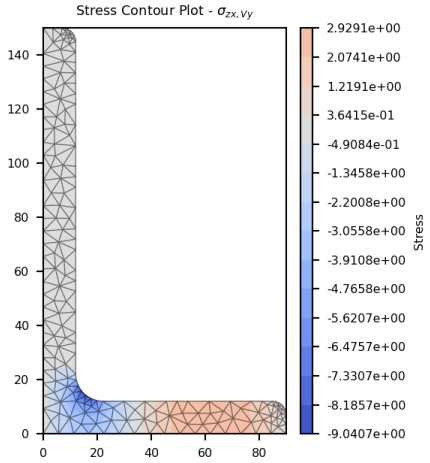

Fig. 122: Contour plot of the shear stress.

plot_stress_vy_zxy(*title='Stress Contour Plot - $\\sigma_{zxy,Vy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress $\sigma_{zxy,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_stress_vy_zxy()
```

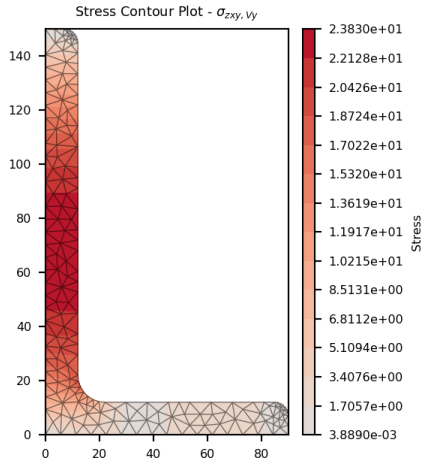

Fig. 123: Contour plot of the shear stress.

plot_stress_vy_zy(*title='Stress Contour Plot - $\\sigma_{zy,Vy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress $\sigma_{zy,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_stress_vy_zy()
```

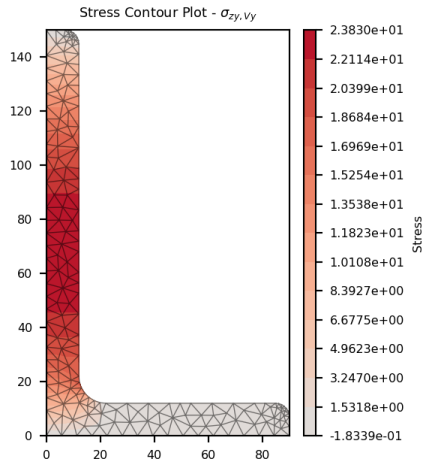

Fig. 124: Contour plot of the shear stress.

plot_stress_zx(*title='Stress Contour Plot - $\\sigma_{zx}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the x-component of the shear stress σ_{zx} resulting from all actions.

Parameters

- **title** (*string*) – Plot title

- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots the x-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_stress_zx()
```

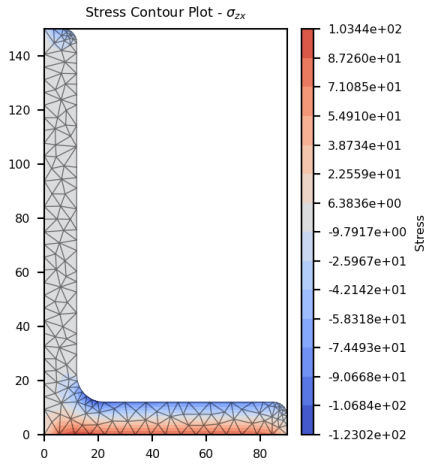

Fig. 125: Contour plot of the shear stress.

plot_stress_zxy(*title='Stress Contour Plot - $\\sigma_{zxy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the resultant shear stress σ_{zxy} resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.

- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots a contour of the resultant shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_stress_zxy()
```

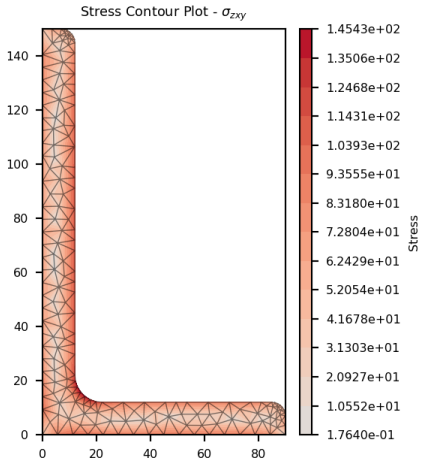

Fig. 126: Contour plot of the shear stress.

plot_stress_zy(*title='Stress Contour Plot - $\\sigma_{zy}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the y-component of the shear stress σ_{zy} resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.

- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots the y-component of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_stress_zy()
```

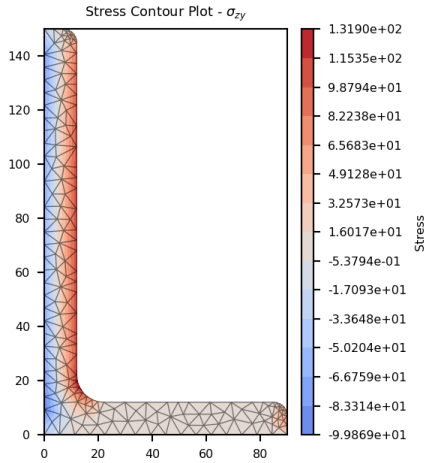

Fig. 127: Contour plot of the shear stress.

plot_stress_zz(*title='Stress Contour Plot - $\\sigma_{zz}$'*, *cmap='coolwarm'*, *normalize=True*, ***kwargs*)

Produces a contour plot of the combined normal stress σ_{zz} resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example plots the normal stress within a 150x90x12 UA section resulting from an axial force of 100 kN, a bending moment about the x-axis of 5 kN.m and a bending moment about the y-axis of 2 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(N=100e3, Mxx=5e6, Myy=2e6)

stress_post.plot_stress_zz()
```

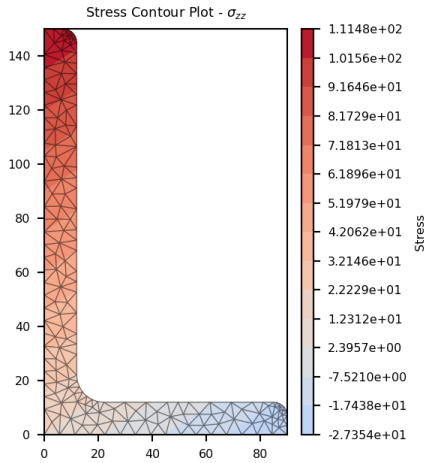

Fig. 128: Contour plot of the normal stress.

plot_vector_mzz_zxy(*title='Stress Vector Plot - $\\sigma_{zxy,Mzz}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,Mzz}$ resulting from the torsion moment M_{zz}.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6)

stress_post.plot_vector_mzz_zxy()
```

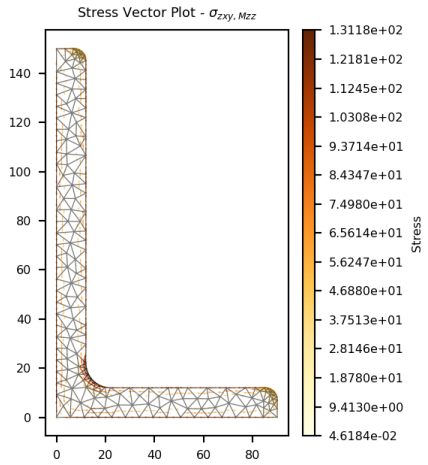

Fig. 129: Vector plot of the shear stress.

plot_vector_v_zxy(*title='Stress Vector Plot - $\\sigma_{zxy,\\Sigma V}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,\Sigma V}$ resulting from the sum of the applied shear forces $V_x + V_y$.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns
Matplotlib axes object

Return type
matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a shear force of 15 kN in the x-direction and 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3, Vy=30e3)

stress_post.plot_vector_v_zxy()
```

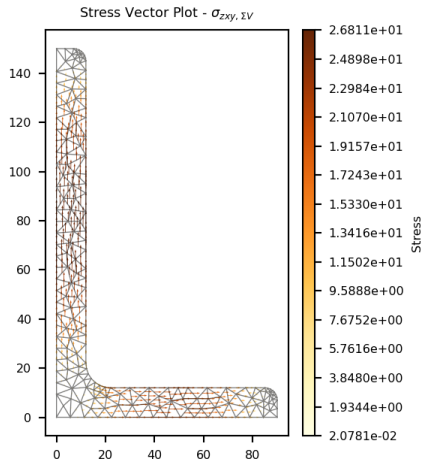

Fig. 130: Vector plot of the shear stress.

plot_vector_vx_zxy(*title='Stress Vector Plot - $\\sigma_{zxy,Vx}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,Vx}$ resulting from the shear force V_x.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a shear force in the x-direction of 15 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vx=15e3)

stress_post.plot_vector_vx_zxy()
```

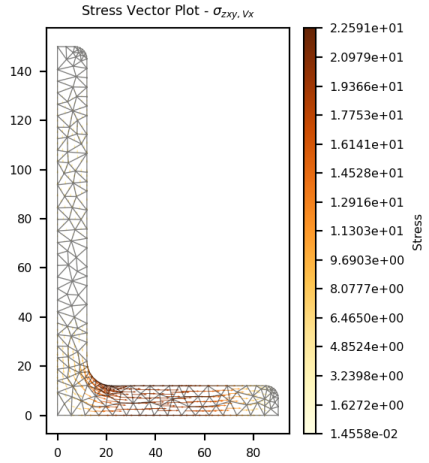

Fig. 131: Vector plot of the shear stress.

plot_vector_vy_zxy(*title='Stress Vector Plot - $\\sigma_{zxy,Vy}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress $\sigma_{zxy,Vy}$ resulting from the shear force V_y.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

`matplotlib.axes`

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a shear force in the y-direction of 30 kN:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Vy=30e3)

stress_post.plot_vector_vy_zxy()
```

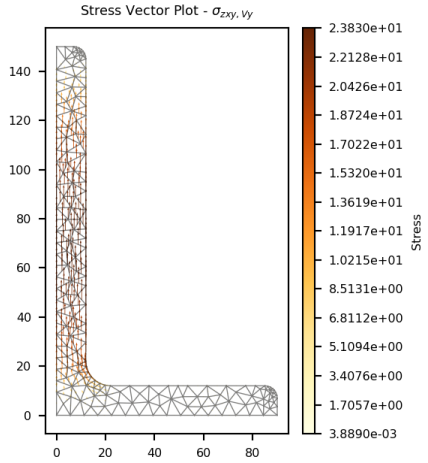

Fig. 132: Vector plot of the shear stress.

plot_vector_zxy(*title='Stress Vector Plot - $\\sigma_{zxy}$'*, *cmap='YlOrBr'*, *normalize=False*, ***kwargs*)

Produces a vector plot of the resultant shear stress σ_{zxy} resulting from all actions.

Parameters

- **title** (*string*) – Plot title
- **cmap** (*string*) – Matplotlib color map.
- **normalize** (*bool*) – If set to true, the CenteredNorm is used to scale the colormap. If set to false, the default linear scaling is used.
- **kwargs** – Passed to *plotting_context()*

Returns

Matplotlib axes object

Return type

matplotlib.axes

The following example generates a vector plot of the shear stress within a 150x90x12 UA section resulting from a torsion moment of 1 kN.m and a shear force of 30 kN in the y-direction:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section
```

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```
geometry = steel_sections.angle_section(d=150, b=90, t=12, r_r=10, r_t=5, n_r=8)
geometry.create_mesh(mesh_sizes=[20])
section = Section(geometry)

section.calculate_geometric_properties()
section.calculate_warping_properties()
stress_post = section.calculate_stress(Mzz=1e6, Vy=30e3)

stress_post.plot_vector_zxy()
```

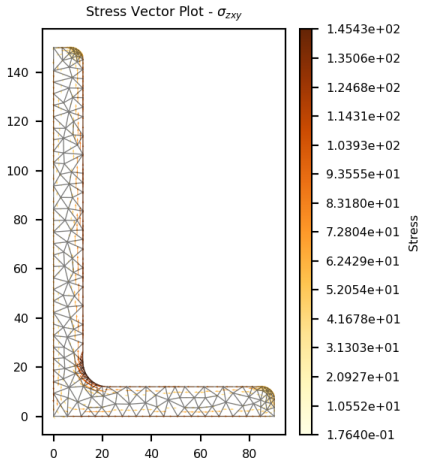

Fig. 133: Vector plot of the shear stress.

MaterialGroup Class

class sectionproperties.analysis.section.**MaterialGroup**(*material*, *num_nodes*)

Bases: `object`

Class for storing elements of different materials.

A MaterialGroup object contains the finite element objects for a specified *material*. The *stress_result* variable provides storage for stresses related each material.

Parameters

- **material** (*Material*) – Material object for the current MaterialGroup
- **num_nods** (*int*) – Number of nodes for the entire cross-section

Variables

- **material** (*Material*) – Material object for the current MaterialGroup
- **stress_result** (*StressResult*) – A StressResult object for saving the stresses of the current material
- **elements** (list[*Tri6*]) – A list of finite element objects that are of the current material type

- **el_ids** (*list[int]*) – A list of the element IDs of the elements that are of the current material type

add_element(*element*)

Adds an element and its element ID to the MaterialGroup.

Parameters

element (*Tri6*) – Element to add to the MaterialGroup

StressResult Class

class sectionproperties.analysis.section.**StressResult**(*num_nodes*)

Bases: object

Class for storing a stress result.

Provides variables to store the results from a cross-section stress analysis. Also provides a method to calculate combined stresses.

Parameters

num_nodes (*int*) – Number of nodes in the finite element mesh

Variables

- **sig_zz_n** (numpy.ndarray) – Normal stress ($\sigma_{zz,N}$) resulting from an axial force
- **sig_zz_mxx** (numpy.ndarray) – Normal stress ($\sigma_{zz,Mxx}$) resulting from a bending moment about the xx-axis
- **sig_zz_myy** (numpy.ndarray) – Normal stress ($\sigma_{zz,Myy}$) resulting from a bending moment about the yy-axis
- **sig_zz_m11** (numpy.ndarray) – Normal stress ($\sigma_{zz,M11}$) resulting from a bending moment about the 11-axis
- **sig_zz_m22** (numpy.ndarray) – Normal stress ($\sigma_{zz,M22}$) resulting from a bending moment about the 22-axis
- **sig_zx_mzz** (numpy.ndarray) – Shear stress ($\sigma_{zx,Mzz}$) resulting from a torsion moment about the zz-axis
- **sig_zy_mzz** (numpy.ndarray) – Shear stress ($\sigma_{zy,Mzz}$) resulting from a torsion moment about the zz-axis
- **sig_zx_vx** (numpy.ndarray) – Shear stress ($\sigma_{zx,Vx}$) resulting from a shear force in the x-direction
- **sig_zy_vx** (numpy.ndarray) – Shear stress ($\sigma_{zy,Vx}$) resulting from a shear force in the x-direction
- **sig_zx_vy** (numpy.ndarray) – Shear stress ($\sigma_{zx,Vy}$) resulting from a shear force in the y-direction
- **sig_zy_vy** (numpy.ndarray) – Shear stress ($\sigma_{zy,Vy}$) resulting from a shear force in the y-direction
- **sig_zz_m** (numpy.ndarray) – Normal stress ($\sigma_{zz,\Sigma M}$) resulting from all bending moments
- **sig_zxy_mzz** (numpy.ndarray) – Resultant shear stress ($\sigma_{zxy,Mzz}$) resulting from a torsion moment in the zz-direction
- **sig_zxy_vx** (numpy.ndarray) – Resultant shear stress ($\sigma_{zxy,Vx}$) resulting from a a shear force in the x-direction

- **sig_zxy_vy** (numpy.ndarray) – Resultant shear stress ($\sigma_{zxy,Vy}$) resulting from a a shear force in the y-direction
- **sig_zx_v** (numpy.ndarray) – Shear stress ($\sigma_{zx,\Sigma V}$) resulting from all shear forces
- **sig_zy_v** (numpy.ndarray) – Shear stress ($\sigma_{zy,\Sigma V}$) resulting from all shear forces
- **sig_zxy_v** (numpy.ndarray) – Resultant shear stress ($\sigma_{zxy,\Sigma V}$) resulting from all shear forces
- **sig_zz** (numpy.ndarray) – Combined normal force (σ_{zz}) resulting from all actions
- **sig_zx** (numpy.ndarray) – Combined shear stress (σ_{zx}) resulting from all actions
- **sig_zy** (numpy.ndarray) – Combined shear stress (σ_{zy}) resulting from all actions
- **sig_zxy** (numpy.ndarray) – Combined resultant shear stress (σ_{zxy}) resulting from all actions
- **sig_1** (numpy.ndarray) – Major principal stress (σ_1) resulting from all actions
- **sig_3** (numpy.ndarray) – Minor principal stress (σ_3) resulting from all actions
- **sig_vm** (numpy.ndarray) – von Mises stress (σ_{VM}) resulting from all actions

calculate_combined_stresses()

Calculates the combined cross-section stresses.

SectionProperties Class

class sectionproperties.analysis.section.**SectionProperties**(*area: Optional[float] = None*, *perimeter: Optional[float] = None*, *mass: Optional[float] = None*, *ea: Optional[float] = None*, *ga: Optional[float] = None*, *nu_eff: Optional[float] = None*, *e_eff: Optional[float] = None*, *g_eff: Optional[float] = None*, *qx: Optional[float] = None*, *qy: Optional[float] = None*, *ixx_g: Optional[float] = None*, *iyy_g: Optional[float] = None*, *ixy_g: Optional[float] = None*, *cx: Optional[float] = None*, *cy: Optional[float] = None*, *ixx_c: Optional[float] = None*, *iyy_c: Optional[float] = None*, *ixy_c: Optional[float] = None*, *zxx_plus: Optional[float] = None*, *zxx_minus: Optional[float] = None*, *zyy_plus: Optional[float] = None*, *zyy_minus: Optional[float] = None*, *rx_c: Optional[float] = None*, *ry_c: Optional[float] = None*, *i11_c: Optional[float] = None*, *i22_c: Optional[float] = None*, *phi: Optional[float] = None*, *z11_plus: Optional[float] = None*, *z11_minus: Optional[float] = None*, *z22_plus: Optional[float] = None*, *z22_minus: Optional[float] = None*, *r11_c: Optional[float] = None*, *r22_c: Optional[float] = None*, *j: Optional[float] = None*, *omega: Optional[ndarray] = None*, *psi_shear: Optional[ndarray] = None*, *phi_shear: Optional[ndarray] = None*, *Delta_s: Optional[float] = None*, *x_se: Optional[float] = None*, *y_se: Optional[float] = None*, *x11_se: Optional[float] = None*, *y22_se: Optional[float] = None*, *x_st: Optional[float] = None*, *y_st: Optional[float] = None*, *gamma: Optional[float] = None*, *A_sx: Optional[float] = None*, *A_sy: Optional[float] = None*, *A_sxy: Optional[float] = None*, *A_s11: Optional[float] = None*, *A_s22: Optional[float] = None*, *beta_x_plus: Optional[float] = None*, *beta_x_minus: Optional[float] = None*, *beta_y_plus: Optional[float] = None*, *beta_y_minus: Optional[float] = None*, *beta_11_plus: Optional[float] = None*, *beta_11_minus: Optional[float] = None*, *beta_22_plus: Optional[float] = None*, *beta_22_minus: Optional[float] = None*, *x_pc: Optional[float] = None*, *y_pc: Optional[float] = None*, *x11_pc:*

Bases: object

Class for storing section properties.

Stores calculated section properties. Also provides methods to calculate section properties entirely derived from other section properties.

Variables

- **area** (*float*) – Cross-sectional area
- **perimeter** (*float*) – Cross-sectional perimeter
- **mass** (*float*) – Cross-sectional mass
- **ea** (*float*) – Modulus weighted area (axial rigidity)
- **ga** (*float*) – Modulus weighted product of shear modulus and area
- **nu_eff** (*float*) – Effective Poisson's ratio
- **e_eff** (*float*) – Effective elastic modulus
- **g_eff** (*float*) – Effective shear modulus
- **qx** (*float*) – First moment of area about the x-axis
- **qy** (*float*) – First moment of area about the y-axis
- **ixx_g** (*float*) – Second moment of area about the global x-axis
- **iyy_g** (*float*) – Second moment of area about the global y-axis
- **ixy_g** (*float*) – Second moment of area about the global xy-axis
- **cx** (*float*) – X coordinate of the elastic centroid
- **cy** (*float*) – Y coordinate of the elastic centroid
- **ixx_c** (*float*) – Second moment of area about the centroidal x-axis
- **iyy_c** (*float*) – Second moment of area about the centroidal y-axis
- **ixy_c** (*float*) – Second moment of area about the centroidal xy-axis
- **zxx_plus** (*float*) – Section modulus about the centroidal x-axis for stresses at the positive extreme value of y
- **zxx_minus** (*float*) – Section modulus about the centroidal x-axis for stresses at the negative extreme value of y
- **zyy_plus** (*float*) – Section modulus about the centroidal y-axis for stresses at the positive extreme value of x
- **zyy_minus** (*float*) – Section modulus about the centroidal y-axis for stresses at the negative extreme value of x
- **rx_c** (*float*) – Radius of gyration about the centroidal x-axis.
- **ry_c** (*float*) – Radius of gyration about the centroidal y-axis.
- **i11_c** (*float*) – Second moment of area about the centroidal 11-axis
- **i22_c** (*float*) – Second moment of area about the centroidal 22-axis
- **phi** (*float*) – Principal axis angle

- **z11_plus** (*float*) – Section modulus about the principal 11-axis for stresses at the positive extreme value of the 22-axis
- **z11_minus** (*float*) – Section modulus about the principal 11-axis for stresses at the negative extreme value of the 22-axis
- **z22_plus** (*float*) – Section modulus about the principal 22-axis for stresses at the positive extreme value of the 11-axis
- **z22_minus** (*float*) – Section modulus about the principal 22-axis for stresses at the negative extreme value of the 11-axis
- **r11_c** (*float*) – Radius of gyration about the principal 11-axis.
- **r22_c** (*float*) – Radius of gyration about the principal 22-axis.
- **j** (*float*) – Torsion constant
- **omega** (numpy.ndarray) – Warping function
- **psi_shear** (numpy.ndarray) – Psi shear function
- **phi_shear** (numpy.ndarray) – Phi shear function
- **Delta_s** (*float*) – Shear factor
- **x_se** (*float*) – X coordinate of the shear centre (elasticity approach)
- **y_se** (*float*) – Y coordinate of the shear centre (elasticity approach)
- **x11_se** (*float*) – 11 coordinate of the shear centre (elasticity approach)
- **y22_se** (*float*) – 22 coordinate of the shear centre (elasticity approach)
- **x_st** (*float*) – X coordinate of the shear centre (Trefftz's approach)
- **y_st** (*float*) – Y coordinate of the shear centre (Trefftz's approach)
- **gamma** (*float*) – Warping constant
- **A_sx** (*float*) – Shear area about the x-axis
- **A_sy** (*float*) – Shear area about the y-axis
- **A_sxy** (*float*) – Shear area about the xy-axis
- **A_s11** (*float*) – Shear area about the 11 bending axis
- **A_s22** (*float*) – Shear area about the 22 bending axis
- **beta_x_plus** (*float*) – Monosymmetry constant for bending about the x-axis with the top flange in compression
- **beta_x_minus** (*float*) – Monosymmetry constant for bending about the x-axis with the bottom flange in compression
- **beta_y_plus** (*float*) – Monosymmetry constant for bending about the y-axis with the top flange in compression
- **beta_y_minus** (*float*) – Monosymmetry constant for bending about the y-axis with the bottom flange in compression
- **beta_11_plus** (*float*) – Monosymmetry constant for bending about the 11-axis with the top flange in compression
- **beta_11_minus** (*float*) – Monosymmetry constant for bending about the 11-axis with the bottom flange in compression

- **beta_22_plus** (*float*) – Monosymmetry constant for bending about the 22-axis with the top flange in compression
- **beta_22_minus** (*float*) – Monosymmetry constant for bending about the 22-axis with the bottom flange in compression
- **x_pc** (*float*) – X coordinate of the global plastic centroid
- **y_pc** (*float*) – Y coordinate of the global plastic centroid
- **x11_pc** (*float*) – 11 coordinate of the principal plastic centroid
- **y22_pc** (*float*) – 22 coordinate of the principal plastic centroid
- **sxx** (*float*) – Plastic section modulus about the centroidal x-axis
- **syy** (*float*) – Plastic section modulus about the centroidal y-axis
- **sf_xx_plus** (*float*) – Shape factor for bending about the x-axis with respect to the top fibre
- **sf_xx_minus** (*float*) – Shape factor for bending about the x-axis with respect to the bottom fibre
- **sf_yy_plus** (*float*) – Shape factor for bending about the y-axis with respect to the top fibre
- **sf_yy_minus** (*float*) – Shape factor for bending about the y-axis with respect to the bottom fibre
- **s11** (*float*) – Plastic section modulus about the 11-axis
- **s22** (*float*) – Plastic section modulus about the 22-axis
- **sf_11_plus** (*float*) – Shape factor for bending about the 11-axis with respect to the top fibre
- **sf_11_minus** (*float*) – Shape factor for bending about the 11-axis with respect to the bottom fibre
- **sf_22_plus** (*float*) – Shape factor for bending about the 22-axis with respect to the top fibre
- **sf_22_minus** (*float*) – Shape factor for bending about the 22-axis with respect to the bottom fibre

asdict()

Returns the SectionProperties dataclass object as a dictionary.

calculate_centroidal_properties(*mesh*)

Calculates the geometric section properties about the centroidal and principal axes based on the results about the global axis.

calculate_elastic_centroid()

Calculates the elastic centroid based on the cross-section area and first moments of area.

9.2.2 *fea* Module

Tri6 Class

class sectionproperties.analysis.fea.**Tri6**(*el_id: int*, *coords: ndarray*, *node_ids: List[int]*, *material:* Material)

Bases: object

Class for a six noded quadratic triangular element.

Provides methods for the calculation of section properties based on the finite element method.

Parameters

- **el_id** (*int*) – Unique element id
- **coords** (numpy.ndarray) – A 2 x 6 array of the coordinates of the tri-6 nodes. The first three columns relate to the vertices of the triangle and the last three columns correspond to the mid-nodes.
- **node_ids** (*list[int]*) – A list of the global node ids for the current element
- **material** (*Material*) – Material object for the current finite element.

Variables

- **el_id** (*int*) – Unique element id
- **coords** (numpy.ndarray) – A 2 x 6 array of the coordinates of the tri-6 nodes. The first three columns relate to the vertices of the triangle and the last three columns correspond to the mid-nodes.
- **node_ids** (*list[int]*) – A list of the global node ids for the current element
- **material** (*Material*) – Material of the current finite element.

element_stress(*N*, *Mxx*, *Myy*, *M11*, *M22*, *Mzz*, *Vx*, *Vy*, *ea*, *cx*, *cy*, *ixx*, *iyy*, *ixy*, *i11*, *i22*, *phi*, *j*, *nu*, *omega*, *psi_shear*, *phi_shear*, *Delta_s*)

Calculates the stress within an element resulting from a specified loading. Also returns the shape function weights.

Parameters

- **N** (*float*) – Axial force
- **Mxx** (*float*) – Bending moment about the centroidal xx-axis
- **Myy** (*float*) – Bending moment about the centroidal yy-axis
- **M11** (*float*) – Bending moment about the centroidal 11-axis
- **M22** (*float*) – Bending moment about the centroidal 22-axis
- **Mzz** (*float*) – Torsion moment about the centroidal zz-axis
- **Vx** (*float*) – Shear force acting in the x-direction
- **Vy** (*float*) – Shear force acting in the y-direction
- **ea** (*float*) – Modulus weighted area
- **cx** (*float*) – x position of the elastic centroid
- **cy** (*float*) – y position of the elastic centroid
- **ixx** (*float*) – Second moment of area about the centroidal x-axis

- **iyy** (*float*) – Second moment of area about the centroidal y-axis
- **ixy** (*float*) – Second moment of area about the centroidal xy-axis
- **i11** (*float*) – Second moment of area about the principal 11-axis
- **i22** (*float*) – Second moment of area about the principal 22-axis
- **phi** (*float*) – Principal bending axis angle
- **j** (*float*) – St. Venant torsion constant
- **nu** (*float*) – Effective Poisson's ratio for the cross-section
- **omega** (numpy.ndarray) – Values of the warping function at the element nodes
- **psi_shear** (numpy.ndarray) – Values of the psi shear function at the element nodes
- **phi_shear** (numpy.ndarray) – Values of the phi shear function at the element nodes
- **Delta_s** (*float*) – Cross-section shear factor

Returns
Tuple containing element stresses and integration weights ($\sigma_{zz,n}$, $\sigma_{zz,mxx}$, $\sigma_{zz,myy}$, $\sigma_{zz,m11}$, $\sigma_{zz,m22}$, $\sigma_{zx,mzz}$, $\sigma_{zy,mzz}$, $\sigma_{zx,vx}$, $\sigma_{zy,vx}$, $\sigma_{zx,vy}$, $\sigma_{zy,vy}$, w_i)

Return type
tuple(numpy.ndarray, numpy.ndarray, ...)

geometric_properties()

Calculates the geometric properties for the current finite element.

Returns
Tuple containing the geometric properties and the elastic and shear moduli of the element: *(area, qx, qy, ixx, iyy, ixy, e, g, rho)*

Return type
tuple(float)

local_coord(*p*)

Map a point *p* = (x, y) in the global coordinate system onto a point (eta, xi, zeta) in the local coordinate system.

Parameters
p (numpy.ndarray) – Global coordinate (x,y)

Returns
Point in local coordinate (eta, xi, zeta)

Return type
numpy.ndarray

local_element_stress(*p*, *N*, *Mxx*, *Myy*, *M11*, *M22*, *Mzz*, *Vx*, *Vy*, *ea*, *cx*, *cy*, *ixx*, *iyy*, *ixy*, *i11*, *i22*, *phi*, *j*, *nu*, *omega*, *psi_shear*, *phi_shear*, *Delta_s*)

Calculates the stress at a point *p* within the element resulting from a specified loading.

Parameters

- **p** (numpy.ndarray) – Point (x,y) in the global coordinate system that is within the element.
- **N** (*float*) – Axial force
- **Mxx** (*float*) – Bending moment about the centroidal xx-axis
- **Myy** (*float*) – Bending moment about the centroidal yy-axis

- **M11** (*float*) – Bending moment about the centroidal 11-axis
- **M22** (*float*) – Bending moment about the centroidal 22-axis
- **Mzz** (*float*) – Torsion moment about the centroidal zz-axis
- **Vx** (*float*) – Shear force acting in the x-direction
- **Vy** (*float*) – Shear force acting in the y-direction
- **ea** (*float*) – Modulus weighted area
- **cx** (*float*) – x position of the elastic centroid
- **cy** (*float*) – y position of the elastic centroid
- **ixx** (*float*) – Second moment of area about the centroidal x-axis
- **iyy** (*float*) – Second moment of area about the centroidal y-axis
- **ixy** (*float*) – Second moment of area about the centroidal xy-axis
- **i11** (*float*) – Second moment of area about the principal 11-axis
- **i22** (*float*) – Second moment of area about the principal 22-axis
- **phi** (*float*) – Principal bending axis angle
- **j** (*float*) – St. Venant torsion constant
- **nu** (*float*) – Effective Poisson's ratio for the cross-section
- **omega** (numpy.ndarray) – Values of the warping function at the element nodes
- **psi_shear** (numpy.ndarray) – Values of the psi shear function at the element nodes
- **phi_shear** (numpy.ndarray) – Values of the phi shear function at the element nodes
- **Delta_s** (*float*) – Cross-section shear factor

Returns
Tuple containing stress values at point *p* ($\sigma_{zz,n}$, $\sigma_{zz,mxx}$, $\sigma_{zz,myy}$, $\sigma_{zz,m11}$, $\sigma_{zz,m22}$, $\sigma_{zx,mzz}$, $\sigma_{zy,mzz}$, $\sigma_{zx,vx}$, $\sigma_{zy,vx}$, $\sigma_{zx,vy}$, $\sigma_{zy,vy}$)

Return type
tuple(float, float, ...)

monosymmetry_integrals(*phi*)

Calculates the integrals used to evaluate the monosymmetry constant about both global axes and both principal axes.

Parameters
phi (*float*) – Principal bending axis angle

Returns
Integrals used to evaluate the monosymmetry constants *(int_x, int_y, int_11, int_22)*

Return type
tuple(float, float, float, float)

point_within_element(*pt*)

Determines whether a point lies within the current element.

Parameters
pt (*list[float, float]*) – Point to check *(x, y)*

Returns
Whether the point lies within an element

Return type
bool

shear_coefficients(*ixx*, *iyy*, *ixy*, *psi_shear*, *phi_shear*, *nu*)

Calculates the variables used to determine the shear deformation coefficients.

Parameters

- **ixx** (*float*) – Second moment of area about the centroidal x-axis
- **iyy** (*float*) – Second moment of area about the centroidal y-axis
- **ixy** (*float*) – Second moment of area about the centroidal xy-axis
- **psi_shear** (numpy.ndarray) – Values of the psi shear function at the element nodes
- **phi_shear** (numpy.ndarray) – Values of the phi shear function at the element nodes
- **nu** (*float*) – Effective Poisson's ratio for the cross-section

Returns
Shear deformation variables *(kappa_x, kappa_y, kappa_xy)*

Return type
tuple(float, float, float)

shear_load_vectors(*ixx*, *iyy*, *ixy*, *nu*)

Calculates the element shear load vectors used to evaluate the shear functions.

Parameters

- **ixx** (*float*) – Second moment of area about the centroidal x-axis
- **iyy** (*float*) – Second moment of area about the centroidal y-axis
- **ixy** (*float*) – Second moment of area about the centroidal xy-axis
- **nu** (*float*) – Effective Poisson's ratio for the cross-section

Returns
Element shear load vector psi *(f_psi)* and phi *(f_phi)*

Return type
tuple(numpy.ndarray, numpy.ndarray)

shear_warping_integrals(*ixx*, *iyy*, *ixy*, *omega*)

Calculates the element shear centre and warping integrals required for shear analysis of the cross-section.

Parameters

- **ixx** (*float*) – Second moment of area about the centroidal x-axis
- **iyy** (*float*) – Second moment of area about the centroidal y-axis
- **ixy** (*float*) – Second moment of area about the centroidal xy-axis
- **omega** (numpy.ndarray) – Values of the warping function at the element nodes

Returns
Shear centre integrals about the x and y-axes *(sc_xint, sc_yint)*, warping integrals *(q_omega, i_omega, i_xomega, i_yomega)*

Return type
tuple(float, float, float, float, float, float)

torsion_properties()

Calculates the element stiffness matrix used for warping analysis and the torsion load vector.

Returns
Element stiffness matrix *(k_el)* and element torsion load vector *(f_el)*

Return type
tuple(numpy.ndarray, numpy.ndarray)

gauss_points

sectionproperties.analysis.fea.**gauss_points**(*n*)

Returns the Gaussian weights and locations for *n* point Gaussian integration of a quadratic triangular element.

Parameters
n (*int*) – Number of Gauss points (1, 3 or 6)

Returns
An *n x 4* matrix consisting of the integration weight and the eta, xi and zeta locations for *n* Gauss points

Return type
numpy.ndarray

shape_function

sectionproperties.analysis.fea.**shape_function**(*coords*, *gauss_point*)

Computes shape functions, shape function derivatives and the determinant of the Jacobian matrix for a tri 6 element at a given Gauss point.

Parameters

- **coords** (numpy.ndarray) – Global coordinates of the quadratic triangle vertices [2 x 6]
- **gauss_point** (numpy.ndarray) – Gaussian weight and isoparametric location of the Gauss point

Returns
The value of the shape functions *N(i)* at the given Gauss point [1 x 6], the derivative of the shape functions in the j-th global direction *B(i,j)* [2 x 6] and the determinant of the Jacobian matrix *j*

Return type
tuple(numpy.ndarray, numpy.ndarray, float)

extrapolate_to_nodes

sectionproperties.analysis.fea.**extrapolate_to_nodes**(*w*)

Extrapolates results at six Gauss points to the six nodes of a quadratic triangular element.

Parameters
w (numpy.ndarray) – Result at the six Gauss points [1 x 6]

Returns
Extrapolated nodal values at the six nodes [1 x 6]

Return type
numpy.ndarray

principal_coordinate

sectionproperties.analysis.fea.**principal_coordinate**(*phi*, *x*, *y*)

Determines the coordinates of the cartesian point *(x, y)* in the principal axis system given an axis rotation angle phi.

Parameters

- **phi** (*float*) – Principal bending axis angle (degrees)
- **x** (*float*) – x coordinate in the global axis
- **y** (*float*) – y coordinate in the global axis

Returns
Principal axis coordinates *(x1, y2)*

Return type
tuple(float, float)

global_coordinate

sectionproperties.analysis.fea.**global_coordinate**(*phi*, *x11*, *y22*)

Determines the global coordinates of the principal axis point *(x1, y2)* given principal axis rotation angle phi.

Parameters

- **phi** (*float*) – Principal bending axis angle (degrees)
- **x11** (*float*) – 11 coordinate in the principal axis
- **y22** (*float*) – 22 coordinate in the principal axis

Returns
Global axis coordinates *(x, y)*

Return type
tuple(float, float)

point_above_line

sectionproperties.analysis.fea.**point_above_line**(*u*, *px*, *py*, *x*, *y*)

Determines whether a point *(x, y)* is a above or below the line defined by the parallel unit vector *u* and the point *(px, py)*.

Parameters

- **u** (numpy.ndarray) – Unit vector parallel to the line [1 x 2]
- **px** (*float*) – x coordinate of a point on the line
- **py** (*float*) – y coordinate of a point on the line
- **x** (*float*) – x coordinate of the point to be tested
- **y** (*float*) – y coordinate of the point to be tested

Returns
This method returns *True* if the point is above the line or *False* if the point is below the line

Return type
bool

9.2.3 *solver* Module

solve_cgs

sectionproperties.analysis.solver.**solve_cgs**(*k*, *f*, *m=None*, *tol=1e-05*)

Solves a linear system of equations (Ku = f) using the CGS iterative method.

Parameters

- **k** (scipy.sparse.csc_matrix) – N x N matrix of the linear system
- **f** (numpy.ndarray) – N x 1 right hand side of the linear system
- **tol** (*float*) – Tolerance for the solver to achieve. The algorithm terminates when either the relative or the absolute residual is below tol.
- **m** (scipy.linalg.LinearOperator) – Preconditioner for the linear matrix approximating the inverse of k

Returns
The solution vector to the linear system of equations

Return type
numpy.ndarray

Raises
RuntimeError – If the CGS iterative method does not converge

solve_cgs_lagrange

sectionproperties.analysis.solver.**solve_cgs_lagrange**(*k_lg*, *f*, *tol=1e-05*, *m=None*)

Solves a linear system of equations (Ku = f) using the CGS iterative method and the Lagrangian multiplier method.

Parameters

- **k** (scipy.sparse.csc_matrix) – (N+1) x (N+1) Lagrangian multiplier matrix of the linear system
- **f** (numpy.ndarray) – N x 1 right hand side of the linear system
- **tol** (*float*) – Tolerance for the solver to achieve. The algorithm terminates when either the relative or the absolute residual is below tol.
- **m** (scipy.linalg.LinearOperator) – Preconditioner for the linear matrix approximating the inverse of k

Returns
The solution vector to the linear system of equations

Return type
numpy.ndarray

Raises
RuntimeError – If the CGS iterative method does not converge or the error from the Lagrangian multiplier method exceeds the tolerance

solve_direct

sectionproperties.analysis.solver.**solve_direct**(*k*, *f*)

Solves a linear system of equations (Ku = f) using the direct solver method.

Parameters

- **k** (scipy.sparse.csc_matrix) – N x N matrix of the linear system
- **f** (numpy.ndarray) – N x 1 right hand side of the linear system

Returns
The solution vector to the linear system of equations

Return type
numpy.ndarray

solve_direct_lagrange

sectionproperties.analysis.solver.**solve_direct_lagrange**(*k_lg*, *f*)

Solves a linear system of equations (Ku = f) using the direct solver method and the Lagrangian multiplier method.

Parameters

- **k** (scipy.sparse.csc_matrix) – (N+1) x (N+1) Lagrangian multiplier matrix of the linear system
- **f** (numpy.ndarray) – N x 1 right hand side of the linear system

Returns
The solution vector to the linear system of equations

Return type
numpy.ndarray

Raises
RuntimeError – If the Lagrangian multiplier method exceeds a tolerance of 1e-5

9.3 Post-Processor Package

9.3.1 *post* Module

plotting_context

sectionproperties.post.post.**plotting_context**(*ax=None*, *pause=True*, *title=''*, *filename=''*, *render=True*, *axis_index=None*, ***kwargs*)

Executes code required to set up a matplotlib figure.

Parameters

- **ax** (matplotlib.axes.Axes) – Axes object on which to plot
- **pause** (*bool*) – If set to true, the figure pauses the script until the window is closed. If set to false, the script continues immediately after the window is rendered.
- **title** (*string*) – Plot title

- **filename** (*string*) – Pass a non-empty string or path to save the image as. If this option is used, the figure is closed after the file is saved.
- **render** (*bool*) – If set to False, the image is not displayed. This may be useful if the figure or axes will be embedded or further edited before being displayed.
- **axis_index** (*Union[None, int, Tuple(int)]*) – If more than 1 axes is created by subplot, then this is the axis to plot on. This may be a tuple if a 2D array of plots is returned. The default value of None will select the top left plot.
- **kwargs** – Passed to `matplotlib.pyplot.subplots()`

draw_principal_axis

`sectionproperties.post.post.`**draw_principal_axis**(*ax*, *phi*, *cx*, *cy*)

Draws the principal axis on a plot.

Parameters

- **ax** (`matplotlib.axes.Axes`) – Axes object on which to plot
- **phi** (*float*) – Principal axis angle in radians
- **cx** (*float*) – x-location of the centroid
- **cy** (*float*) – y-location of the centroid

print_results

`sectionproperties.post.post.`**print_results**(*cross_section*, *fmt*)

Prints the results that have been calculated to the terminal.

Parameters

- **cross_section** (`CrossSection`) – structural cross-section object
- **fmt** (*string*) – Number format

CHAPTER

TEN

THEORETICAL BACKGROUND

10.1 Introduction

The analysis of homogenous cross-sections is particularly relevant in structural design, in particular for the design of steel structures, where complex built-up sections are often utilised. Accurate warping independent properties, such as the second moment of area and section moduli, are crucial input for structural analysis and stress verification. Warping dependent properties, such as the Saint-Venant torsion constant and warping constant are essential in the verification of slender steel structures when lateral-torsional buckling is critical.

Warping independent properties for basic cross-sections are relatively simple to calculate by hand. However accurate warping independent properties, even for the most basic cross-section, require solving several boundary value partial differential equations. This necessitates numerical methods in the calculation of these properties, which can be extended to arbitrary complex sections.

This section of the documentation describes the theory and application of the finite element method to cross-sectional analysis used by *sectionproperties*. The goal of *sectionproperties* is to perform cross-sectional and stress analysis on arbitrary cross-sections, see below figure. In its simplest form, an arbitrary cross-section can be defined by a series of points, segments and holes.

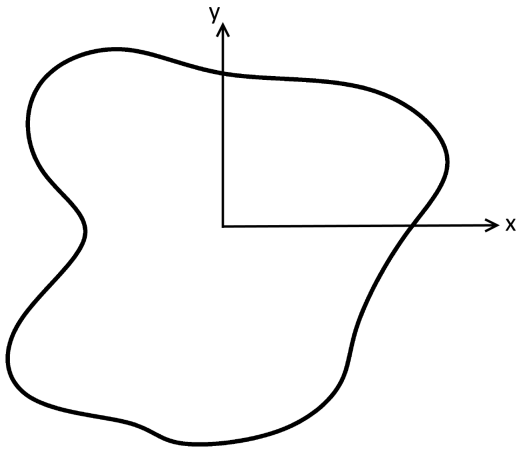

Fig. 1: Arbitrary cross-section with adopted axis convention.

10.2 Mesh Generation

The cross-section is meshed using quadratic superparametric triangular elements (Tri6) using the triangle library for Python. Superparametric quadratic elements are defined as having straight edges and mid-nodes located at the mid-point between adjacent corner nodes. *triangle* implements Triangle, which is a two dimensional quality mesh generator and Delaunay triangulator written by Jonathan Shewchuk in C.

For the calculation of warping independent properties (i.e. area properties), the mesh quality is not important as superparametric elements have a constant Jacobian and will result in an exact solution independent of mesh quality. However, this is not the case for the calculation of warping dependent properties. As a result, mesh quality and refinement is critical and thus the user is encouraged to ensure an adequate mesh is generated.

10.3 Finite Element Preliminaries

10.3.1 Element Type

Quadratic six noded triangular elements were implemented in *sectionproperties* in order to utilise the finite element formulations for calculating section properties. The figure below shows a generic six noded triangular element. As previously mentioned, *sectionproperties* implements superparametric elements, therefore the edges in the below image will always be straight and not curved.

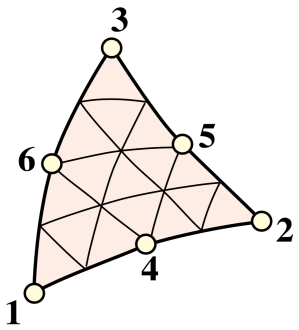

Fig. 2: Six noded triangular element [1].

The quadratic triangular element was used due to the ease of mesh generation and convergence advantages over the linear triangular element.

10.3.2 Isoparametric Representation

An isoparametric coordinate system has been used to evaluate the shape functions of the parent element and map them to a generalised triangular element within the mesh. Three independent isoparametric coordinates (η, ξ, ζ) are used to map the six noded triangular element as shown in the figure below.

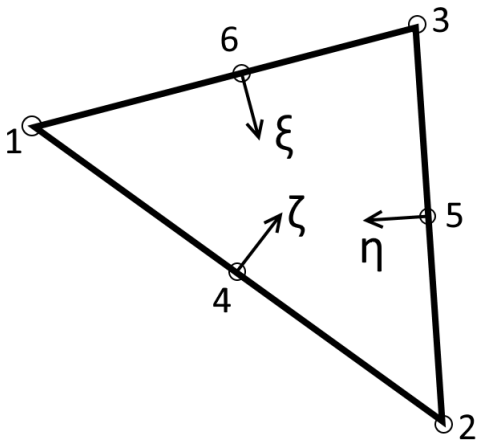

Fig. 3: Isoparametric coordinates for the two dimensional triangular element.

Shape Functions

The shape functions for the six noded triangular element in terms of the isoparametric coordinates are as follows:

$$
\begin{aligned}
N_1 &= \eta(2\eta - 1) \\
N_2 &= \xi(2\xi - 1) \\
N_3 &= \zeta(2\zeta - 1) \\
N_4 &= 4\eta\xi \\
N_5 &= 4\xi\zeta \\
N_6 &= 4\eta\zeta
\end{aligned}
$$

The above shape functions can be combined into the shape function row vector:

$$
\mathbf{N} = [N_1 \; N_2 \; N_3 \; N_4 \; N_5 \; N_6]
$$

Cartesian Partial Derivatives

The partial derivatives of the shape functions with respect to the cartesian coordinates, denoted as the **B** matrix, are required in the finite element formulations of various section properties. Felippa [1] describes the multiplication of the *Jacobian matrix* (**J**) and the partial derivative matrix (**P**):

$$
\mathbf{J}\,\mathbf{P} = \begin{bmatrix} 1 & 1 & 1 \\ \sum x_i \frac{\partial N_i}{\partial \eta} & \sum x_i \frac{\partial N_i}{\partial \xi} & \sum x_i \frac{\partial N_i}{\partial \zeta} \\ \sum y_i \frac{\partial N_i}{\partial \eta} & \sum y_i \frac{\partial N_i}{\partial \xi} & \sum y_i \frac{\partial N_i}{\partial \zeta} \end{bmatrix} \begin{bmatrix} \frac{\partial \eta}{\partial x} & \frac{\partial \eta}{\partial y} \\ \frac{\partial \xi}{\partial x} & \frac{\partial \xi}{\partial y} \\ \frac{\partial \zeta}{\partial x} & \frac{\partial \zeta}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

The determinant of the *Jacobian matrix* scaled by one half is equal to the Jacobian:

$$
J = \frac{1}{2} \det \mathbf{J}
$$

The equation for textbf{J P} can be re-arranged to evaluate the partial derivate matrix (**P**):

$$
\mathbf{P} = \mathbf{J}^{-1} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}
$$

As described in [1], the derivates of the shape functions can be evaluated using the below expressions:

$$\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial \eta} & \frac{\partial N_i}{\partial \xi} & \frac{\partial N_i}{\partial \zeta} \end{bmatrix} [\mathbf{P}]$$

where the derivatives of the shape functions with respect to the isoparametric parameters can easily be evaluated from the equation for the shape functions, resulting in the following expression for the **B** matrix:

$$\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} 4\eta - 1 & 0 & 0 \\ 0 & 4\xi - 1 & 0 \\ 0 & 0 & 4\zeta - 1 \\ 4\xi & 4\eta & 0 \\ 0 & 4\zeta & 4\xi \\ 4\zeta & 0 & 4\eta \end{bmatrix} \mathbf{J}^{-1} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

10.3.3 Numerical Integration

Three different integration schemes are utilised in the cross-section analysis in order to evaluate the integrals of varying order polynomials. The one point, three point and six point integration schemes are summarised in the figure below:

Table 1: Six noded triangle integration schemes with maximum degree of polynomial that is evaluated exactly [1].

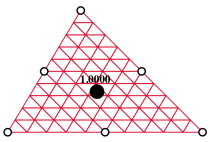

Fig. 4: 1 pt. integration; p-degree = 1.

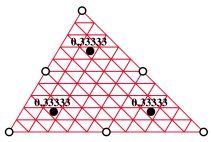

Fig. 5: 3 pt. integration; p-degree = 2.

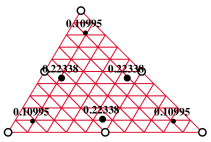

Fig. 6: 6 pt. integration; p-degree = 4.

The locations and weights of the Gauss points are summarised in the table below [1]:

Scheme	η-location	ξ-location	ζ-location	weight
1 pt.	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1
3 pt.	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$
	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{3}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{3}$
6 pt.	$1 - 2g_2$	g_2	g_2	w_2
	g_2	$1 - 2g_2$	g_2	w_2
	g_2	g_2	$1 - 2g_2$	w_2
	g_1	g_1	$1 - 2g_1$	w_1
	$1 - 2g_1$	g_1	g_1	w_1
	g_1	$1 - 2g_1$	g_1	w_1

The parameters for the six point numerical integration are shown below:

$$g_{1,2} = \frac{1}{18}\left(8 - \sqrt{10} \pm \sqrt{38 - 44\sqrt{\frac{2}{5}}}\right)$$

$$w_{1,2} = \frac{620 \pm \sqrt{213125 - 53320\sqrt{10}}}{3720}$$

Bringing together the isoparametric representation of the six noded triangular element and numerical integration, the integration of a function $f(\eta, \xi, \zeta)$ proves to be simpler than integrating the corresponding function $f(x, y)$ over the cartesian element [2]. The transformation formula for integrals is:

$$\begin{aligned}\int_\Omega f(x,y)\,dx\,dy &= \int_{\Omega_r} f(\eta,\xi,\zeta)\,J\,d\eta\,d\xi\,d\zeta \\ &= \sum_i^n w_i f(\eta_i,\xi_i,\zeta_i)\,J_i\end{aligned}$$

where the sum is taken over the integration points, w_i is the weight of the current integration point and J_i is the Jacobian at the current integration point (recall that the Jacobian is constant for the superparametric six noded triangular element).

10.3.4 Extrapolation to Nodes

The most optimal location to sample stresses are at the integration points, however the results are generally plotted using nodal values. As a result, the stresses at the integration points need to be extrapolated to the nodes of the element. The extrapolated stresses at the nodes ($\tilde{\boldsymbol{\sigma}}_g$) can be calculated through the multiplication of a smoothing matrix ($\mathbf{H}$) and the stresses at the integration points ($\boldsymbol{\sigma}_g$) [2]:

$$\tilde{\boldsymbol{\sigma}}_g = \mathbf{H}^{-1}\,\boldsymbol{\sigma}_g$$

where the $\mathbf{H}$ matrix contains the row vectors of the shape functions at each integration point:

$$\mathbf{H} = \begin{bmatrix} \mathbf{N}(\eta_1,\xi_1,\zeta_1) \\ \mathbf{N}(\eta_2,\xi_2,\zeta_2) \\ \mathbf{N}(\eta_3,\xi_3,\zeta_3) \\ \mathbf{N}(\eta_4,\xi_4,\zeta_4) \\ \mathbf{N}(\eta_5,\xi_5,\zeta_5) \\ \mathbf{N}(\eta_6,\xi_6,\zeta_6) \end{bmatrix}$$

Where two or more elements share the same node, nodal averaging is used to evaluate the nodal stress.

10.3.5 Lagrangian Multiplier

As described in the calculation of the *Torsion Constant* and *Shear Properties*, partial differential equations are to be solved with purely Neumann boundary conditions. In the context of the torsion and shear problem, this involves the inversion of a nearly singular global stiffness matrix. After shifting the domain such that the centroid coincides with the global origin, the Lagrangian multiplier method is used to solve the set of linear equations of the form $\mathbf{Ku} = \mathbf{F}$ by introducing an extra constraint on the solution vector whereby the mean value is equal to zero. Larson et. al [3] describe the resulting modified stiffness matrix, and solution and load vector:

$$\begin{bmatrix} \mathbf{K} & \mathbf{C}^{\mathrm{T}} \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ 0 \end{bmatrix}$$

where $\mathbf{C}$ is a row vector of ones and λ may be though of as a force acting to enforce the constraints, which should be relatively small when compared to the values in the force vector and can be omitted from the solution vector.

10.4 Calculation of Cross-Section Properties

10.4.1 Cross-Sectional Area

The area A of the cross-section is given by [2]:

$$A = \int_A dx\, dy = \sum_e A_e = \sum_e \int_\Omega J_e\, d\eta\, d\xi\, d\zeta$$

As the Jacobian is constant over the element, the integration over the element domain in the above equation can be performed using one point integration:

$$A = \sum_e \sum_{i=1}^{1} w_i J_i$$

10.4.2 First Moments of Area

The first moments of area are defined by [2]:

$$Q_x = \int_A y\, dA = \sum_e \int_\Omega \mathbf{N}\mathbf{y}_e J_e\, d\eta\, d\xi\, d\zeta$$
$$Q_y = \int_A x\, dA = \sum_e \int_\Omega \mathbf{N}\mathbf{x}_e J_e\, d\eta\, d\xi\, d\zeta$$

where $\mathbf{x}_e$ and $\mathbf{y}_e$ are column vectors containing the cartesian coordinates of the element nodes. The above equations can be evaluated using three point integration as the shape functions ($\mathbf{N}$) are quadratic:

$$Q_x = \sum_e \sum_{i=1}^{3} w_i \mathbf{N}_i \mathbf{y}_e J_e$$
$$Q_y = \sum_e \sum_{i=1}^{3} w_i \mathbf{N}_i \mathbf{x}_e J_e$$

10.4.3 Centroids

The coordinates of the centroid are found from [2]:

$$x_c = \frac{Q_y}{A}$$
$$y_c = \frac{Q_x}{A}$$

10.4.4 Second Moments of Area

The second moments of area are defined by [2]:

$$I_{xx} = \int_A y^2\, dA = \sum_e \int_\Omega (\mathbf{N}\mathbf{y}_e)^2 J_e\, d\eta\, d\xi\, d\zeta$$
$$I_{yy} = \int_A x^2\, dA = \sum_e \int_\Omega (\mathbf{N}\mathbf{x}_e)^2 J_e\, d\eta\, d\xi\, d\zeta$$
$$I_{xy} = \int_A xy\, dA = \sum_e \int_\Omega \mathbf{N}\mathbf{y}_e \mathbf{N}\mathbf{x}_e J_e\, d\eta\, d\xi\, d\zeta$$

The above equations can be evaluated using six point integration as the square of the shape functions are quartic:

$$I_{xx} = \sum_e \sum_{i=1}^{6} w_i (\mathbf{N}_i \mathbf{y}_e)^2 J_e$$
$$I_{yy} = \sum_e \sum_{i=1}^{6} w_i (\mathbf{N}_i \mathbf{x}_e)^2 J_e$$
$$I_{xy} = \sum_e \sum_{i=1}^{6} w_i \mathbf{N}\mathbf{y}_e \mathbf{N}\mathbf{x}_e J_e$$

The above equations list the second moments of area about the global coordinate system axis, which is chosen arbitrarily by the user. These properties can be found about the centroidal axis of the cross-section by using the parallel axis theorem:

$$I_{\overline{xx}} = I_{xx} - {y_c}^2 A = I_{xx} - \frac{{Q_x}^2}{A}$$
$$I_{\overline{yy}} = I_{yy} - {x_c}^2 A = I_{yy} - \frac{{Q_y}^2}{A}$$
$$I_{\overline{xy}} = I_{xy} - x_c y_c A = I_{xy} - \frac{Q_x Q_y}{A}$$

10.4.5 Radii of Gyration

The radii of gyration can be calculated from the second moments of area and the cross-sectional area as follows [2]:

$$r_x = \sqrt{\frac{I_{xx}}{A}}$$
$$r_y = \sqrt{\frac{I_{yy}}{A}}$$

10.4.6 Elastic Section Moduli

The elastic section modulii can be calculated from the second moments of area and the extreme (min. and max.) coordinates of the cross-section in the x and y-directions [2]:

$$Z_{xx}^{+} = \frac{I_{\overline{xx}}}{y_{max} - y_c}$$
$$Z_{xx}^{-} = \frac{I_{\overline{xx}}}{y_c - y_{min}}$$
$$Z_{yy}^{+} = \frac{I_{\overline{yy}}}{x_{max} - x_c}$$
$$Z_{yy}^{-} = \frac{I_{\overline{yy}}}{x_c - x_{min}}$$

10.4.7 Plastic Section Moduli

For a homogenous section, the plastic centroid can be determined by finding the intersection of the two lines that evenly divide the cross-sectional area in both the x and y directions. A suitable procedure could not be found in literature and thus an algorithm involving the iterative incrementation of the plastic centroid was developed. The algorithm uses Brent's method to efficiently locate the plastic centroidal axes in the global and principal directions.

Once the plastic centroid has been located, the plastic section moduli can be readily computed using the following expression:

$$S_{xx} = \frac{A}{2} |y_{c,t} - y_{c,b}|$$
$$S_{yy} = \frac{A}{2} |x_{c,t} - x_{c,b}|$$

where A is the cross-sectional area, and $x_{c,t}$ and $x_{c,b}$ refer to the centroids of the top half section and bottom half section respectively.

10.4.8 Principal Axis Properties

The principal bending axes are determined by calculating the principal moments of inertia[2]:

$$I_{11} = \frac{I_{\overline{xx}} + I_{\overline{yy}}}{2} + \Delta$$
$$I_{22} = \frac{I_{\overline{xx}} + I_{\overline{yy}}}{2} - \Delta$$

where:

$$\Delta = \sqrt{\left(\frac{I_{\overline{xx}} - I_{\overline{yy}}}{2}\right)^2 + {I_{\overline{xy}}}^2}$$

The angle between the $\bar{x}$ axis and the axis belonging to the largest principal moment of inertia can be computed as follows:

$$\phi = \tan^{-1} \frac{I_{\overline{xx}} - I_{11}}{I_{\overline{xy}}}$$

The prinicpal section moduli require the calculation of the perpendicular distance from the principal axes to the extreme fibres. All the nodes in the mesh are considered with vector algebra used to compute the perpendicular distances and the minimum and maximum distances identified. The perpendicular distance from a point P to a line parallel to $\overrightarrow{u}$ that passes through Q is given by:

$$d = | \overrightarrow{PQ} \times \overrightarrow{u} |$$

The location of the point is checked to see whether it is above or below the principal axis. Again vector algebra is used to check this condition. The condition in the below equation will result in the point being above the $\overrightarrow{u}$ axis.

$$\overrightarrow{QP} \times \overrightarrow{u} < 0$$

Using the above equations, the principal section moduli can be computed similar to that in the calculation of the *Elastic Section Moduli* and *Plastic Section Moduli*.

10.4.9 Torsion Constant

The Saint-Venant torsion constant (J) can be obtained by solving the below partial differential equation for the warping function, ω:

$$\nabla^2 \omega = 0$$

subject to the boundary condition described below:

$$\frac{\partial \omega}{\partial x} n_x + \frac{\partial \omega}{\partial y} n_y = y n_x - x n_y$$

Pilkey [2] shows that by using the finite element method, this problem can be reduced to a set of linear equations of the form:

$$\mathbf{K}\boldsymbol{\omega} = \mathbf{F}$$

where $\mathbf{K}$ and $\mathbf{F}$ are assembled through summation at element level. The element equations for the e^{th} element are:

$$\mathbf{k}^e \boldsymbol{\omega}^e = \mathbf{f}^e$$

with the stiffness matrix defined as:

$$\mathbf{k}^e = \int_\Omega \mathbf{B}^{\mathrm{T}} \mathbf{B} J_e \, d\eta \, d\xi \, d\zeta$$

and the load vector defined as:

$$\mathbf{f}^e = \int_\Omega \mathbf{B}^{\mathrm{T}} \begin{bmatrix} \mathbf{N}\mathbf{y} \\ -\mathbf{N}\mathbf{x} \end{bmatrix} J_e \, d\eta \, d\xi \, d\zeta$$

Applying numerical integration to the stiffness matrix and load vector results in the following expressions:

$$\mathbf{k}^e = \sum_{i=1}^{3} w_i \mathbf{B}_i^{\mathrm{T}} \mathbf{B}_i J_e$$

$$\mathbf{f}^e = \sum_{i=1}^{6} w_i \mathbf{B}_i^{\mathrm{T}} \begin{bmatrix} \mathbf{N}_i \mathbf{y}_e \\ -\mathbf{N}_i \mathbf{x}_e \end{bmatrix} J_e$$

Once the warping function has been evaluated, the Saint-Venant torsion constant can be calculated as follows:

$$J = I_{xx} + I_{yy} - \boldsymbol{\omega}^{\mathrm{T}} \mathbf{K} \boldsymbol{\omega}$$

10.4.10 Shear Properties

The shear behaviour of the cross-section can be described by Saint-Venant's elasticity solution for a homogenous prismatic beam subjected to transverse shear loads [2]. Through cross-section equilibrium and linear-elasticity, an expression for the shear stresses resulting from a transverse shear load can be derived. Pilkey [2] explains that this is best done through the introduction of shear functions, Ψ and Φ, which describe the distribution of shear stress within a cross-section resulting from an applied transverse load in the x and y directions respectively. These shear functions can be obtained by solving the following uncoupled partial differential equations:

$$\nabla^2 \Psi = 2(I_{\overline{xy}} y - I_{\overline{xx}} x)$$

$$\nabla^2 \Phi = 2(I_{\overline{xy}} x - I_{\overline{yy}} y)$$

subject to the respective boundary conditions:

$$\frac{\partial \Psi}{\partial n} = \mathbf{n} \cdot \mathbf{d}$$
$$\frac{\partial \Phi}{\partial n} = \mathbf{n} \cdot \mathbf{h}$$

where $\mathbf{n}$ is the normal unit vector at the boundary and $\mathbf{d}$ and $\mathbf{h}$ are defined as follows:

$$\mathbf{d} = \nu \left(I_{\overline{xx}} \frac{x^2 - y^2}{2} - I_{\overline{xy}} xy \right) \mathbf{i} + \nu \left(I_{\overline{xx}} xy + I_{\overline{xy}} \frac{x^2 - y^2}{2} \right) \mathbf{j}$$
$$\mathbf{h} = \nu \left(I_{\overline{yy}} xy - I_{\overline{xy}} \frac{x^2 - y^2}{2} \right) \mathbf{i} - \nu \left(I_{\overline{xy}} xy + I_{\overline{yy}} \frac{x^2 - y^2}{2} \right) \mathbf{j}$$

Pilkey [2] shows that the shear equations subject to the boundary conditions can be solved using the finite element method. This results in a set of linear equations at element level of the form:

$$\mathbf{k}^e \mathbf{\Psi}^e = \mathbf{f}_x^e$$
$$\mathbf{k}^e \mathbf{\Phi}^e = \mathbf{f}_y^e$$

The local stiffness matrix, $\mathbf{k}^e$, is identical to the matrix used to determine the torsion constant:

$$\mathbf{k}^e = \int_\Omega \mathbf{B}^{\mathrm{T}} \mathbf{B} J_e \, d\eta \, d\xi \, d\zeta$$

The load vectors are defined as:

$$\mathbf{f}_x^e = \int_\Omega \left[\frac{\nu}{2} \mathbf{B}^{\mathrm{T}} \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} + 2(1+\nu) \mathbf{N}^{\mathrm{T}} (I_{\overline{xx}} \mathbf{Nx} - I_{\overline{xy}} \mathbf{Ny}) \right] J_e \, d\eta \, d\xi \, d\zeta$$
$$\mathbf{f}_y^e = \int_\Omega \left[\frac{\nu}{2} \mathbf{B}^{\mathrm{T}} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + 2(1+\nu) \mathbf{N}^{\mathrm{T}} (I_{\overline{yy}} \mathbf{Ny} - I_{\overline{xy}} \mathbf{Nx}) \right] J_e \, d\eta \, d\xi \, d\zeta$$

where:

$$d_1 = I_{\overline{xx}} r - I_{\overline{xy}} q$$
$$d_2 = I_{\overline{xy}} r + I_{\overline{xx}} q$$
$$h_1 = -I_{\overline{xy}} r + I_{\overline{yy}} q$$
$$h_2 = -I_{\overline{yy}} r - I_{\overline{xy}} q$$
$$r = (\mathbf{Nx})^2 - (\mathbf{Ny})^2$$
$$q = 2\mathbf{NxNy}$$

Applying numerical integration to the stiffness matrix and load vector results in the following expressions:

$$\mathbf{k}^e = \sum_{i=1}^{3} w_i \mathbf{B}_i^{\mathrm{T}} \mathbf{B}_i J_e$$
$$\mathbf{f}_x^e = \sum_{i=1}^{6} w_i \left[\frac{\nu}{2} \mathbf{B}_i^{\mathrm{T}} \begin{bmatrix} d_{1,i} \\ d_{2,i} \end{bmatrix} + 2(1+\nu) \mathbf{N}_i^{\mathrm{T}} (I_{\overline{xx}} \mathbf{N}_i \mathbf{x}_e - I_{\overline{xy}} \mathbf{N}_i \mathbf{y}_e) \right] J_e$$
$$\mathbf{f}_y^e = \sum_{i=1}^{6} w_i \left[\frac{\nu}{2} \mathbf{B}_i^{\mathrm{T}} \begin{bmatrix} h_{1,i} \\ h_{2,i} \end{bmatrix} + 2(1+\nu) \mathbf{N}_i^{\mathrm{T}} (I_{\overline{yy}} \mathbf{N}_i \mathbf{y}_e - I_{\overline{xy}} \mathbf{N}_i \mathbf{x}_e) \right] J_e$$

Shear Centre

The shear centre can be computed consistently based on elasticity, or through Trefftz's definition, which is based on thin-wall assumptions [2].

Elasticity: Pilkey [2] demonstrates that the coordinates of the shear centre are given by the following expressions:

$$x_s = \frac{1}{\Delta_s}\left[\frac{\nu}{2}\int_\Omega (I_{\overline{yy}}x + I_{\overline{xy}}y)\left(x^2+y^2\right)\,d\Omega - \int_\Omega \mathbf{g}\cdot\boldsymbol{\nabla}\boldsymbol{\Phi}\,d\Omega\right]$$
$$y_s = \frac{1}{\Delta_s}\left[\frac{\nu}{2}\int_\Omega (I_{\overline{xx}}y + I_{\overline{xy}}x)\left(x^2+y^2\right)\,d\Omega + \int_\Omega \mathbf{g}\cdot\boldsymbol{\nabla}\boldsymbol{\Psi}\,d\Omega\right]$$

where:

$$\Delta_s = 2(1+\nu)(I_{\overline{xx}}I_{\overline{yy}} - {I_{\overline{xy}}}^2)$$
$$\mathbf{g} = y\mathbf{i} - x\mathbf{j}$$

The first integral in shear centre equations can be evaluated using quadrature for each element. The second integral can be simplified once the shear functions, Ψ and Φ, have been obtained:

$$\int_\Omega \mathbf{g}\cdot\boldsymbol{\nabla}\boldsymbol{\Phi}\,d\Omega = \mathbf{F}^{\mathrm{T}}\boldsymbol{\Phi}$$
$$\int_\Omega \mathbf{g}\cdot\boldsymbol{\nabla}\boldsymbol{\Psi}\,d\Omega = \mathbf{F}^{\mathrm{T}}\boldsymbol{\Psi}$$

where $\mathbf{F}$ is the global load vector determined for the torsion problem in *Torsion Constant*. The resulting expression for the shear centre therefore becomes:

$$x_s = \frac{1}{\Delta_s}\left[\left(\frac{\nu}{2}\sum_{i=1}^{6} w_i(I_{\overline{yy}}\mathbf{N}_i\mathbf{x}_e + I_{\overline{xy}}\mathbf{N}_i\mathbf{y}_e)\Big((\mathbf{N}_i\mathbf{x}_e)^2 + (\mathbf{N}_i\mathbf{y}_e)^2\Big)J_e\right) - \mathbf{F}^{\mathrm{T}}\boldsymbol{\Phi}\right]$$
$$y_s = \frac{1}{\Delta_s}\left[\left(\frac{\nu}{2}\sum_{i=1}^{6} w_i(I_{\overline{xx}}\mathbf{N}_i\mathbf{y}_e + I_{\overline{xy}}\mathbf{N}_i\mathbf{x}_e)\Big((\mathbf{N}_i\mathbf{x}_e)^2 + (\mathbf{N}_i\mathbf{y}_e)^2\Big)J_e\right) + \mathbf{F}^{\mathrm{T}}\boldsymbol{\Psi}\right]$$

Trefftz's Definition: Using thin walled assumptions, the shear centre coordinates according to Trefftz's definition are given by:

$$x_s = \frac{I_{\overline{xy}}I_{x\omega} - I_{\overline{yy}}I_{y\omega}}{I_{\overline{xx}}I_{\overline{yy}} - {I_{\overline{xy}}}^2}$$
$$y_s = \frac{I_{\overline{xx}}I_{x\omega} - I_{\overline{xy}}I_{y\omega}}{I_{\overline{xx}}I_{\overline{yy}} - {I_{\overline{xy}}}^2}$$

where the sectorial products of area are defined as:

$$I_{x\omega} = \int_\Omega x\omega(x,y)\,d\Omega$$
$$I_{y\omega} = \int_\Omega y\omega(x,y)\,d\Omega$$

The finite element implementation of the above integrals are shown below:

$$I_{x\omega} = \sum_e \sum_{i=1}^{6} w_i\mathbf{N}_i\mathbf{x}_e\mathbf{N}_i\boldsymbol{\omega}_e J_e$$
$$I_{y\omega} = \sum_e \sum_{i=1}^{6} w_i\mathbf{N}_i\mathbf{y}_e\mathbf{N}_i\boldsymbol{\omega}_e J_e$$

Shear Deformation Coefficients

The shear deformation coefficients are used to calculate the shear area of the section as a result of transverse loading. The shear area is defined as $A_s = k_s A$. Pilkey [2] describes the finite element formulation used to determine the shear deformation coefficients:

$$\kappa_x = \sum_e \int_\Omega \left(\boldsymbol{\Psi}^{e\mathrm{T}}\mathbf{B}^{\mathrm{T}} - \mathbf{d}^{\mathrm{T}}\right)(\mathbf{B}\boldsymbol{\Psi}^e - \mathbf{d})\, J_e\, d\Omega$$

$$\kappa_y = \sum_e \int_\Omega \left(\boldsymbol{\Phi}^{e\mathrm{T}}\mathbf{B}^{\mathrm{T}} - \mathbf{h}^{\mathrm{T}}\right)(\mathbf{B}\boldsymbol{\Phi}^e - \mathbf{h})\, J_e\, d\Omega$$

$$\kappa_{xy} = \sum_e \int_\Omega \left(\boldsymbol{\Psi}^{e\mathrm{T}}\mathbf{B}^{\mathrm{T}} - \mathbf{d}^{\mathrm{T}}\right)(\mathbf{B}\boldsymbol{\Phi}^e - \mathbf{h})\, J_e\, d\Omega$$

where the shear areas are related to κ_x and κ_y by:

$$k_{s,x}A = \frac{{\Delta_s}^2}{\kappa_x}$$

$$k_{s,y}A = \frac{{\Delta_s}^2}{\kappa_y}$$

$$k_{s,xy}A = \frac{{\Delta_s}^2}{\kappa_{xy}}$$

The finite element formulation of the shear deformation coefficients is described below:

$$\kappa_x = \sum_e \sum_{i=1}^{6} w_i \left(\boldsymbol{\Psi}^{e\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} - \frac{\nu}{2}\begin{bmatrix} d_{1,i} \\ d_{2,i} \end{bmatrix}^{\mathrm{T}}\right)\left(\mathbf{B}_i\boldsymbol{\Psi}^e - \frac{\nu}{2}\begin{bmatrix} d_{1,i} \\ d_{2,i} \end{bmatrix}\right) J_e$$

$$\kappa_y = \sum_e \sum_{i=1}^{6} w_i \left(\boldsymbol{\Phi}^{e\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} - \frac{\nu}{2}\begin{bmatrix} h_{1,i} \\ h_{2,i} \end{bmatrix}^{\mathrm{T}}\right)\left(\mathbf{B}_i\boldsymbol{\Phi}^e - \frac{\nu}{2}\begin{bmatrix} h_{1,i} \\ h_{2,i} \end{bmatrix}\right) J_e$$

$$\kappa_{xy} = \sum_e \sum_{i=1}^{6} w_i \left(\boldsymbol{\Psi}^{e\mathrm{T}}\mathbf{B}_i^{\mathrm{T}} - \frac{\nu}{2}\begin{bmatrix} d_{1,i} \\ d_{2,i} \end{bmatrix}^{\mathrm{T}}\right)\left(\mathbf{B}_i\boldsymbol{\Phi}^e - \frac{\nu}{2}\begin{bmatrix} h_{1,i} \\ h_{2,i} \end{bmatrix}\right) J_e$$

Warping Constant

The warping constant, Γ, can be calculated from the warping function (ω) and the coordinates of the shear centre [2]:

$$\Gamma = I_\omega - \frac{{Q_\omega}^2}{A} - y_s I_{x\omega} + x_s I_{y\omega}$$

where the warping moments are calculated as follows:

$$Q_\omega = \int_\Omega \omega\, d\Omega = \sum_e \sum_{i=1}^{3} w_i \mathbf{N}_i \boldsymbol{\omega}_e J_e$$

$$I_\omega = \int_\Omega \omega^2\, d\Omega = \sum_e \sum_{i=1}^{6} w_i (\mathbf{N}_i \boldsymbol{\omega}_e)^2 J_e$$

10.4.11 Monosymmetry Constants

The monosymmetry constants are used to evaluate buckling in sections with unequal flanges. The constants are calculated in accordance with the formula provided is AS4100-1998 [4]:

$$\beta_x = \frac{1}{I_{xx}} \int_\Omega x^2 y + y^3 d\Omega - 2y_s$$
$$\beta_y = \frac{1}{I_{yy}} \int_\Omega xy^2 + x^3 d\Omega - 2x_s$$

The finite element formulation of the above integrals is described below:

$$\int_\Omega x^2 y + y^3 d\Omega = \sum_e \sum_{i=1}^{6} w_i \left[(\mathbf{N}_i \mathbf{x}_e)^2 \mathbf{N}_i \mathbf{y}_e + (\mathbf{N}_i \mathbf{y}_e)^3 \right] J_e$$
$$\int_\Omega xy^2 + x^3 d\Omega = \sum_e \sum_{i=1}^{6} w_i \left[\mathbf{N}_i \mathbf{x}_e (\mathbf{N}_i \mathbf{y}_e)^2 + (\mathbf{N}_i \mathbf{x}_e)^3 \right] J_e$$

10.5 Cross-Section Stresses

Cross-section stresses resulting from an axial force, bending moments, a torsion moment and shear forces, can be evaluated at the integration points within each element. *Extrapolation to Nodes* describes the process of extrapolating the stresses to the element nodes and the combination of the results with the adjacent elements through nodal averaging.

10.5.1 Axial Stresses

The normal stress resulting from an axial force N_{zz} at any point i is given by:

$$\sigma_{zz} = \frac{N_{zz}}{A}$$

10.5.2 Bending Stresses

Global Axis Bending

The normal stress resulting from a bending moments M_{xx} and M_{yy} at any point i is given by [2]:

$$\sigma_{zz} = -\frac{I_{\overline{xy}} M_{xx} + I_{\overline{xx}} M_{yy}}{I_{\overline{xx}} I_{\overline{yy}} - {I_{\overline{xy}}}^2} \overline{x}_i + \frac{I_{\overline{yy}} M_{xx} + I_{\overline{xy}} M_{yy}}{I_{\overline{xx}} I_{\overline{yy}} - {I_{\overline{xy}}}^2} \overline{y}_i$$

Principal Axis Bending

Similarly, the normal stress resulting from a bending moments M_{11} and M_{22} at any point i is given by:

$$\sigma_{zz} = -\frac{M_{22}}{I_{\overline{22}}} \overline{x}_{1,i} + \frac{M_{11}}{I_{\overline{11}}} \overline{y}_{2,i}$$

10.5.3 Torsion Stresses

The shear stresses resulting from a torsion moment M_{zz} at any point i within an element e are given by [2]:

$$\boldsymbol{\tau}^e = \begin{bmatrix} \tau_{zx} \\ \tau_{zy} \end{bmatrix}^e = \frac{M_{zz}}{J} \left(\mathbf{B}_i \boldsymbol{\omega}^e - \begin{bmatrix} \mathbf{N}_i \mathbf{y}_e \\ -\mathbf{N}_i \mathbf{x}_e \end{bmatrix} \right)$$

10.5.4 Shear Stresses

The shear stresses resulting from transverse shear forces V_{xx} and V_{yy} at any point i within an element e are given by [2]:

$$\begin{bmatrix} \tau_{zx} \\ \tau_{zy} \end{bmatrix}^e = \frac{V_{xx}}{\Delta_s} \left(\mathbf{B}_i \boldsymbol{\Psi}^e - \frac{\nu}{2} \begin{bmatrix} d_{1,i} \\ d_{2,i} \end{bmatrix} \right) + \frac{V_{yy}}{\Delta_s} \left(\mathbf{B}_i \boldsymbol{\Phi}^e - \frac{\nu}{2} \begin{bmatrix} h_{1,i} \\ h_{2,i} \end{bmatrix} \right)$$

10.5.5 von Mises Stresses

The von Mises stress can be determined from the net axial and shear stress as follows [2]:

$$\sigma_{vM} = \sqrt{{\sigma_{zz}}^2 + 3({\tau_{zx}}^2 + {\tau_{zy}}^2)}$$

10.5.6 Principal Stresses

For a cross section subjected to axial force, shear in the x and y axes which are perpendicular to the centroidal (z) axis, and moments about all three axes, there are no axial stresses in the x or y axes, and so the stress tensor is given by:

$$\boldsymbol{\sigma} = \begin{bmatrix} 0 & 0 & \tau_{zx} \\ 0 & 0 & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix}$$

and of course the complementary shear stresses are equal, $\tau_{zx} = \tau_{xz}$, $\tau_{zy} = \tau_{yz}$.

By definition, the principal stresses are those for which the stress tensor becomes a diagonal matrix through a coordinate transformation. Since this is the basic eigenvalue problem, the principal stresses are then given by:

$$\det(\boldsymbol{\sigma} - \lambda \mathbf{I}) = 0$$

Of which the characteristic polynomial can then be written:

$$\lambda^3 - I_1 \lambda^2 + I_2 \lambda - I_3 = 0$$

where the stress invariants I are then given by [5]:

$$\begin{aligned} I_1 &= \text{tr}(\boldsymbol{\sigma}) = \sigma_{zz} \\ I_2 &= \frac{1}{2} \left[\text{tr}(\boldsymbol{\sigma})^2 - \text{tr}(\boldsymbol{\sigma}^2) \right] = -\tau_{zx}^2 - \tau_{zy}^2 \\ I_3 &= \det(\boldsymbol{\sigma}) = 0 \end{aligned}$$

and thus, the cubic polynomial reduces to a quadratic, the two roots of which are then the first and third principal stresses (with $\sigma_2 = 0$):

$$\sigma_{1,3} = \frac{\sigma_{zz}}{2} \pm \sqrt{\left(\frac{\sigma_{zz}}{2} \right)^2 + \tau_{zx}^2 + \tau_{zy}^2}$$

10.6 Composite Cross-Sections

Pilkey [2] explains that composite cross-sections can be analysed using a modulus-weighted approach in which the differential area element is multiplied by the elastic modulus for the element, E_e:

$$d\widetilde{A} = E_e \, dA$$

The expression for section properties after the application of numerical integration then becomes:

$$\int f(x, y) \, d\widetilde{A} = \sum_{i}^{n} w_i f(\eta_i, \xi_i, \zeta_i) \, J_i E_e$$

Pilkey [2] also notes that an assumption of the elastic cross-sectional analysis of warping and shear is that:

$$\sigma_x = \sigma_y = \tau_{xy} = 0$$

However, for composite sections with variable Poisson's ratios, this assumption does not hold. Pilkey [2] does mention that engineering materials often have very similar Poisson's ratios and therefore the difference can be negligible.

Note: If the Poisson's ratio of two materials used in a composite analysis are vastly different, the assumptions used in *sectionproperties* may not hold, see Chapter 5 & 6 of [2].

For the warping and shear analysis of composite cross-sections, *sectionproperties* defines an area based on an effective Poisson's ratio that is used to calculate the relevant properties described above:

$$\nu_{eff} = \frac{(E.A)_g}{2(G.A)_g} - 1$$

10.7 References

1. C. A. Felippa, Introduction to Finite Element Methods, Department of Aerospace Engineering Sciences and Center for Aerospace Structures University of Colorado, Boulder, Colorado, 2004.
2. W. D. Pilkey, Analysis and Design of Elastic Beams: Computational Methods, John Wiley & Sons, Inc., New York, 2002.
3. M. G. Larson, F. Bengzon, The Finite Element Method: Theory, Implementation, and Applications, Vol. 10, Springer, Berlin, Heidelberg, 2013. doi:10.1007/978-3-642-33287-6.
4. AS 4100 - 1998: Steel Structures. (1998, June). Standards Australia.
5. Oñate, E. (2009), Structural Analysis with the Finite Element Method. Linear Statics. Volume 1: The Basis and Solids, Springer Netherlands

CHAPTER

ELEVEN

TESTING AND RESULTS VALIDATION

sectionproperties has a (slowly) growing suite of tests. The testing suite serves to verify functionality and find exceptions from edge cases, but also validate the results against known cases. Since this library performs engineering calculations, it should have some manner of proving its accuracy. Each analyst who uses it is responsible for their own projects and calculations, but having a high level of confidence that the software can produce *correct* results, *given the correct inputs*, is a boon to all users. Some test results and explanations from the latter category will be outlined on this page, since the former really serves no use to the end user.

11.1 Textbook Examples

An obvious starting location is replicating examples from academic texts. "Aircraft Structures" by David J. Peery is a highly regarded text in the field of aerospace structures[1].

11.1.1 Peery - Symmetric Sections

The simplest example of a realistic section problem is the symmetric I-Beam, with a free-fixed boundary condition and a transverse tip load. The free-body-diagram and shear and moment diagrams are shown in the problem statement, referenced below. This problem is Example 1 in Section 6.2.

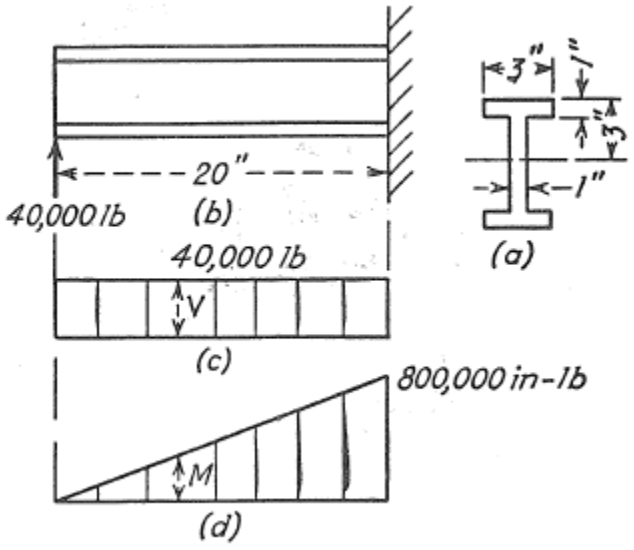

FIG. 6.5.

[1] D. J. Peery, *Aircraft Structures.* New York: Dover Publications, 2011. ISBN-10: 0-486-48580-3

Check #1:

In *sectionproperties*, there are multiple ways to set this problem up. We could different shapely geometries and merge together, or a set of custom points, or a built-in constructor. For the sake of simplicity, this simpler I-section is identical to the Nastran I-section definition, so it makes sense to utilize the built-in constructor from `pre.library.nastran_sections.nastran_i`.

Using an arbitrarily coarse mesh, the properties can then be directly calculated from the class method `Section.calculate_geomtric_properties()`.

Peery lists the second moment of area about the primary bending axis as a value of $43.3[in^4]$. For the automated tests in this library, we check against this hardcoded value, with a tolerance of ± 0.1

Check #2:

As a final check against this example, we can calculate the maximum bending stress on the I-beam. From simple statics, the maximum moment from the FBD will be $800,000[in-lbs]$ at the fixed end. Applying this moment to our section from before will allow computation of stress over the FEM.

Peery quotes the peak value at $55.5[ksi]$, which is rounded to the nearest decimal place. From the equatio listed in the text, the theoretical value is actually $55,427.3[psi]$.

$$f = \frac{My}{I} = 55,427.3 = 55,400$$

Again, the automated test against this checks the hardcoded value with a tolerance of $\pm 0.1 rounded value$.

For full details and the most updated code of this exampe, see the *examples page* in the documentation gallery. For the exact test code execution, check the source.

11.1.2 Peery - Unsymmetric Sections

For a more complex example, we can turn to Example 1 in Section 7.2 of Peery. Here, we have a still-simplified Z-section, but bending about two axes. Note axes definitions in the problem statement. Beam axial direction in *sectionproperties* is always referenced as the z-axis, and loads must be applied in this coordinate system.

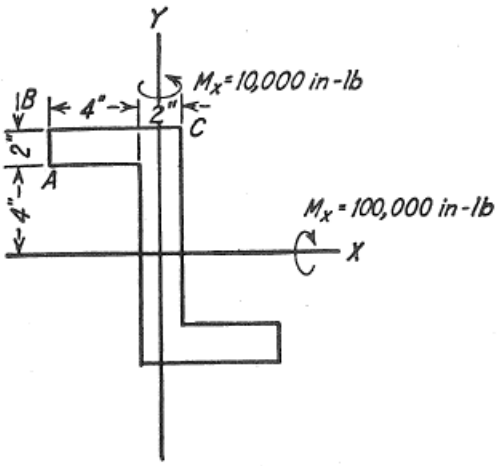

FIG. 7.6.

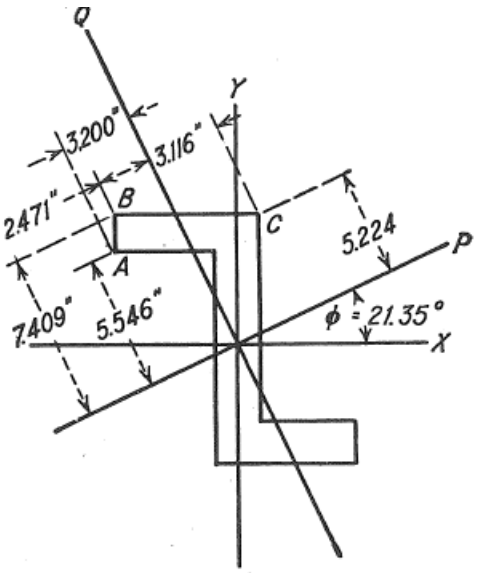

FIG. 7.7.

The construction of this geometry takes a similar approach to Ex 6.2.1, and utilizes a built-in factory: `pre.library.nastran_sections.nastran_zed`. The only difference you may notice in the test code is usage of a custom class for ease of initialization. This is not necessary.

Using an arbitrarily coarse mesh, the properties can then be directly calculated from the class method `Section.calculate_geomtric_properties()`. Each property listed directly by Peery is taken as a hardcoded value and checked against, within the testing suite.

Property	Peery Value
I_x	693.3 [in^4]
I_y	173.3 [in^4]
I_xy	-240 [in^4]
I_p	787.1 [in^4]
I_q	79.5 [in^4]
theta	21.35 [deg]

For stress results, the theoretical values follow the biaxial bending equation. These values are checked against automatically in the testing suite. Note that again Peery rounds the values quoted directly, for simplicity. The testing suite also verifies that the theoretical value as per the equation matches the theoretical value quoted in the text, which also matches the computed value from the *sectionproperties* FEM.

$$f_b = \frac{M_x I_{xy} - M_y I_x}{I_x I_y - I_{xy}^2} x + \frac{M_y I_{xy} - M_x I_y}{I_x I_y - I_{xy}^2} y$$

Point	x	y	-494x	-315y	f_b, [psi]
A	-5	4	2470	-1260	1210.0 = 1210
B	-5	6	2470	-1890	580.0 = 580
C	1	6	-494	-1890	-2384.0 = -2380

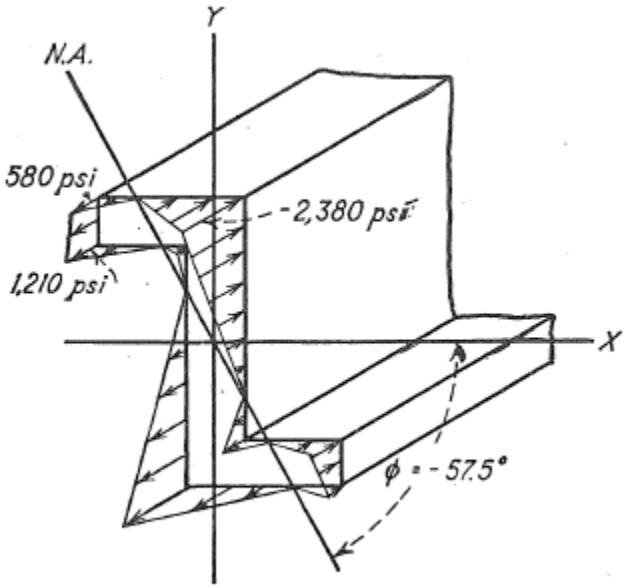

FIG. 7.8.

For full details and the most updated code of this example, see the *examples page* in the documentation gallery. For the exact test code execution, check the source.

Here's a quick example that harnesses some of the power of *sectionproperties* and shows its simplicity:

```
import sectionproperties.pre.library.steel_sections as steel_sections
from sectionproperties.analysis.section import Section

# create geometry of the cross-section
geometry = steel_sections.i_section(d=203, b=133, t_f=7.8, t_w=5.8, r=8.9, n_r=8)

# generate a finite element mesh
geometry.create_mesh(mesh_sizes=[10])

# create a Section object for analysis
section = Section(geometry)

# calculate various cross-section properties
section.calculate_geometric_properties()
section.calculate_warping_properties()

# print some of the calculated section properties
print(section.get_area())  # cross-section area
>>>3231.80
print(section.get_ic())  # second moments of area about the centroidal axis
>>>(23544664.29, 3063383.07, 0.00)
print(section.get_j())  # torsion constant
>>>62954.43
print(section.get_As())  # shear areas in the x & y directions
>>>(1842.24, 1120.19)
```

CHAPTER

TWELVE

SUPPORT

Raise an issue on the GitHub issue tracker or contact me at robbie.vanleeuwen@gmail.com. If you have a request for a feature to be added to the *sectionproperties* package, please don't hesitate to get in touch

CHAPTER

THIRTEEN

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